

ADJACENCY MATRICES OF GENERALIZED COMPOSITION AND GENERALIZED DISJUNCTION OF GRAPHS

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ABSTRACT. Corresponding to any graph, one can obtain its adjacency matrix. Many graph theorists defined different operations on graphs. In this article we try to generalize the disjunction operation in graphs introduced by Harary [2]. Further we wish to analyse the nature of adjacency matrices corresponding to the graphs obtained by generalized disjunction and generalized composition defined in [5].

1. INTRODUCTION

Several graph theoretic operations were introduced by various authors to get new graphs from the given ones. The operations ‘Composition and disjunction’ are two among them. In 2015 Acharya and Mehta introduced the concept of generalized Cartesian product [7]. In 2019 Manju and Suresh Singh, generalized the concept of composition of two graphs with respect to distance as a parameter and studied various properties of 2-composition of path graphs [5].

Definition 1.1. [3] Let $G = (V, E)$ be a graph. The distance $d_G(u, v)$ between two points u and v in G is the length of the shortest path joining u and v .

Definition 1.2. [3] Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two graphs. The composition of G_1 and G_2 denoted by $G_1[G_2]$ is a graph $G = (V, E)$ with vertex

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set $V = V_1 \times V_2$ and two vertices (u_1, u_2) and (v_1, v_2) are adjacent in $G_1[G_2]$ if u_1 is adjacent to v_1 in G_1 (that is $d_{G_1}(u_1, v_1) = 1$) or $u_1 = v_1$ and u_2 is adjacent to v_2 in G_2 (that is, $d_{G_1}(u_1, v_1) = 0$ and $d_{G_2}(u_2, v_2) = 1$).

Definition 1.3. [5] The generalized composition, say k -composition of two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is the graph $(G_1[G_2])_k = (V, E)$ with vertex set $V = V_1 \times V_2$ and the edge set E defined as follows; two vertices (u_1, u_2) and (v_1, v_2) are adjacent in $(G_1[G_2])_k$ if

- (i) either $d_{G_1}(u_1, v_1) = k$, or
- (ii) $d_{G_1}(u_1, v_1) = 0$ and $d_{G_2}(u_2, v_2) = k$.

It is clear that for $k = 1$, we get usual composition of graphs. When $k = 2$ we have the following definition.

Definition 1.4. [5] The 2-composition of two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is the graph $(G_1[G_2])_2 = (V, E)$ with vertex set $V = V_1 \times V_2$ and the edge set E defined as follows; (u_1, u_2) and (v_1, v_2) are adjacent in $(G_1[G_2])_2$ if

- (i) either $d_{G_1}(u_1, v_1) = 2$, or
- (ii) $d_{G_1}(u_1, v_1) = 0$ and $d_{G_2}(u_2, v_2) = 2$.

Definition 1.5. [2] The disjunction of two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is the graph $G_1 \vee G_2 = (V, E)$ where $V = V_1 \times V_2$ and the edge set E defined as follows; two vertices $u = (u_1, u_2)$ and $v = (v_1, v_2)$ are adjacent in $G_1 \vee G_2$, if u_1 is adjacent to v_1 in G_1 or u_2 is adjacent to v_2 in G_2 or both.

Definition 1.6. [1] The adjacency matrix $A = [a_{ij}]$ of a labelled graph G with p vertices is a $p \times p$ matrix in which $a_{ij} = 1$ if v_i is adjacent to v_j and $a_{ij} = 0$ otherwise.

Definition 1.7. [4] The degree matrix $D = [d_{ij}]$ of a labelled graph G with p vertices is a $p \times p$ matrix in which $d_{ij} = 0$ if $i \neq j$ and $d_{ii} = d(v_i)$.

Definition 1.8. [2] Let $A = [a_{ij}]$ be an $m \times n$ matrix and $B = [b_{ij}]$ be a $p \times q$ matrix, then their tensor product denoted by $A \otimes B$ is an $mp \times nq$ matrix and is defined as

$$A \otimes B = \begin{bmatrix} a_{11}B & a_{12}B & \cdots & a_{1n}B \\ a_{21}B & a_{22}B & \cdots & a_{2n}B \\ \vdots & \vdots & & \vdots \\ a_{m1}B & a_{m2}B & \cdots & a_{mn}B \end{bmatrix}.$$

2. GENERALIZED DISJUNCTION OF TWO GRAPHS

Now, we try to generalize the definition of disjunction given by Harary [2].

Definition 2.1. The generalized disjunction say k -disjunction of two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is the graph $G_1 \vee_k G_2 = (V, E)$ where $V = V_1 \times V_2$ and the edge set E defined as follows; two vertices $u = (u_1, u_2)$ and $v = (v_1, v_2)$ are adjacent in $G_1 \vee_k G_2$, if $d_{G_1}(u_1, v_1) = k$ or $d_{G_2}(u_2, v_2) = k$ or both.

It is clear that for $k = 1$, we get usual disjunction of graphs. When $k = 2$ we have the following.

Definition 2.2. The 2-disjunction of two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is the graph $G_1 \vee_2 G_2 = (V, E)$ where $V = V_1 \times V_2$ and the edge set E defined as follows; two vertices $u = (u_1, u_2)$ and $v = (v_1, v_2)$ are adjacent in $G_1 \vee_2 G_2$, if $d_{G_1}(u_1, v_1) = 2$ or $d_{G_2}(u_2, v_2) = 2$ or both.

Example 1. See figure 1.

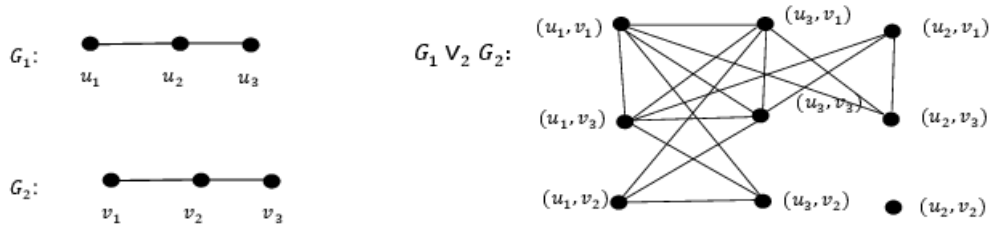


FIGURE 1. 2-disjunction of G_1 and G_2

3. ADJACENCY MATRIX OF $G_1 \vee_2 G_2$, 2-DISJUNCTION OF G_1 AND G_2

In this section we obtain the adjacency matrix of $G_1 \vee_2 G_2$ and generalize the result for k -disjunction.

Proposition 3.1. [2] Let G_1 and G_2 be two simple connected graphs of order n and m respectively. Then,

$$A(G_1 \vee_2 G_2) = A(G_1) \otimes J_m \oplus J_n \otimes A(G_2) \oplus A(G_1) \otimes A(G_2),$$

where J_m and J_n are the $m \times m$ and $n \times n$ matrices with every entry 1, \oplus denotes addition modulo 2 and \otimes denotes the tensor product.

Definition 3.1. [6] Let $G = (V, E)$ be a simple graph with the vertex set $\{u_1, u_2, \dots, u_n\}$. The second stage adjacency matrix $A_2(G) = [a_{ij}]$ is defined as follows

$$a_{ij} = \begin{cases} 1 & \text{if } d_G(u_i, u_j) = 2, \\ 0 & \text{otherwise.} \end{cases}$$

Proposition 3.2. Let G_1 and G_2 be two simple connected graphs of order n and m respectively. Then,

$$A(G_1 \vee_2 G_2) = A_2(G_1) \otimes J_m \oplus J_n \otimes A_2(G_2) \oplus A_2(G_1) \otimes A_2(G_2),$$

where J_m and J_n are the $m \times m$ and $n \times n$ matrices with every entry 1, \oplus denotes addition modulo 2 and \otimes denotes the tensor product.

Proof. Suppose $V(G_1) = \{u_1, u_2, \dots, u_n\}$ and $V(G_2) = \{v_1, v_2, \dots, v_m\}$. Then consider $A(G_1 \vee_2 G_2)$,

$$A(G_1 \vee_2 G_2) = \begin{bmatrix} A_{11} & A_{12} & \dots & A_{1n} \\ \vdots & \vdots & & \vdots \\ A_{r1} & A_{r2} & \dots & A_{rn} \\ \vdots & \vdots & \ddots & \vdots \\ A_{n1} & A_{n2} & \dots & A_{nn} \end{bmatrix},$$

where

$$A_{rs} = \begin{matrix} & (u_s, v_1) & (u_s, v_2) & \dots & (u_s, v_m) \\ \begin{matrix} (u_r, v_1) \\ (u_r, v_2) \\ \vdots \\ (u_r, v_m) \end{matrix} & \begin{bmatrix} S_{11} & S_{12} & \dots & S_{1m} \\ \vdots & \vdots & \ddots & \vdots \\ S_{m1} & S_{m2} & \dots & S_{mm} \end{bmatrix} \end{matrix}.$$

Suppose $d_{G_1}(u_r, u_s) = 2$ then $S_{ij} = 1 \forall i$ and j . So, in this case $A_{rs} = J_m$.

Next suppose that $d_{G_1}(u_r, u_s) \neq 2$. Then $S_{ij} = \begin{cases} 1 & \text{if } d_{G_2}(v_i, v_j) = 2 \\ 0 & \text{otherwise} \end{cases}$.

Therefore, in this case $A_{rs} = A_2(G_2)$. Hence,

$$A_{rs} = \begin{cases} J_m & \text{if } d_{G_1}(u_r, u_s) = 2, \\ A_2(G_2) & \text{if } d_{G_1}(u_r, u_s) \neq 2 \text{ and } d_{G_2}(v_i, v_j) = 2. \end{cases}$$

Assume $A_2(G_1) = [b_{ij}]_{n \times n}$, $A_2(G_2) = [c_{ij}]_{m \times m}$ and consider

$$A_2(G_1) \otimes J_m = \begin{bmatrix} b_{11}J_m & b_{12}J_m & \cdots & b_{1n}J_m \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1}J_m & b_{n2}J_m & \cdots & b_{nn}J_m \end{bmatrix}.$$

Then the $(i, j)^{\text{th}}$ block of $A_2(G_1) \otimes J_m = [b_{ij}J_m]$ and

$$[b_{ij}J_m] = \begin{cases} J_m & \text{if } b_{ij} = 1 \text{ (that is, if } d_{G_1}(u_i, u_j) = 2), \\ (0)_{m \times m} & \text{otherwise.} \end{cases}$$

Now,

$$\begin{aligned} J_n \otimes A_2(G_2) &= \begin{bmatrix} 1 & 1 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \cdots & 1 \end{bmatrix} \otimes A_2(G_2) \\ &= \begin{bmatrix} A_2(G_2) & A_2(G_2) & \cdots & A_2(G_2) \\ \vdots & \vdots & \ddots & \vdots \\ A_2(G_2) & A_2(G_2) & \cdots & A_2(G_2) \end{bmatrix} \end{aligned}$$

so, the $(i, j)^{\text{th}}$ block of $J_n \otimes A_2(G_2)$ is $A_2(G_2) \forall i$ and j .

Next,

$$A_2(G_1) \otimes A_2(G_2) = \begin{bmatrix} b_{11}A_2(G_2) & b_{12}A_2(G_2) & \cdots & b_{1n}A_2(G_2) \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1}A_2(G_2) & b_{n2}A_2(G_2) & \cdots & b_{nn}A_2(G_2) \end{bmatrix}.$$

Hence the $(i, j)^{\text{th}}$ block of $A_2(G_1) \otimes A_2(G_2)$ is $b_{ij}A_2(G_2)$ and,

$$b_{ij}A_2(G_2) = \begin{cases} A_2(G_2) & \text{if } b_{ij} = 1 \text{ (that is, if } d_{G_1}(u_i, u_j) = 2), \\ (0)_{m \times m} & \text{otherwise.} \end{cases}$$

It follows that, the $(i, j)^{\text{th}}$ block of $A_2(G_1) \otimes J_m \oplus J_n \otimes A_2(G_2) \oplus A_2(G_1) \otimes A_2(G_2)$

$$\begin{aligned} &= \begin{cases} J_m \oplus A_2(G_2) \oplus A_2(G_2) & \text{if } d_{G_1}(u_i, u_j) = 2 \\ 0 + A_2(G_2) + 0 & \text{if } d_{G_1}(u_i, u_j) \neq 2 \end{cases} \\ &= \begin{cases} J_m & \text{if } d_{G_1}(u_i, u_j) = 2 \\ A_2(G_2) & \text{if } d_{G_1}(u_i, u_j) \neq 2 \end{cases}. \end{aligned}$$

Therefore, the $(i, j)^{\text{th}}$ block of $A_2(G_1) \otimes J_m \oplus J_n \otimes A_2(G_2) \oplus A_2(G_1) \otimes A_2(G_2)$ is equal to the $(i, j)^{\text{th}}$ block of $A(G_1 \vee_2 G_2)$. \square

Definition 3.2. Let $G = (V, E)$ be a simple graph with the vertex set $\{u_1, u_2, \dots, u_n\}$. The r^{th} stage adjacency matrix $A_r(G) = [a_{ij}]$ is defined as follows

$$a_{ij} = \begin{cases} 1 & \text{if } d_G(u_1, u_2) = r, \\ 0 & \text{otherwise.} \end{cases}$$

Proposition 3.3. Let G_1 and G_2 be two simple connected graphs of order n and m respectively. Then,

$$A(G_1 \vee_k G_2) = A_k(G_1) \otimes J_m \oplus J_n \otimes A_k(G_2) \oplus A_k(G_1) \otimes A_k(G_2),$$

where $A_k(G)$ is the k^{th} stage adjacency matrix of G .

Proof. It can be proved similar to proposition 3.2. \square

4. ADJACENCY MATRIX OF $(G_1[G_2])_2$

Next, we discuss the adjacency matrix of $(G_1[G_2])_2$ of the given two graphs G_1 and G_2 .

Proposition 4.1. [2] Let G_1 and G_2 be two simple connected graphs of order n and m respectively. Then,

$$A(G_1[G_2]) = A(G_1) \otimes J_m \oplus I_n \otimes A(G_2),$$

where J_m is the $m \times m$ matrix with every entry 1 and I_n is the identity matrix of order $n \times n$, \oplus denotes addition modulo 2 and \otimes denotes the tensor product.

Proposition 4.2. Let G_1 and G_2 be two simple connected graphs of order n and m respectively. Then,

$$A((G_1[G_2])_2) = A_2(G_1) \otimes J_m \oplus I_n \otimes A_2(G_2),$$

where J_m is the $m \times m$ matrix with every entry 1 and I_n is the identity matrix of order $n \times n$, \oplus denotes addition modulo 2 and \otimes denotes the tensor product.

Proof. Suppose $V(G_1) = \{u_1, u_2, \dots, u_n\}$ and $V(G_2) = \{v_1, v_2, \dots, v_m\}$. Then consider $A((G_1[G_2])_2)$,

$$A((G_1[G_2])_2) = \begin{bmatrix} A_{11} & A_{12} & \dots & A_{1n} \\ \vdots & \vdots & & \vdots \\ A_{r1} & A_{r2} & \dots & A_{rn} \\ \vdots & \vdots & \ddots & \vdots \\ A_{n1} & A_{n2} & \dots & A_{nn} \end{bmatrix},$$

where

$$A_{rs} = \begin{matrix} & (u_s, v_1) & (u_s, v_2) & \dots & (u_s, v_m) \\ \begin{matrix} (u_r, v_1) \\ (u_r, v_2) \\ \vdots \\ (u_r, v_m) \end{matrix} & \begin{bmatrix} S_{11} & S_{12} & \dots & S_{1m} \\ \vdots & \vdots & \ddots & \vdots \\ S_{m1} & S_{m2} & \dots & S_{mm} \end{bmatrix} \end{matrix}.$$

Suppose $d_{G_1}(u_r, u_s) = 0$, then $S_{ij} = \begin{cases} 1 & \text{if } d_{G_2}(v_i, v_j) = 2 \\ 0 & \text{if } d_{G_2}(v_i, v_j) \neq 2 \end{cases}$.

Therefore in this case $A_{rs} = A_2(G_2)$.

Next suppose that $d_{G_1}(u_r, u_s) = 2$, then $S_{ij} = 1 \forall i$ and j . Therefore in this case $A_{rs} = J_m$.

Also, suppose $d_{G_1}(u_r, u_s) \neq 0$ or 2 . Then A_{rs} is zero matrix of order $m \times m$. Thus,

$$A_{rs} = \begin{cases} A_2(G_2) & \text{if } d_{G_1}(u_r, u_s) = 0 \\ J_m & \text{if } d_{G_1}(u_r, u_s) = 2 \\ (0)_{m \times m} & \text{otherwise.} \end{cases}$$

Suppose $A_2(G_1) = [b_{ij}]_{n \times n}$, $A_2(G_2) = [c_{ij}]_{m \times m}$, then

$$A_2(G_1) \otimes J_m = \begin{bmatrix} b_{11}J_m & b_{12}J_m & \dots & b_{1n}J_m \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1}J_m & b_{n2}J_m & \dots & b_{nn}J_m \end{bmatrix}.$$

Then the $(i, j)^{\text{th}}$ block of $A_2(G_1) \otimes J_m = [b_{ij}J_m]$ and

$$[b_{ij}J_m] = \begin{cases} J_m & \text{if } b_{ij} = 1 \text{ (that is, if } d_{G_1}(u_i, u_j) = 2) \\ (0)_{m \times m} & \text{otherwise.} \end{cases}$$

$$\text{Now, } I_n \otimes A_2(G_2) = \begin{bmatrix} A_2(G_2) & 0 & \cdots & 0 \\ 0 & A_2(G_2) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & A_2(G_2) \end{bmatrix}.$$

$$\text{Therefore, the } (i, j)^{\text{th}} \text{ block of } I_n \otimes A_2(G_2) = \begin{cases} A_2(G_2) & \text{if } i = j \\ (0)_{m \times m} & \text{otherwise} \end{cases}.$$

It follows that the $(i, j)^{\text{th}}$ block of $A_2(G_1) \otimes J_m \oplus I_n \otimes A_2(G_2)$

$$= \begin{cases} J_m \oplus (0)_{m \times m} & \text{if } i \neq j \text{ with } d_{G_1}(u_i, u_j) = 2 \\ (0)_{m \times m} \oplus A_2(G_2) & \text{if } i = j \text{ (that is, if } d_{G_1}(u_i, u_j) = 0) \\ (0)_{m \times m} & \text{otherwise,} \end{cases}$$

$$= \begin{cases} J_m & \text{if } i \neq j \text{ with } d_{G_1}(u_i, u_j) = 2 \\ A_2(G_2) & \text{if } i = j \text{ (that is, if } d_{G_1}(u_i, u_j) = 0) \\ (0)_{m \times m} & \text{otherwise.} \end{cases}$$

This gives, $(i, j)^{\text{th}}$ block of $A_2(G_1) \otimes J_m \oplus I_n \otimes A_2(G_2)$ is equal to the $(i, j)^{\text{th}}$ block of $A((G_1[G_2])_2)$.

□

Proposition 4.3. *Let G_1 and G_2 be two simple connected graphs of order n and m respectively. Then,*

$$A((G_1[G_2])_k) = A_k(G_1) \otimes J_m \oplus I_n \otimes A_k(G_2).$$

Proof. Proof is similar to the proposition 4.2

□

Proposition 4.4. [8] *Let G be a simple connected, triangle free and square free graph. Then*

$$A_2(G) = A^2(G) - D(G).$$

In the light of proposition 4.4, we can easily state the following two propositions

Proposition 4.5. *Let G_1 and G_2 be two simple connected graphs of order n and m respectively. Then,*

$$A((G_1[G_2])_2) = A^2(G_1) \otimes J_m - D(G_1) \otimes J_m \oplus I_n \otimes A^2(G_2) - I_n \otimes D(G_2).$$

Proof. We have

$$\begin{aligned} A((G_1[G_2])_2) &= A_2(G_1) \otimes J_m \oplus I_n \otimes A_2(G_2) \\ &= (A^2(G_1) - D(G_1)) \otimes J_m \oplus I_n \otimes (A^2(G_2) - D(G_2)) \\ &= A^2(G_1) \otimes J_m - D(G_1) \otimes J_m \oplus I_n \otimes A^2(G_2) - I_n \otimes D(G_2). \end{aligned}$$

□

Proposition 4.6. *Let G_1 and G_2 be two simple connected graphs of order n and m respectively. Then,*

$$\begin{aligned} A(G_1 \vee_2 G_2) &= A^2(G_1) \otimes J_m - D(G_1) \otimes J_m \oplus J_n \otimes A^2(G_2) \\ &\quad - J_n \otimes D(G_2) \oplus A^2(G_1) \otimes A^2(G_2) \\ &\quad - A^2(G_1) \otimes D(G_2) - D(G_1) \otimes A^2(G_2) \oplus D(G_1) \otimes D(G_2). \end{aligned}$$

Proof. We have,

$$\begin{aligned} A(G_1 \vee_2 G_2) &= A_2(G_1) \otimes J_m \oplus J_n \otimes A_2(G_2) \oplus A_2(G_1) \otimes A_2(G_2) \\ &= (A^2(G_1) - D(G_1)) \otimes J_m \oplus J_n \otimes (A^2(G_2) - D(G_2)) \\ &\quad \oplus (A^2(G_1) - D(G_1)) \otimes (A^2(G_2) - D(G_2)). \end{aligned}$$

Hence the result.

□

5. RELATION BETWEEN THE ADJACENCY MATRICES OF A GRAPH G AND THE POWERS OF G

Definition 5.1. [3] *Let G be a graph. The n^{th} power of G , denoted by G^n has the same vertex set as that of G and two vertices u and v are adjacent in G^n , whenever $d_G(u, v) \leq n$.*

When $n = 2$, we get the square of a graph G , denoted by G^2 has the same vertex set as that of G and two vertices u and v are adjacent in G^2 , whenever $d_G(u, v) \leq 2$.

Proposition 5.1. *Let G be a simple connected graph with adjacency matrix $A(G)$. Then, $A(G^2) - A(G) = A_2(G)$, Where $A(G^2)$ is the adjacency matrix of G^2 .*

Proof. Suppose $V(G) = \{v_1, v_2, \dots, v_m\}$. Then,

$$\begin{aligned} \text{the } (i, j)^{\text{th}} \text{ entry of } A(G^2) &= \begin{cases} 1 & \text{if } d_{G^2}(v_i, v_j) = 1 \\ 0 & \text{otherwise} \end{cases} \\ &= \begin{cases} 1 & \text{if } d_G(v_i, v_j) = 1 \text{ or } 2 \\ 0 & \text{otherwise} \end{cases} \end{aligned}$$

$$\text{The } (i, j)^{\text{th}} \text{ entry of } A(G) = \begin{cases} 1 & \text{if } d_G(v_i, v_j) = 1 \\ 0 & \text{otherwise} \end{cases}$$

Therefore, the $(i, j)^{\text{th}}$ entry of

$$\begin{aligned} A(G^2) - A(G) &= \begin{cases} 0 & \text{if } d_G(v_i, v_j) = 1 \\ 1 & \text{if } d_G(v_i, v_j) = 2 \\ 0 & \text{otherwise} \end{cases} \\ &= \begin{cases} 1 & \text{if } d_G(v_i, v_j) = 2 \\ 0 & \text{otherwise} \end{cases} \\ &= (i, j)^{\text{th}} \text{ entry of } A_2(G). \end{aligned}$$

□

Proposition 5.2. *Let G be a simple connected graph with adjacency matrix $A(G)$. Then, $A(G^k) - A(G^{k-1}) = A_k(G)$, Where $A(G^k)$ is the adjacency matrix of G^k .*

In the light of the proposition 5.2, proposition 4.3 can be restated as

Proposition 5.3. *Let G_1 and G_2 be two simple connected graphs of order n and m respectively. Then,*

$$A((G_1[G_2])_k) = A(G_1^k) \otimes J_m - A(G_1^{k-1}) \otimes J_m \oplus I_n \otimes A(G_2^k) - I_n \otimes A(G_2^{k-1}).$$

Proof. It can be proved by using propositions 4.3 and 5.2. □

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