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FUZZY IRRESOLUTE FUNCTIONS VIA FUZZY \mathscr{C} -OPEN SETS

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ABSTRACT. The fuzzy continuity of $f:X\to Y$ is defined as the pre image of a fuzzy open subset of Y is fuzzy open in X. The fuzzy irresolute function of $f:X\to Y$ is defined as the pre image of a fuzzy nearly open subset of Y is fuzzy nearly open in X. In this paper a new class of fuzzy irresoluteness is introduced using fuzzy $\mathscr C$ -open sets and their properties are investigated.

1. Introduction

The notions fuzzy irresoluteness are studied by Yalvac [1], Mukherjee and Sinha [2], Malakar [3], Caldas [4], [5], and Seenivasan [6], [7]. In 2016 [8], Xavier and Thangavelu introduced fuzzy $\mathscr C$ -open sets using arbitrary complement function $\mathscr C:[0,1]\to[0,1]$. In 2017 [9], the fuzzy $\mathscr C$ -interior operator are defined by fuzzy $\mathscr C$ -open sets. Further fuzzy nearly $\mathscr C$ -open sets are investigated using fuzzy $\mathscr C$ -interior operator and fuzzy closure operator. Later fuzzy $\mathscr C$ -continuity are defined using fuzzy nearly $\mathscr C$ -open sets. In this paper the fuzzy irresolute functions are defined using fuzzy nearly $\mathscr C$ -open sets and their properties are studied.

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2. Preliminaries

Definition 2.1. [10]

- (i) λ is fuzzy regular \mathscr{C} -open $(RO_{\mathscr{C}})$, if $int_{\mathscr{C}}(cl(\lambda)) = \lambda$;
- (ii) λ is fuzzy α - \mathscr{C} -open($\alpha O_{\mathscr{C}}$), if $\lambda \leq int_{\mathscr{C}}(cl(int_{\mathscr{C}}(\lambda)))$;
- (iii) λ is fuzzy semi \mathscr{C} -open $(SO_{\mathscr{C}})$, if $\lambda \leq cl(int_{\mathscr{C}}(\lambda))$;
- (iv) λ is fuzzy pre \mathscr{C} -open $(PO_{\mathscr{C}})$, if $\lambda \leq int_{\mathscr{C}}(cl(\lambda))$;
- (v) λ is fuzzy semi pre \mathscr{C} -open $(\beta O_{\mathscr{C}})$, if $\lambda \leq cl(int_{\mathscr{C}}(cl(\lambda)))$;
- (vi) λ is fuzzy b- \mathscr{C} -open $(BO_{\mathscr{C}})$, if $\lambda \leq cl(int_{\mathscr{C}}(\lambda)) \vee int_{\mathscr{C}}(cl(\lambda))$;
- (vii) λ is fuzzy $b^{\#}$ - \mathscr{C} -open $(b^{\#}O_{\mathscr{C}})$, if $\lambda = cl(int_{\mathscr{C}}(\lambda)) \vee int_{\mathscr{C}}(cl(\lambda))$.

The standard complement of fuzzy regular open (fuzzy α -open, fuzzy semi open, fuzzy pre open, fuzzy semi pre open, fuzzy b-open, fuzzy $b^\#$ -open) is fuzzy regular closed (fuzzy α -closed, fuzzy semi closed, fuzzy pre closed, fuzzy semi pre closed, fuzzy b-closed, fuzzy $b^\#$ -closed). The analogues result is not true for the above said fuzzy nearly $\mathscr C$ -open sets.

Definition 2.2. [10]

- (i) λ is fuzzy regular \mathscr{C} -closed, if $cl(int_{\mathscr{C}}(\lambda)) = \lambda$;
- (ii) λ is fuzzy α - \mathscr{C} -closed, if $cl(int_{\mathscr{C}}(cl(\lambda))) \leq \lambda$;
- (iii) λ is fuzzy semi \mathscr{C} -closed, if $int_{\mathscr{C}}(cl(\lambda)) < \lambda$;
- (iv) λ is fuzzy pre \mathscr{C} -closed, if $cl(int_{\mathscr{C}}(\lambda)) \leq \lambda$;
- (v) λ is fuzzy semi pre \mathscr{C} -closed, if $int_{\mathscr{C}}(cl(int_{\mathscr{C}}(\lambda))) \leq \lambda$;
- (vi) λ is fuzzy b- \mathscr{C} -closed, if $\lambda > cl(int_{\mathscr{C}}(\lambda)) \wedge int_{\mathscr{C}}(cl(\lambda))$;
- (vii) λ is fuzzy $b^{\#}$ - \mathscr{C} -closed, if $int_{\mathscr{C}}(cl(\lambda)) \wedge cl(int_{\mathscr{C}}(\lambda)) = \lambda$.

Definition 2.3. [10] A function $f:(X,\tau)\to (Y,\sigma)$ be a function and $\mathscr C$ be a complement function. Then f is said to be

- (i) fuzzy \mathscr{C} -continuous if $f^{-1}(\mu)$ is fuzzy \mathscr{C} -open for each fuzzy open subset μ of Y.
- (ii) fuzzy regular \mathscr{C} -continuous if $f^{-1}(\mu)$ is fuzzy regular \mathscr{C} -open for each fuzzy open subset μ of Y.
- (iii) fuzzy α - \mathscr{C} -continuous if $f^{-1}(\mu)$ is fuzzy α - \mathscr{C} -open for each fuzzy open subset μ of Y.
- (iv) fuzzy semi \mathscr{C} -continuous if $f^{-1}(\mu)$ is fuzzy semi \mathscr{C} -open for each fuzzy open subset μ of Y.

- (v) fuzzy pre \mathscr{C} -continuous if $f^{-1}(\mu)$ is fuzzy pre \mathscr{C} -open for each fuzzy open subset μ of Y.
- (vi) fuzzy semi pre \mathscr{C} -continuous if $f^{-1}(\mu)$ is fuzzy semi pre \mathscr{C} -open for each fuzzy open subset μ of Y.
- (vii) fuzzy b-C-continuous if $f^{-1}(\mu)$ is fuzzy b-C-open for each fuzzy open subset μ of Y.
- (viii) fuzzy $b^{\#}$ - \mathscr{C} -continuous if $f^{-1}(\mu)$ is fuzzy $b^{\#}$ - \mathscr{C} -open for each fuzzy open subset μ of Y.

Lemma 2.1. [8] A function of $f:(X,\tau)\to (Y,\sigma)$ is fuzzy continuous if and only if $f^{-1}(\lambda)$ is a fuzzy $\mathscr C$ -open subset of X for each fuzzy $\mathscr C$ -open set λ of Y, where $\mathscr C$ satisfies the monotonic and involutive conditions.

3. Fuzzy &-IRRESOLUTENESS

Definition 3.1. Let $f:(X,\tau)\to (Y,\sigma)$ be a function and $\mathscr C$ be a complement function. Then f is

- (i) fuzzy regular \mathscr{C} -irresolute if it induces a function $f^{-1}: RO_{\mathscr{C}}(Y, \sigma) \to RO_{\mathscr{C}}(X, \tau)$.
- (ii) fuzzy α - \mathscr{C} -irresolute if it induces a function $f^{-1}: \alpha O_{\mathscr{C}}(Y, \sigma) \to \alpha O_{\mathscr{C}}(X, \tau)$.
- (iii) fuzzy semi $\mathscr C$ -irresolute if it induces a function $f^{-1}:SO_{\mathscr C}(Y,\sigma)\to SO_{\mathscr C}(X,\tau)$.
- (iv) fuzzy pre \mathscr{C} -irresolute if it induces a function $f^{-1}:PO_{\mathscr{C}}(Y,\sigma)\to PO_{\mathscr{C}}(X,\tau)$.
- (v) fuzzy semi pre \mathscr{C} -irresolute if it induces a function $f^{-1}: \beta O_{\mathscr{C}}(Y,\sigma) \to \beta O_{\mathscr{C}}(X,\tau)$.
- (vi) fuzzy b- $\mathscr C$ -irresolute if it induces a function $f^{-1}:BO_{\mathscr C}(Y,\sigma)\to BO_{\mathscr C}(X,\tau)$.
- (vii) fuzzy $b^\#$ - $\mathscr C$ -irresolute if it induces a function $f^{-1}:b^\#O_\mathscr C(Y,\sigma)\to b^\#O_\mathscr C(X,\tau)$.

Theorem 3.1. Suppose $f:(X,\tau)\to (Y,\sigma)$ be a function and $\mathscr C$ be a complement function that satisfies monotonic and involutive conditions. Then f is fuzzy regular $\mathscr C$ -irresolute if and only if $f^{-1}(\lambda)$ is fuzzy regular $\mathscr C$ -closed in X, for every fuzzy regular $\mathscr C$ -closed in Y.

Proof. Suppose f is fuzzy regular- $\mathscr C$ -irresolute. Let λ be fuzzy regular- $\mathscr C$ -closed in Y. Since $\mathscr C$ satisfies monotonic and involutive conditions, $\mathscr C\lambda$ is fuzzy regular- $\mathscr C$ -open in Y. Then $f^{-1}(\mathscr C\lambda)$ is fuzzy regular $\mathscr C$ -open in $X\Rightarrow\mathscr C(f^{-1}(\lambda))$ is fuzzy regular $\mathscr C$ -closed in X.

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Conversely, assume that $f^{-1}(\lambda)$ is fuzzy regular $\mathscr C$ -closed in X. Let λ be fuzzy regular $\mathscr C$ -open in Y. Since $\mathscr C$ satisfies involutive condition, $\mathscr C(\mathscr C\lambda)$ is fuzzy regular $\mathscr C$ -open. Then $\mathscr C\lambda$ is fuzzy regular $\mathscr C$ -closed in Y. By assumption $f^{-1}(\mathscr C\lambda)$ is fuzzy regular $\mathscr C$ -closed in X. Then $\mathscr C(f^{-1}(\lambda))$ is fuzzy regular $\mathscr C$ -closed in X is fuzzy regular $\mathscr C$ -open in X. Since $\mathscr C$ satisfies involutive condition, $f^{-1}(\lambda)$ is fuzzy regular $\mathscr C$ -open in X. Thus f is fuzzy regular $\mathscr C$ -irresolute. \square

Theorem 3.2. Suppose $f:(X,\tau)\to (Y,\sigma)$ be a function and $\mathscr C$ be a complement function that satisfies monotonic and involutive conditions. Then f is fuzzy α - $\mathscr C$ -closed in X, for every fuzzy α - $\mathscr C$ -closed in Y.

Proof. Suppose f is fuzzy α - \mathscr{C} -irresolute. Let λ be fuzzy α - \mathscr{C} -closed in Y. Since \mathscr{C} satisfies monotonic and involutive conditions, $\mathscr{C}\lambda$ is fuzzy α - \mathscr{C} -open in Y. Then $f^{-1}(\mathscr{C}\lambda)$ is fuzzy α - \mathscr{C} -open in $X \Rightarrow \mathscr{C}(f^{-1}(\lambda))$ is fuzzy α - \mathscr{C} -open in $X \Rightarrow f^{-1}(\lambda)$ is fuzzy α - \mathscr{C} -closed in X.

Conversely, assume that $f^{-1}(\lambda)$ is fuzzy α - \mathscr{C} -closed in X. Let λ be fuzzy α - \mathscr{C} -open in Y. Since \mathscr{C} satisfies involutive condition, $\mathscr{C}(\mathscr{C}\lambda)$ is fuzzy α - \mathscr{C} -open. Then $\mathscr{C}\lambda$ is fuzzy α - \mathscr{C} -closed in Y. By assumption $f^{-1}(\mathscr{C}\lambda)$ is fuzzy α - \mathscr{C} -closed in X. Then $\mathscr{C}(f^{-1}(\lambda))$ is fuzzy α - \mathscr{C} -closed in X \Rightarrow $\mathscr{C}(\mathscr{C}(f^{-1}(\lambda)))$ is fuzzy α - \mathscr{C} -open in X. Since \mathscr{C} satisfies involutive condition, $f^{-1}(\lambda)$ is fuzzy α - \mathscr{C} -open in X. Thus f is fuzzy α - \mathscr{C} -irresolute. \square

Theorem 3.3. Suppose $f:(X,\tau)\to (Y,\sigma)$ be a function and $\mathscr C$ be a complement function that satisfies monotonic and involutive conditions. Then f is fuzzy semi $\mathscr C$ -irresolute if and only if $f^{-1}(\lambda)$ is fuzzy semi $\mathscr C$ -closed in X, for every fuzzy semi $\mathscr C$ -closed in Y.

Proof. Suppose f is fuzzy semi $\mathscr C$ -irresolute. Let λ be fuzzy semi $\mathscr C$ -closed in Y. Since $\mathscr C$ satisfies monotonic and involutive conditions, $\mathscr C\lambda$ is fuzzy semi $\mathscr C$ -open in Y. Then $f^{-1}(\mathscr C\lambda)$ is fuzzy semi $\mathscr C$ -open in $X\Rightarrow \mathscr C(f^{-1}(\lambda))$ is fuzzy semi $\mathscr C$ -open in $X\Rightarrow f^{-1}(\lambda)$ is fuzzy semi $\mathscr C$ -closed in X.

Conversely, assume that $f^{-1}(\lambda)$ is fuzzy semi \mathscr{C} -closed in X. Let λ be fuzzy semi \mathscr{C} -open in Y. Since \mathscr{C} satisfies involutive condition, $\mathscr{C}(\mathscr{C}\lambda)$ is fuzzy semi \mathscr{C} -open. Then $\mathscr{C}\lambda$ is fuzzy semi \mathscr{C} -closed in Y. By assumption $f^{-1}(\mathscr{C}\lambda)$ is fuzzy semi \mathscr{C} -closed in X. Then $\mathscr{C}(f^{-1}(\lambda))$ is fuzzy semi \mathscr{C} -closed in X

 $\Rightarrow \mathscr{C}(\mathscr{C}(f^{-1}(\lambda)))$ is fuzzy semi \mathscr{C} -open in X. Since \mathscr{C} satisfies involutive condition, $f^{-1}(\lambda)$ is fuzzy semi \mathscr{C} -open in X. Thus f is fuzzy semi \mathscr{C} -irresolute.

Theorem 3.4. Suppose $f:(X,\tau)\to (Y,\sigma)$ be a function and $\mathscr C$ be a complement function that satisfies monotonic and involutive conditions. Then f is fuzzy pre $\mathscr C$ -irresolute if and only if $f^{-1}(\lambda)$ is fuzzy pre $\mathscr C$ -closed in X, for every fuzzy pre $\mathscr C$ -closed in Y.

Proof. Suppose f is fuzzy pre $\mathscr C$ -irresolute. Let λ be fuzzy pre $\mathscr C$ -closed in Y. Since $\mathscr C$ satisfies monotonic and involutive conditions, $\mathscr C\lambda$ is fuzzy pre $\mathscr C$ -open in Y. Then $f^{-1}(\mathscr C\lambda)$ is fuzzy pre $\mathscr C$ -open in $X\Rightarrow \mathscr C(f^{-1}(\lambda))$ is fuzzy pre $\mathscr C$ -open in $X\Rightarrow f^{-1}(\lambda)$ is fuzzy pre $\mathscr C$ -closed in X.

Conversely, assume that $f^{-1}(\lambda)$ is fuzzy pre \mathscr{C} -closed in X. Let λ be fuzzy pre \mathscr{C} -open in Y. Since \mathscr{C} satisfies involutive condition, $\mathscr{C}(\mathscr{C}\lambda)$ is fuzzy pre \mathscr{C} -open. Then $\mathscr{C}\lambda$ is fuzzy pre \mathscr{C} -closed in Y. By assumption $f^{-1}(\mathscr{C}\lambda)$ is fuzzy pre \mathscr{C} -closed in X. Then $\mathscr{C}(f^{-1}(\lambda))$ is fuzzy pre \mathscr{C} -closed in $X \Rightarrow \mathscr{C}(\mathscr{C}(f^{-1}(\lambda)))$ is fuzzy pre \mathscr{C} -open in X. Since \mathscr{C} satisfies involutive condition, $f^{-1}(\lambda)$ is fuzzy pre \mathscr{C} -open in X. Thus f is fuzzy pre \mathscr{C} -irresolute. \square

Theorem 3.5. Suppose $f:(X,\tau)\to (Y,\sigma)$ be a function and $\mathscr C$ be a complement function that satisfies monotonic and involutive conditions. Then f is fuzzy semi pre $\mathscr C$ -irresolute if and only if $f^{-1}(\lambda)$ is fuzzy semi pre $\mathscr C$ -closed in X, for every fuzzy semi pre $\mathscr C$ -closed in Y.

Proof. Suppose f is fuzzy semi pre $\mathscr C$ -irresolute. Let λ be fuzzy semi pre $\mathscr C$ -closed in Y. Since $\mathscr C$ satisfies monotonic and involutive conditions, $\mathscr C\lambda$ is fuzzy semi pre $\mathscr C$ -open in Y. Then $f^{-1}(\mathscr C\lambda)$ is fuzzy semi pre $\mathscr C$ -open in $X \Rightarrow \mathscr C(f^{-1}(\lambda))$ is fuzzy semi pre $\mathscr C$ -open in $X \Rightarrow f^{-1}(\lambda)$ is fuzzy semi pre $\mathscr C$ -closed in X.

Conversely, assume that $f^{-1}(\lambda)$ is fuzzy semi pre \mathscr{C} -closed in X. Let λ be fuzzy semi pre \mathscr{C} -open in Y. Since \mathscr{C} satisfies involutive condition, $\mathscr{C}(\mathscr{C}\lambda)$ is fuzzy semi pre \mathscr{C} -open. Then $\mathscr{C}\lambda$ is fuzzy semi pre \mathscr{C} -closed in Y. By assumption $f^{-1}(\mathscr{C}\lambda)$ is fuzzy semi pre \mathscr{C} -closed in X. Then $\mathscr{C}(f^{-1}(\lambda))$ is fuzzy semi pre \mathscr{C} -open in X. Since \mathscr{C} satisfies involutive condition, $f^{-1}(\lambda)$ is fuzzy semi pre \mathscr{C} -open in X. Thus f is fuzzy semi pre \mathscr{C} -irresolute. \square

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Theorem 3.6. Suppose $f:(X,\tau)\to (Y,\sigma)$ be a function and $\mathscr C$ be a complement function that satisfies monotonic and involutive conditions. Then f is fuzzy b- $\mathscr C$ -closed in X, for every fuzzy b- $\mathscr C$ -closed in Y.

Proof. Suppose f is fuzzy b- $\mathscr C$ -irresolute. Let λ be fuzzy b- $\mathscr C$ -closed in Y. Since $\mathscr C$ satisfies monotonic and involutive conditions, $\mathscr C\lambda$ is fuzzy b- $\mathscr C$ -open in Y. Then $f^{-1}(\mathscr C\lambda)$ is fuzzy b- $\mathscr C$ -open in $X \Rightarrow \mathscr C(f^{-1}(\lambda))$ is fuzzy b- $\mathscr C$ -open in $X \Rightarrow f^{-1}(\lambda)$ is fuzzy b- $\mathscr C$ -closed in X.

Conversely, assume that $f^{-1}(\lambda)$ is fuzzy b- \mathscr{C} -closed in X. Let λ be fuzzy b- \mathscr{C} -open in Y. Since \mathscr{C} satisfies involutive condition, $\mathscr{C}(\mathscr{C}\lambda)$ is fuzzy b- \mathscr{C} -open. Then $\mathscr{C}\lambda$ is fuzzy b- \mathscr{C} -closed in Y. By assumption $f^{-1}(\mathscr{C}\lambda)$ is fuzzy b- \mathscr{C} -closed in X. Then $\mathscr{C}(f^{-1}(\lambda))$ is fuzzy b- \mathscr{C} -closed in X \Rightarrow $\mathscr{C}(\mathscr{C}(f^{-1}(\lambda)))$ is fuzzy b- \mathscr{C} -open in X. Since \mathscr{C} satisfies involutive condition, $f^{-1}(\lambda)$ is fuzzy b- \mathscr{C} -open in X. Thus f is fuzzy b- \mathscr{C} -irresolute. \square

Theorem 3.7. Suppose $f:(X,\tau)\to (Y,\sigma)$ be a function and $\mathscr C$ be a complement function that satisfies monotonic and involutive conditions. Then f is fuzzy $b^\#$ - $\mathscr C$ -irresolute if and only if $f^{-1}(\lambda)$ is fuzzy $b^\#$ - $\mathscr C$ -closed in X, for every fuzzy $b^\#$ - $\mathscr C$ -closed in Y.

Proof. Suppose f is fuzzy $b^\#$ - $\mathscr C$ -irresolute. Let λ be fuzzy $b^\#$ - $\mathscr C$ -closed in Y. Since $\mathscr C$ satisfies monotonic and involutive conditions, $\mathscr C\lambda$ is fuzzy $b^\#$ - $\mathscr C$ -open in Y. Then $f^{-1}(\mathscr C\lambda)$ is fuzzy $b^\#$ - $\mathscr C$ -open in $X\Rightarrow \mathscr C(f^{-1}(\lambda))$ is fuzzy $b^\#$ - $\mathscr C$ -open in $X\Rightarrow f^{-1}(\lambda)$ is fuzzy $b^\#$ - $\mathscr C$ -closed in X.

Conversely, assume that $f^{-1}(\lambda)$ is fuzzy $b^\#$ - \mathscr{C} -closed in X. Let λ be fuzzy $b^\#$ - \mathscr{C} -open in Y. Since \mathscr{C} satisfies involutive condition, $\mathscr{C}(\mathscr{C}\lambda)$ is fuzzy $b^\#$ - \mathscr{C} -open. Then $\mathscr{C}\lambda$ is fuzzy $b^\#$ - \mathscr{C} -closed in Y. By assumption $f^{-1}(\mathscr{C}\lambda)$ is fuzzy $b^\#$ - \mathscr{C} -closed in X. Then $\mathscr{C}(f^{-1}(\lambda))$ is fuzzy $b^\#$ - \mathscr{C} -open in X. Since \mathscr{C} satisfies involutive condition, $f^{-1}(\lambda)$ is fuzzy $b^\#$ - \mathscr{C} -open in X. Thus f is fuzzy $b^\#$ - \mathscr{C} -irresolute. \square

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