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ON CONTRA SOFT A$_R$S CLOSED SETS IN SOFT TOPOLOGICAL SPACES

P. ANBARASI RODRIGO AND K. RAJENDRA SUBA [1]

ABSTRACT. In this paper we introduce soft A_RS closed on soft topological spaces and study some of their properties. We also investigate the concepts of contra soft A$_R$S closed mappings, contra soft A_RS open mappings and also discuss their relationship with other soft mappings. Counter examples are given to show the non coincidence of these functions.

1. INTRODUCTION

The soft set theory is a rapidly processing field of mathematics. Molodtsov, see [1] shown several applications of this theory in solving many practical problems in economics, engineering, social science, medical science, and so on. In 2010 Muhammad Shabir and Munazza Naz used soft sets to define a topology namely soft topology. soft generalized closed set was introduced by K. Kannan in 2012. The investigation of generalized closed sets has led to several new and interesting concepts like new covering properties and new separation axioms. Some of these separation axioms have been found to be useful in computer science and digital topology. In this paper we defined soft ARS - closed mapping, soft A_RS - open mapping and a detailed study of some of its properties in soft

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[1] *corresponding author*

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topological spaces. With the help of counter examples, we show that the non coincidence of these various types of mappings.

2. PRELIMINARIES

Throughout this paper (X, τ, E) or $\tilde{X}$ denotes the soft topological spaces. For a subset (A, E) of $\tilde{X}$, the closure, the interior and the complement of (A, E) are denoted by $cl(A, E)$, $int(A, E)$ and $(A, E)^C$ respectively. We recall some basic definitions that are used in the sequel.

Definition 2.1. *Let (X, τ, E) be a soft topological space. A soft set (F, E) is called soft A_RS closed set if $\beta cl(F, E) \subseteq (U, E)$ and (U, E) is soft ω-open.*
The set of all soft A_RS closed sets is denoted by $SA_RSC(X)$.

Definition 2.2. *A map $f : (X, \tau, E) \to (Y, \sigma, K)$ is said to be aoft A_RS continuous if inverse image of every soft closed set in (Y, σ, K) is soft A_RS closed in (X, τ, E).*

Definition 2.3. *A map $f : (X, \tau, E) \to (Y, \sigma, K)$ is said to be soft A_RS open map if image of each soft open set in (X, τ, E) is soft A_RS open in (Y, σ, K).*

Definition 2.4. *A map $f : (X, \tau, E) \to (Y, \sigma, K)$ is said to be soft slightly A_RS continuous if the inverse image of every soft closed set in (Y, σ, K) is soft A_RS open in (X, τ, E).*

Proposition 2.1.
(1) *If the map $f : (X, \tau, E) \to (Y, \sigma, K)$ is soft continuous (or soft semi continuous or soft α-continuous or soft JP-continuous) then it is a soft A_RS continuous.*
(2) *If the map $f : (X, \tau, E) \to (Y, \sigma, K)$ is soft A_RS continuous function then it is soft gsp-continuous.*

3. CONTRA SOFT A_RS CONTINUOUS FUNCTIONS

Definition 3.1. *A map $f : (X, \tau, E) \to (Y, \sigma, K)$ is said to be contra soft A_RS continuous if the inverse image of every soft open set in Y is soft A_RS closed in X.*

Example 1. *Let X={x_1,x_2}, Y={y_1,y_2} and E={e_1,e_2}, K={k_1,k_2} where $\tau = \{F_3, F_{11}, F_{15}, F_{16}\}$, $\tau^c = \{F_6, F_5, F_{15}, F_{16}\}$, $A_R SC(Y, \sigma, K) = \{F_1, F_4, F_5, F_6, F_7, F_8, F_9, F_{10}, F_{12}, F_{13}, F_{14}, F_{15}, F_{16}\}$ and $\sigma = \{F_5, F_{12}, F_{15}, F_{16}\}$, $\sigma^c = \{F_{11}, F_4, F_{15}, F_{16}\}$, is* *defined as* $f(F_1) = F_1$, $f(F_2) = F_2$, $f(F_3) = F_3$, $f(F_4) = F_4$, $f(F_5) = F_5$, $f(F_6) = F_6$, $f(F_7)$ F_7, $f(F_8) = F_8$, $f(F_9) = F_9$, $f(F_{10}) = F_{10}$,$f(F_{11}) = F_{11}$,$f(F_{12}) = F_{12}$,$f(F_{13}) = F_{13}$,$f(F_{14}) = F_{14}$,$f(F_{15}) = F_{15}$,$f(F_{16}) = F_{16}$. *Clearly* f *is contra soft* A_RS *continuous.*

Theorem 3.1. *Every contra soft continuous map is a contra soft* A_RS *continuous.*

Proof. Let $f : (X, \tau, E) \to (Y, \sigma, K)$ be a contra soft continuous function and (U, K) be a soft open set in (Y, σ, K), $f^{-1}((U, K))$ is soft closed in (X, τ, E). Since every soft closed set is soft A_RS closed, $f^{-1}((U, K))$ is soft A_RS closed in (X, τ, E). Therefore f is contra soft A_RS continuous. □

The converse statement is not true.

Example 2. *Let* τ $=\{F_3, F_{11}, F_{15}, F_{16}\}$, $\tau^c = \{F_6, F_5, F_{15}, F_{16}\}$, $A_R SC(Y, \sigma, K) = \{F_1, F_4, F_5, F_6, F_7, F_8, F_9, F_{10}, F_{12}, F_{13}, F_{14}, F_{15}, F_{16}\}$ *and* $\sigma = \{F_5, F_{12}, F_{15}, F_{16}\}$, σ^c $=\{F_{11}, F_4, F_{15}, F_{16}\}$, *is defined as* $f(F_1) = F_1$, $f(F_2) = F_2$, $f(F_3) = F_3$, $f(F_4) = F_4$, $f(F_5) = F_5$, $f(F_6) = F_6$, $f(F_7) = F_7$, $f(F_8) = F_8$, $f(F_9) = F_9$, $f(F_{10}) = F_{10}$, $f(F_{11}) = F_{11}$, $f(F_{12}) = F_{12}$,$f(F_{13}) = F_{13}$,$f(F_{14}) = F_{14}$, $f(F_{15}) = F_{15}$, $f(F_{16}) = F_{16}$. *Clearly f is contra soft* A_R*S continuous. But* F_{12} *is not in* τ^c *of* (X, τ, E). *Hence f is not contra soft continuous.*

Theorem 3.2. *Every contra soft semi continuous map is a contra soft* A_RS *continuous.*

Proof. Obvious from the definition and proposition 2.1. □

The converse statement is not true.

Example 3. *Let* $\tau = \{F_1, F_4, F_7 F_{13} F_{15}, F_{16}\}$, $\tau^c = \{F_{14}, F_{12}, F_{10}, F_2, F_{15}, F_{16}\}$, $A_R SC(X, \tau, E) = \{F_1, F_2, F_3, F_4, F_5, F_6, F_9, F_{10}, F_{11}, F_{12}, F_{14}, F_{15}, F_{16}\}$, $(X, \tau, E) = \{F_{14}, F_6, F_{10}, F_9, F_5, F_4, F_2, F_{15}, F_{16}\}$ *and* $\sigma = \{F_5, F_{12}, F_{15}, F_{16}\}$, $\sigma^c = \{F_{11}, F_4, F_{15}, F_{16}\}$, *is defined as* $f(F_1) = F_1$, $f(F_2) = F_2$, $f(F_3) = F_3$, $f(F_4) = F_4$, $f(F_5) = F_5$, $f(F_6) = F_6$, $f(F_7) = F_7$, $f(F_8) = F_8$, $f(F_9) = F_9$, $f(F_{10}) = F_{10}$, $f(F_{11}) = F_{11}$, $f(F_{12}) = F_{12}$, $f(F_{13}) = F_{13}$, $f(F_{14}) = F_{14}$, $f(F_{15}) = F_{15}$, $f(F_{16}) = F_{16}$. *Clearly f is contra soft* A_R*S continuous. But*F_{12} *is not in soft semi closed of X. Hence f is not contra soft semi continuous.*

Theorem 3.3. *Every contra soft* $alpha$*-continuous map is a contra soft* A_RS *continuous.*

Proof. Obvious from the definition and proposition 2.1. □

The converse statement is not true.

Example 4. *Let* X ={x_1,x_2}, Y ={y_1,y_2} *and* E ={e_1,e_2}, *K*={k_1,k_2}, $f:(X,\tau,E)\to(Y,\sigma,K)$ *where* τ = {F_4,F_7,F_{15},F_{16}}, τ^c = {F_{10},F_{12},F_{15},F_{16}}, A_R*SC*(X,τ,E) = {F_1,$F_2F_3F_4F_5$,F_6,F_8F_9,F_{10},F_{11},F_{12}, $F_{13}F_{14}$,F_{15},F_{16}} , $\alpha c(X,\tau,E)$ = {F_{12},$F_3F_{10}F_8F_5$,F_2,F_1, F_{15},F_{16}} *and* σ = {F_3,F_{11},F_{15},F_{16}}, σ^c = {F_6,F_5,F_{15},F_{16}}, *is defined as f(*F_1*)* = F_1, *f(*F_2*)* = F_2, *f(*F_3*)* = F_3, *f(*F_4*)* = F_4, *f(*F_5*)* = F_5, *f(*F_6*)* = F_6, *f(*F_7*)* = F_7, *f(*F_8*)* = F_8, *f(*F_9*)* = F_9, *f(*F_{10}*)* = F_{10},*f(*F_{11}*)* = F_{11},*f(*F_{12}*)* = F_{12},*f(*F_{13}*)* = F_{13},*f(*F_{14}*)* = F_{14},*f(*F_{15}*)* = F_{15},*f(*F_{16}*)* = F_{16}. *Clearly f is contra soft* A_R*S continuous. But*F_{11} *is not in soft* α *closed of X. Hence f is not contra soft* $alpha$ *continuous.*

Theorem 3.4. *Every contra soft* JP *continuous map is a contra soft* A_RS *continuous.*

Proof. Obvious from the definition and proposition 2.1. □

The converse statement is not true.

Example 5. *Let* $f:(X,\tau,E)\to(Y,\sigma,K)$ *where* τ = {F_1,F_4,F_7,$F_{13}F_{15}F_{16}$}, τ^c = {F_{14},F_{12},F_{10},F_2,F_{15},F_{16}}, A_R*SC*(X,τ,E) = {F_1,$F_2F_3F_4F_5$,F_6,F_9,F_{10},F_{11}, F_{12}, F_{14},F_{15},F_{16}} , *SJPc*(X,τ,E) ={F_2,$F_3F_4F_5F_6$,F_9,F_{10},F_{11},F_{12},F_{14}, F_{15},F_{16}} *and* σ = {F_1,F_{13}, F_{15},F_{16}}, σ^c = {F_{14},F_2,F_{15},F_{16}}, *is defined as f(*F_1*)* = F_1, *f(*F_2*)* = F_2, *f(*F_3*)* = F_3, *f(*F_4*)* = F_4, *f(*F_5*)* = F_5, *f(*F_6*)* = F_6, *f(*F_7*)* = F_7, *f(*F_8*)* = F_8, *f(*F_9*)* = F_9, *f(*F_{10}*)* = F_{10},*f(*F_{11}*)* = F_{11},*f(*F_{12}*)* = F_{12},*f(*F_{13}*)* = F_{13},*f(*F_{14}*)* = F_{14},*f(*F_{15}*)* = F_{15},*f(*F_{16}*)* = F_{16}. *Clearly f is contra soft* A_R*S continuous. But* F_1 *is not in soft* JP *closed of X. Hence f is not contra soft* JP *continuous.*

Theorem 3.5. *Every contra soft* A_RS *continuous map is a contra soft* gsp *continuous.*

Proof. Obvious from the definition and proposition 2.1. □

The converse statement is not true.

Example 6. *Let* $f:(X,\tau,E)\to(Y,\sigma,K)$ *where* τ = {F_1,F_4,F_7,F_{13},F_{15},F_{16}}, τ^c = {F_{14},F_{12},F_{10},F_2,F_{15},F_{16}}, A_R*SC*(X,τ,E) = {F_1,F_2F_3,F_4,F_5,F_6,F_9,F_{10},F_{11}, F_{12}, F_{14},F_{15},F_{16}} , *Sgspc*(X,τ,E) ={F_1,F_2,$F_3F_4F_5F_6$,F_7, F_8,F_9,F_{10},F_{11},F_{12}, F_{13},F_{14}, F_{15},F_{16}} *and* σ ={F_1,F_4,F_7,F_{13},F_{15},F_{16}}, σ^c = {F_{14},F_{12},F_{10},F_2,F_{15},F_{16}}, *is defined as f(*F_1*)* = F_1, *f(*F_2*)* = F_2, *f(*F_3*)* = F_3, *f(*F_4*)* = F_4, *f(*F_5*)* = F_5, *f(*F_6*)* =

F_6, f(F_7) = F_7, f(F_8) = F_8, f(F_9) = F_9, f(F_{10}) = F_{10},f(F_{11}) = F_{11}, f(F_{12}) = F_{12}, f(F_{13}) = F_{13}, f(F_{14}) = F_{14}, f(F_{15}) = F_{15}, f(F_{16}) = F_{16}. Clearly f is contra soft gsp continuous. ButF_7,F_{13} is not in soft A_RS closed of X. Hence f is not contra soft A_RS continuous.

Remark 3.1. *The composition of two contra soft A_RS continuous maps need not be contra soft A_RS continuous.*

Example 7. *In the soft topological space $(X, \tau, E), (Y, \sigma, K), (Z, \eta,$ R) and X ={x_1,x_2}, Y ={y_1,y_2}, Z ={z_1,z_2}, and E ={e_1,e_2}, K ={k_1,k_2}, R={r_1,r_2}*
$f : (X, \tau, E) \to (Y, \sigma, K)$, $g : (Y, \sigma, K) \to (Z, \eta, R)$ where
τ = {F_4,F_7,F_{15},F_{16}}, τ^c = {F_{10},F_{12},F_{15},F_{16}},
A_RSC(X, τ, E) = {F_1,F_2,F_3,F_5,F_6,F_8,F_9,F_{10},F_{11},F_{12},F_{13},F_{14},F_{15},F_{16}} and
σ = {F_3,F_{11},F_{15},F_{16}}, σ^c = {F_6,F_5,F_{15},F_{16}}, A_RSC(Y, σ, K) =
{F_1,F_4,F_5,F_6,F_7,F_8,$F_9$$F_{10}$,$F_{12}$,$F_{13}$,$F_{14}$, F_{15},F_{16}} and η ={F_5,F_{12},F_{15},F_{16}}, η^c = {F_{11},F_4,F_{15},F_{16}} is defined as $(g \circ f)$ F_1= F_1, $(g \circ f)$ F_2= F_2, $(g \circ f)$ F_3= F_3, $(g \circ f)$ F_4= F_4, $(g \circ f)$ F_5= F_5, $(g \circ f)$$F_6$= F_6, $(g \circ f)$ F_7= F_7, $(g \circ f)$ F_8= F_8, $(g \circ f)$ F_9= F_9, $(g \circ f)$ F_{10}= F_{10}, $(g \circ f)$ F_{11}= F_{11}, $(g \circ f)$ F_{12}= F_{12}, $(g \circ f)$ F_{13}= F_{13}, $(g \circ f)$ F_{14}= F_{14}, $(g \circ f)$ F_{15}= F_{15}, $(g \circ f)$ F_{16}= F_{16}. Clearly f and g is contra soft A_RS continuous. But $(g \circ f)$: $(X, \tau, E) \to (Z, \eta, R)$ is not contra soft A_RS continuous. Since $(g \circ f)^(-1)$ (F_5) =F_4, F_4 is not soft A_RS closed in (X, τ, E). Hence $(g \circ f)$ is not contra soft A_RS continuous.

Definition 3.2. *A space X is said to be locally soft A_RS indiscrete if every soft A_RS open set of X is soft closed in X.*

Theorem 3.6. *If $f : (X, \tau, E) \to (Y, \sigma, K)$ is soft A_RS continuous and if Y is soft locally indiscrete, then f is contra soft A_RS continuous.*

Proof. Let (G, F) be an soft open set of (Y, σ, K). Since Y is soft locally discrete, (G, F) is soft closed. Since f is soft A_RS continuous, $(f^{-1})((G, F))$ is soft A_RS closed in X. Therefore, f is contra soft A_RS continuous. □

Theorem 3.7. *If $f : (X, \tau, E) \to (Y, \sigma, K)$ is soft continuous and X is a locally indiscrete space, then f is contra soft A_RS continuous.*

Proof. Let (G, F) be any soft open set of (Y, σ, K) . Since f is continuous $(f^{-1})((G, F))$ is soft open in X. Since$X$ is soft locally discrete, $(f^{-1})((G, F))$ is soft closed in

X. Every soft closed set is soft A_RS closed, $(f^{-1})((G,F))$ is soft A_RS closed in$X$. Therefore,f is contra soft A_RS continuous. □

Theorem 3.8. *Let $f:(X,\tau,E)\to(Y,\sigma,K)$ is soft A_RS irresolute map with Yas locally soft A_RS indiscrete space and $g:(Y,\sigma,K)\to(Z,\eta,R)$ is contra soft A_RS continuous, then ($g\circ f$) is soft A_RS continuous.*

Proof. Let (G,F) be any soft closed set in (Z,η,R). Since g is contra soft A_RS continuous, $g^{-1}((G,F))$ is soft A_RS open in Y. But Y is locally soft A_RS indiscrete, $g^{-1}((G,F))$ is soft closed in Y. Hence, $g^{-1}((G,F))$ is soft A_RS closed in Y. Since, f is soft A_RS irresolute, $f^{-1}(g^{-1}((G,F)))=(g\circ f)^{-1}((G,F))$ is soft A_RS closed in X. Therefore, $(g\circ f)$ is soft A_RS continuous. □

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DEPARTMENT OF MATHEMATICS
ST.MARY'S COLLEGE (AUTONOMOUS)
THOOTHUKUDI
MANONMANIAM SUNDARANAR UNIVERSITY
ABISHIEKAPATTI, TIRUNELVELI, INDIA

DEPARTMENT OF MATHEMATICS
ST.MARY'S COLLEGE (AUTONOMOUS)
THOOTHUKUDI
MANONMANIAM SUNDARANAR UNIVERSITY
ABISHIEKAPATTI, TIRUNELVELI, INDIA
E-mail address: rajendrasuba88@gmail.com