ADV MATH SCI JOURNAL
Advances in Mathematics: Scientific Journal **9** (2020), no.2, 561–567
ISSN: 1857-8365 (printed); 1857-8438 (electronic)
Special Issue on ICCSPAM-2020; https://doi.org/10.37418/amsj.9.2.4

# SEPERATION AXIOMS IN SUPRA TOPOLOGICAL SPACES

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ABSTRACT. This aim of this paper is to introduce and investigate the properties of supra $g^{\#}\alpha$ seperation axioms in supra topological spaces and obtain some relationship between the existing sets.

## 1. INTRODUCTION

In 1983, A. S. Mashhour et al. [3] introduced the supra topological spaces. In 2010, O. R. Sayed et al. [5] introduced and studied a class of sets and maps between topological spaces called supra b-open sets and supra b-continuous functions respectively. So the supra open sets are defined where the supra topological spaces are presented. We have known that every topological space is a supra topological space, so as every open set is a supra open set, but the converse is not always true. Consideration the intersection condition is not necessary to have a supra topological space. Njastad at [4] in 1965 introduced $\alpha$-open sets. In 2008, R. Devi, S. Sampathkumar and M. Caldas [1] introduced the supra $\alpha$-open sets and supra-$T_{i-1}$ spaces where $i = 1, 2, 3$. In this paper we study the relationships between supra separation axioms and supra $g^{\#}\alpha$ and study some of characterizations of them. Now we study the notions of supra T$_i$ spces i = 0,1,3. Also we introduce and study the concepts of supra D$_i$ spaces for i = 0,1,2 and investigate several properties for these concepts.

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2010 *Mathematics Subject Classification.* 54B05.
*Key words and phrases.* supra-$T_i$ spaces, supra-$\alpha T_i$ spaces and supra-$g^{\#}\alpha T_i$ spaces .

## 2. Preliminaries

**Definition 2.1.** *[3] A subfamily of $\mu$ of X is said to be a supra topology on X, if*

1. *X, $\phi \in \mu$*
2. *if $A_i \in \mu$ for all i $\in$ J then $\cup A_i \in \mu$.*

*The pair (X, $\mu$) is called supra topological space. The elements of $\mu$ are called supra open sets in (X, $\mu$) and complement of a supra open set is called a supra closed set.*

**Definition 2.2.** *[3]*

1. *The supra closure of a set A is denoted by $cl^\mu(A)$ and is defined as $cl^\mu(A) = \cap\{B : B$ is a supra closed set and $A \subseteq B\}$.*
2. *The supra interior of a set A is denoted by $int^\mu(A)$ and defined as $int^\mu(A) = \cup\{B : B$ is a supra open set and $A \supseteq B\}$.*

**Definition 2.3.** *[3] Let (X, $\tau$) be a topological spaces and $\mu$ be a supra topology on X. We call $\mu$ a supra topology associated with $\tau$ if $\tau \subset \mu$.*

**Definition 2.4.** *[1] Let (X,$\mu$) be a supra topological space. A subset A of X is called supra $\alpha$-open set if $A \subseteq int^\mu(cl^\mu(int^\mu (A))$. The complement of supra $\alpha$ - open set is supra $\alpha$-closed set.*

**Definition 2.5.** *[2] Let $(X, \mu)$ be a supra topological space. A subset $A$ of $X$ is called supra $g^{\#}\alpha$-closed set if $\alpha cl^\mu(A) \subseteq U$ ,whenever A$\subseteq$ U and U is supra g-open set of X. The complement of supra $g^{\#}\alpha$ - closed set is called supra $g^{\#}\alpha$-open set.*

**Definition 2.6.** *[3] Let $(X, \mu)$ be a supra topological space, then:*

1. *$X$ is $S$-$T_0$ if for every two distinct points $x$ and $y$ in $X$ there exists a supra open set $U$ that contains only one of the points $x$ and $y$.*
2. *$X$ is $S$-$T_1$ if for every two distinct points $x$ and $y$ in $X$ there exists two supra open sets $U$ and $V$ such that $x \in U$, $y \notin U$ and $y \notin V$, $x \notin V$.*
3. *$X$ is $S$-$T_2$ if for every two distinct points $x$ and $y$ in $X$ there exists two disjoint supra open sets $U$ and $V$ such that $x \in U$ and $y \in V$.*

**Remark 2.1.** *[3] Every supra-$T_i$ space is supra $T_{i-1}$ space.*

## 3. Separation axioms on supra $g^{\#}\alpha$-open sets

**Definition 3.1.** *If $(X, \tau)$ is a supra topological space, for all $x, y \in X, x \neq y$, and there exist a supra $g^{\#}\alpha$-open set $G$ such that $x \in G$ and $y \notin G$. Then $(X, \tau)$ is called a supra $g^{\#}\alpha$-$T_0$-space.*

**Definition 3.2.** *If $(X, \tau)$ is a supra topological space, $A \subseteq X$, $A \neq \emptyset$, $\tau_A$ is the class of all intersection of A with each element in $\tau$, then $(A, \tau_A)$ is called a supra topological subspace of$(X, \tau)$.*

**Theorem 3.1.** *Every supra $T_0$-space is a supra $g^{\#}\alpha$ $T_0$ space.*

*Proof.* Let $(X, \tau)$ be a supra $T_0$-space and let $x, y \in X$, $x \neq y$ then there exist a supra open set $G \subseteq X$ such that $x \in G$ and $y \notin G$. Since every supra open set is a supra $g^{\#}\alpha$-open set. Then $G \subseteq X$ is a supra $g^{\#}\alpha$ -open set such that $x \in G$ and $y \notin G$. Hence $(X, \tau)$ is a supra $g^{\#}\alpha$ $T_0$-space. □

**Example 1.** *Let $X = \{a, b, c\}$, $\mu = \{X, \phi, \{b\}, \{a, b\}\}$ and $g^{\#}\alpha$-open set of $X, \mu) = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}\}$. Let $x = \{a\}$ and $y = \{b\}$ then $G = \{a, c\}$ is $g^{\#}\alpha$ $T_0$ space but not supra $T_0$-space.*

**Theorem 3.2.** *If $(X, \tau)$, $(X^*, \tau^*)$ are two supra topological spaces, $(X, \tau)$ is a supra $T_0$-space and f is a supra open function and bijective then $(X^*, \tau^*)$ is a supra $T_0$-space.*

*Proof.* Suppose that $(X, \tau)$ is a supra $T_0$-space. Now we have to prove that $(X^*, \tau^*)$ is a supra $T_0$-space. Let $x^*, y^* \in X^*$, $x^* \neq y^*$, since f is a bijective function then there exist $x, y \in X$ such that : $x^* = f(x)$, $y^* = f(y)$ and $x \neq y$. Since $(X, \tau)$ is a supra $T_0$-space then there exists $G \subseteq X$ is a supra open set such that $x \in G$ and $y \notin G$. We obtain that$f(G) \subseteq X^*$ is a supra open sets in $X^*$ because f is a supra open function. So $x^* \in f(G)$ and $y^* \notin f(G)$. Then $(X^*, \tau^*)$ is a supra $T_0$-space. □

**Theorem 3.3.** *If $(X, \tau)$ is a supra $g^{\#}\alpha$ $T_0$-space and $(Y, \tau)$ is a supra topological subspace of $(X, \tau)$ then $(Y, \tau_y)$ is a supra $g^{\#}\alpha$ $T_0$-space.*

*Proof.* Suppose that $x, y \in Y$, $x \neq y$, since $Y \subseteq X$ then $x, y \in X$. Since $(X, \tau)$ is a supra $g^{\#}\alpha$ $T_0$-space means that there exist a supra $g^{\#}\alpha$ -open set $G \subseteq X$ such that $x \in G$ and $y \notin G$. We have that $G_y = Y \cap G$, $G_y$ is a supra $g^{\#}\alpha$ -open set in Y and $x \in G_y$ but $y \notin G_y$, so we found a supra $g^{\#}\alpha$ -open set $G_y \subseteq Y$ which it contained x and not contained y. Hence $(Y, \tau_y)$ is a supra $g^{\#}\alpha$ $T_0$-space. □

**Theorem 3.4.** *If $(X,\tau)$, $(X^*,\tau^*)$ are two supra topological spaces, $(X,\tau)$ is a supra $g^{\#}\alpha$ $T_0$-space and f is a supra open function and bijective then $(X^*,\tau^*)$ is a supra $g^{\#}\alpha$ $T_0$-space.*

**Definition 3.3.** *If $(X,\tau)$ is a supra topological space, $G,H \subseteq X$ are supra $g^{\#}\alpha$ -open sets and if $x \in G$, $x \notin H$ and $y \notin G$, $y \in H$ then $(X,\tau)$ is called a supra $g^{\#}\alpha$ $T_1$-space.*

**Theorem 3.5.** *Every supra $T_1$-space is a supra $g^{\#}\alpha$ $T_1$.*

*Proof.* Let $(X,\tau)$ be a supra $T_1$-space, and let $x,y \in X$, $x \neq y$ then there exist two supra open sets $G,H \subseteq X$ such that $x \in G$ and $x \notin H$, $y \notin G$, $y \in H$. Since every supra open set is a supra $g^{\#}\alpha$ -open set. Then $G,H \subseteq X$ are two supra $g^{\#}\alpha$ -open sets such that $x \in G$ and $x \notin H$, $y \notin G$, $y \in H$. Hence $(X,\tau)$ is a supra $g^{\#}\alpha$ $T_1$-space. □

**Example 2.** *From example 1, let $x = \{a\}$ and $y = \{b\}$ then $G = \{a,c\}$ and $H = \{b,c\}$ is $g^{\#}\alpha$ $T_1$ space but not supra $T_1$-space.*

**Theorem 3.6.** *If $(X,\tau)$ is a supra $g^{\#}\alpha$ $T_1$-space and $(Y,\tau)$ is a supra topological subspace of $(X,\tau)$ then $(Y,\tau_y)$ is a supra $g^{\#}\alpha$ $T_1$-space.*

**Theorem 3.7.** *If $(X,\tau)$, $(X^*,\tau^*)$ are two supra topological spaces, $(X,\tau)$ is a supra $g^{\#}\alpha$ $T_1$-space and f is a supra open function and bijective then $(X^*,\tau^*)$ is a supra $g^{\#}\alpha$ $T_1$-space.*

**Theorem 3.8.** *If $(X,\tau)$ is a supra topological space then $(X,\tau)$ is a supra $g^{\#}\alpha T_1$-space if and only if for every $x \in X$, {x} is a supra closed set.*

*Proof.* Let $(X,\tau)$ be a supra topological space, we show that $\{x\}^c$ is a supra open set in X. Suppose that $a \in \{x\}^c$, $a \neq x$ then by (def. 3) there exist $G_a$ is a supra open set in X where $G_a$ does not contain x. Hence; $a \in G_a \in \{x\}^c$ and $\{x\}^c = \{G_a : a \in \{x\}^c$. This means $\{x\}^c$ is a union of all supra open sets and by (axiom two from the def. of supra topological space). $\{x\}^c$ is a supra open set. Then $\{x\}$ is a supra closed set.

Conversely, Suppose that $\{x\}$ is a supra closed set in X and let $a,b \in X$ where $a \neq b$ then $a \in \{b\}^c$, $b \in \{a\}^c$ and $\{b\}^c$, $\{a\}^c$ are supra open sets in X. Hence $(X,\tau)$ is a supra $g^{\#}\alpha T_1$-space. □

**Definition 3.4.** *If $(X, \tau)$ is a supra topological space, for all $x, y \in X$, $x \neq y$, then there exist $G, H \subseteq X$ are supra $g^{\#}\alpha$ -open sets such that $x \in G$, $y \in H$, $G \cap H = \emptyset$, then $(X, \tau)$ is called a supra $g^{\#}\alpha$ $T_2$-space.*

**Theorem 3.9.** *Every supra $T_2$-space is a supra $g^{\#}\alpha$ $T_2$-space.*

*Proof.* Let $(X, \tau)$ be a supra $g^{\#}\alpha$ $T_2$-space, and let $x, y \in X$, $x \neq y$ then there exist two supra open sets $G, H \subseteq X$ such that $x \in G$, $y \in H$, $G \cap H\emptyset$. Since every supra open set is a supra $g^{\#}\alpha$ -open set. Then $G, H \subseteq X$are two supra $g^{\#}\alpha$ -open sets such that $x \in G$, $y \in H$, $G \cap H\emptyset$. Hence $(X, \tau)$ is a supra $g^{\#}\alpha$ $T_2$-space. ☐

**Example 3.** *From example 1, let $x = \{a\}$ and $y = \{b\}$ then $G = \{a, c\}$ and $H = \{b\}$ is $g^{\#}\alpha$ $T_2$ space but not supra $T_2$-space.*

**Theorem 3.10.** *Every supra $g^{\#}\alpha$ $T_2$-space is a supra $g^{\#}\alpha$ $T_0$-space.*

*Proof.* Let $(X, \tau)$ be a $\alpha$ $T_0$ space and let $x, y \in X$, $x \neq y$ then there exist two supra $g^{\#}\alpha$ -open sets $G, H \subseteq X$ such that $x \in G$, $y \in H$, $G \cap H = \emptyset$. Since $G \cap H = \emptyset$, that is mean $x \in G$ and $x \notin H$, $y \notin G$, $y \in H$. Hence $(X, \tau)$ is a supra $g^{\#}\alpha$ $T_0$-space. ☐

**Theorem 3.11.** *Every supra $g^{\#}\alpha$ $T_2$-space is a supra $g^{\#}\alpha$ $T_1$-space.*

*Proof.* Let $(X, \tau)$ be a supra $g^{\#}\alpha$ $T_2$-space and let $x, y \in X$, $x \neq y$ then there exist two supra $g^{\#}\alpha$ open sets $G, H \subseteq X$ such that $x \in G$ and $x \notin H$, $y \notin G$, $y \in H$. That is mean there exists a supra $g^{\#}\alpha$ -open set $G \subseteq X$ such that $x \in G$ and $y \notin G$. Hence $(X, \tau)$ is a supra $g^{\#}\alpha$ $T_0$-space. ☐

**Remark 3.1.**

*supra $T_2$-space $\Rightarrow$ supra $\alpha T_2$-space $\Rightarrow$ supra $g^{\#}\alpha T_2$-space*
$\Downarrow$ $\Downarrow$ $\Downarrow$
*supra $T_1$-space $\Rightarrow$ supra $\alpha T_1$-space $\Rightarrow$ supra $g^{\#}\alpha T_1$-space*
$\Downarrow$ $\Downarrow$ $\Downarrow$
*supra $T_0$-space $\Rightarrow$ supra $\alpha T_0$-space $\Rightarrow$ supra $g^{\#}\alpha T_0$-space*

The converse of the above remark is need not be true in general, is shown in the Example.

**Example 4.** *From example 1, let $x = \{a\}$ and $y = \{b\}$ then $G = \{a, c\}$ and is $g^{\#}\alpha$ $T_0$ space but not supra $g^{\#}\alpha$ $T_2$ (resp. $g^{\#}\alpha$ $T_1$)-space.*

As we proved in the supra $g^{\#}\alpha$ $T_0$-space, and supra $g^{\#}\alpha$ $T_1$-space, here also the hereditary property of a supra $g^{\#}\alpha$ $T_2$-space will be proved.

**Theorem 3.12.** *If $(X, \tau)$ is a supra $g^{\#}\alpha$ $T_2$-space and $(Y, \tau)$ is a supra topological subspace of $(X, \tau)$ then $(Y, \tau_y)$ is a supra $g^{\#}\alpha$ $T_2$-space.*

*Proof.* Suppose that $x, y \in Y$, $x \neq y$, since $Y \subseteq X$ then $x, y \in X$ which means that there exist two supra $g^{\#}\alpha$ -open sets $G, H \subseteq X$ such that $x \in G$ and $y \in H$, $G \cap H = \emptyset$. Now $G_y = G \cap Y$, $H_y = H \cap Y$ are two supra $g^{\#}\alpha$ -open sets in Y such that $x \in G_y$ and $y \in H_y$. Since $G \cap H = \emptyset$, then $G_y \cap H_y = \emptyset$. So $(Y, \tau_y)$ is a supra $g^{\#}\alpha$ $T_2$-space. □

**Theorem 3.13.** *If $(X, \tau)$, $(X^*, \tau^*)$ are two supra topological spaces, $(X, \tau)$ is a supra $g^{\#}\alpha$ $T_2$-space and f is a supra open function and bijective then $(X^*, \tau^*)$ is a supra $g^{\#}\alpha$ $T_2$-space.*

**Theorem 3.14.** *A supra topological space $(X, \tau)$ is $S$-$g^{\#}\alpha$ $T_0$ if and only if for each pair of distinct points $x$, $y$ in $X$, $g^{\#}\alpha Cl^{\mu}(\{x\}) \neq g^{\#}\alpha Cl^{\mu}(\{y\})$.*

**Definition 3.5.** *A supra topological space $(X, \mu)$ is called a supra $g^{\#}\alpha$-symmetric space if for $x$ and $y$ in $X$, $x \in Cl^{\mu}(\{y\})$ implies $y \in Cl^{\mu}(\{x\})$.*

**Theorem 3.15.** *Let $(X, \mu)$ be a supra $g^{\#}\alpha$-symmetric space. Then the following are equivalent:*

(1) *$(X, \mu)$ is $Sg^{\#}\alpha$-$T_0$;*
(2) *$(X, \mu)$ is $Sg^{\#}\alpha$-$T_1$.*

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