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MINIMIZATION OF TOTAL WAITING TIME OF JOBS IN SPECIALLY STRUCTURED TWO STAGE FLOWSHOP SCHEDULING INCLUDING TRANSPORTATION TIME WITH DISJOINT JOB BLOCK CRITERIA

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ABSTRACT. The present paper is aimed to provide algorithm for minimizing the total waiting time of jobs for specially structured two stage flowshop scheduling. The model includes the transportation time with disjoint job block criteria. The algorithm is made clear by numerical illustration. The lemma has been provided on which study is based.

1. INTRODUCTION

Scheduling means to obtain a sequence of jobs for a set of machines such that certain performance measures are optimized. Scheduling is generally considered to be one of the most important issues in the planning and operation of a manufacturing system. Better scheduling system has vital influence in decreasing the cost, increasing the output, client contentment and on the whole provides competitive assistance to the organization. Manufacturing units and service centers play an important part in the economic growth of nation. Productivity can be increased if existing assets are used in an optimal method. In the routine working of production houses and service providers numerous applied and experimental situations exist relating to flow shop scheduling. In today's manufacturing and distribution systems, scheduling have significant role

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to meet customer requirements as quickly as possible while maximizing the profits. In a flow shop scheduling problem n-jobs are processed on m-machines and the processing order i.e. the order in which various machines are required for completing the job is given. The common objectives in flow shop scheduling problems are to minimize some performance measures such as make-span, mean flow time, mean tardiness, mean setup time, number of tardy jobs and mean number of setups.

2. Preliminaries

In order to find optimal sequence of jobs the fundamental study was made by Johnson [1] using heuristic approach for n jobs 2 and restrictive case 3 machines flow-shop scheduling . Ignall and Schrage [2] developed branch and bound algorithms for the permutation flow-shop problem minimizing make-span . Lockett et.al. [3] studied sequencing problems which involves sequence dependent changeover times. Maggu and Das et. al. [4] introduced the equivalent job concept for job block in scheduling problems. Singh T.P. [5] extended the study by introducing various parameters like transportation time, break down interval, weightage of jobs etc. The work was further extended by Gupta J.N.D. [6], Rajendran C. et.al. [7], Singh T.P. et.al. [8] considering criteria other than make-span. Further Singh T.P., Gupta D. et.al. [9]- [10] made an attempt to minimize the rental cost of machines including job block through simple heuristic approach. Gupta D. and Bharat Goyal [11] studied specially structured two stage Flow Shop scheduling models with the objective to optimize the total waiting time of jobs. This paper is an extension of study done by Gupta D. and Bharat Goyal [12] in the sense job block concept is taken into consideration. The concept of job block is significant in scheduling systems where certain orderings of jobs are prescribed either by technological constraints or by externally imposed policy. The basic concept of equivalent job for job block in job sequencing was investigated by Maggu, P. L. and Das, G. [13]. The string of disjoint job blocks consist of two disjoint job blocks such that in one job block the order of jobs is fixed and in second job block the order of jobs is arbitrary. Heydari [14] studied flow shop scheduling problem with processing of jobs in a string of disjoint job blocks. Singh T.P., Kumar, R. and Gupta, D. [15] studied $n \times 2$ flow-shop scheduling problem in which processing time, set up time each associated with probabilities along with jobs in a string of disjoint job blocks. The objective of the study is to obtain an optimal sequence of jobs to minimize the total waiting time of the machines. An algorithm is proposed to solve the problem and is validated with the help of a numerical example.

3. Practical Situation

Manufacturing units/industries play a momentous role in the economic progress of a country. Flow shop scheduling occurs in various offices, service stations, banks, airports etc. In our routine working in industrial and manufacturing units diverse jobs are practiced on a variety of machines. In textile industry different types of fabric is produced using different types of yarn. Here, the maximum equivalent time taken in dying of yarn on first machine is always less than or equal to the minimum equivalent time taken in weaving of yarn on the second machine. Flowshop scheduling occurs in various offices, service stations, airports etc. Routine working in industries and factories have diverse jobs which are to be processed on various machines.Sometimes the manufacturer has a minimum time contract with the customers to complete their job. This condition leads to enquire about the best way to schedule the task so that waiting times for the jobs are reduced and greater satisfaction is achieved. The idea of minimizing the waiting time may be a reasonable aspect from managers of factories/ industries perspective when he has minimum time bond with a profit making party to complete the jobs.

Lemma 3.1. *Let p jobs 1,2,3,...,p are to be processed through two machines M and N in the order MN with no passing allowed. Let job* $i(i = 1, 2, 3, \ldots, p)$ *has processing times* M_i *and*N_i *on each machine respectively assuming their respective probabilities* s_i *and* t_i *such that* $0 \leq s_i \leq 1$;$0 \leq t_i \leq 1$*and*$\sum s_i = \sum t_i = 1$. *Expected processing times are defined as* $M_i' = M_i * s_i$ *and* $N_i' = Ni * t_i$ *satisfying expected processing times structural relationships:*

$$MaxM_i' \leq MinN_i',$$

then for the p job sequence S: $\alpha_1, \alpha_2, \ldots, \alpha_p$,

$$T_{\alpha_p N} = M_{\alpha_1}' + N_{\alpha_2}' + N_{\alpha_2}' + \ldots + N_{\alpha_p}',$$

where $T_{\alpha N}$ *is the completion time of job* α *on machine N.*

Proof. Applying mathematical induction hypothesis on p.

Let the statement S(p):

$$T_{\alpha_p N} = M'_{\alpha_1} + N'_{\alpha_2} + N'_{\alpha_2} + \ldots + N'_{\alpha_p},$$
$$T_{\alpha_1 M} = M'_{\alpha_1}$$
$$T_{\alpha_1 N} = M'_{\alpha_1} + N'_{\alpha_1}.$$

Hence for $p = 1$ the statement S(1) is true.

Let for $p = k$, the statement S(k) be true, i.e.,

$$T_{\alpha_k N} = M'_{\alpha_1} + N'_{\alpha_2} + N'_{\alpha_2} + \ldots + N'_{\alpha_k}.$$

Now,

$$T_{\alpha_{k+1} N} = Max(T_{\alpha_{k+1} M}, T_{\alpha_k N}) + N'_{\alpha_{k+1}},$$
$$= Max(M'_{\alpha_1} + M'_{\alpha_2} + \ldots + M'_{\alpha_{k+1}}, M'_{\alpha_1} + N'_{\alpha_1} + N'_{\alpha_2} + \ldots + N'_{\alpha_k}) + N'_{\alpha_{k+1}},$$

as $Max M'_i \leq Min_{N_i}$. Hence,

$$T_{\alpha_{k+1} N} = M'_{\alpha_1} + N'_{\alpha_1} + N'_{\alpha_2} + \ldots + N'_{\alpha_k} + N'_{\alpha_{k+1}},$$

and further, for $p = k + 1$ the statement S(k+1) holds true. Since S(p) is true for $p = 1, p = k, p = k + 1$ and k being arbitrary.

Therefore, S(p): $T_{\alpha_p N} = M'_{\alpha_1} + N'_{\alpha_2} + N'_{\alpha_2} + \ldots + N'_{\alpha_p}$ is true. □

Lemma 3.2. *With the same notations as that of Lemma 3.1 , for p job sequence S:* $\alpha_1, \alpha_2, \ldots, \alpha_p$:

$$W_{\alpha_1} = 0,$$
$$W_{\alpha_k} = M'_{\alpha_1} + \sum_{r=1}^{k-1} x_{\alpha_r} - M'_{\alpha_k},$$

where W_{α_k} *is the waiting time of job* α_k *for the sequence(* $\alpha_1, \alpha_2, \ldots, \alpha_p$*) and* $x_{\alpha_r} = N'_{\alpha_r} - M'_{\alpha_r}, \alpha_r \in (1, 2, 3, \ldots, p)$.

Proof.

$$W_{\alpha_1} = 0$$
$$\begin{aligned} W_{\alpha_k} &= Max(T_{\alpha_k M}, T_{\alpha_{k-1} N}) - T_{\alpha k M} \\ &= Max(M'_{\alpha_1} + M'_{\alpha_2} \ldots + M'_{\alpha_k}, M'_{\alpha_1} + N'_{\alpha 1} + N'_{\alpha_2} \ldots + N'_{\alpha_{k-1}}) \\ &\quad -(M'_{\alpha_1} + M'_{\alpha_2} \ldots + M'_{\alpha_k}) \\ &= M'_{\alpha_1} + N'_{\alpha 1} + N'_{\alpha_2} \ldots + N'_{\alpha_{k-1}} - M'_{\alpha_1} - M'_{\alpha_2} \ldots - M'_{\alpha_k} \\ &= M'_{\alpha_1} + (N'_{\alpha 1} - M'_{\alpha_1}) + (N'_{\alpha_2} - M'_{\alpha_2}) + \ldots (N'_{\alpha_{k-1}} - M'_{\alpha_{k-1}}) - M'_{\alpha_k} \\ &= M'_{\alpha_1} + \sum_{r=1}^{k-1} (N'_{\alpha r} - M'_{\alpha_r}) - M'_{\alpha_k} \\ &= M'_{\alpha_1} + \sum_{r=1}^{k-1} x_{\alpha_r} - M'_{\alpha_k} \end{aligned}$$

□

Theorem 3.1. *Let p jobs 1,2,3,. . . ,p are to be processed through two machines M and N in the order MN with no passing allowed. Let job i*$(i = 1, 2, 3, \ldots, p)$ *has processing times* M_i *and*N_i *on each machine respectively assuming their respective probabilities* s_i *and* t_i *such that* $0 \leq s_i \leq 1$;$0 \leq t_i \leq 1$*and*$\sum s_i = \sum t_i = 1$. *Expected processing times are defined as* $M_i' = M_i * s_i$ *and* $N_i' = Ni * t_i$ *satisfying expected processing times structural relationships:*

$$MaxM_i' \leq MinN_i',$$

then for the p job sequence S:$\alpha_1, \alpha_2, \ldots, \alpha_p$ *the total waiting time* T_w*(say)*

$$T_w = pM'_{\alpha_1} + \sum_{r=1}^{p-1} z_{\alpha_r} - \sum_{i=1}^{p} M_i',$$

$$z_{\alpha_r} = (p - r)x_{\alpha_r}; \alpha_r \in (1, 2, 3, ..p).$$

Proof. From Lemma 3.2 we have,

$$\begin{aligned}
W_{\alpha_1} &= 0 \\
k &= 2, k - 1 = 1 \\
W_{\alpha_2} &= M'_{\alpha_1} + \sum_{r=1}^{1} x_{\alpha_r} - M'_{\alpha_2} \\
&= M'_{\alpha_1} + x_{\alpha_1} - M'_{\alpha_2} \\
k &= 3, k - 1 = 2 \\
W_{\alpha_3} &= M'_{\alpha_1} + \sum_{r=1}^{2} x_{\alpha_r} - M'_{\alpha_3} \\
&= M'_{\alpha_1} + x_{\alpha_1} + x_{\alpha_2} - M'_{\alpha_3}.
\end{aligned}$$

Continuing in this way

$$\begin{aligned}
k &= p, k - 1 = p - 1 \\
W_{\alpha_p} &= M'_{\alpha_1} + \sum_{r=1}^{p-1} x_{\alpha_r} - M'_{\alpha_P} \\
&= M'_{\alpha_1} + x_{\alpha_1} + x_{\alpha_2} + \ldots + x_{\alpha_{p-1}} - M'_{\alpha_p}.
\end{aligned}$$

Hence, total waiting time

$$T_w = W_{\alpha_1} + W_{\alpha_2} + W_{\alpha_3} + \ldots + W_{\alpha_p}$$
$$T_w = 0 + (M'_{\alpha_1} + x_{\alpha_1} - M'_{\alpha_2}) + (M'_{\alpha_1} + x_{\alpha_1} + x_{\alpha_2} - M'_{\alpha_3})$$
$$+ \ldots + (M'_{\alpha_1} + x_{\alpha_1} + x_{\alpha_2} + \ldots + x_{\alpha_{p-1}} - M'_{\alpha_p})$$
$$T_w = (M'_{\alpha_1} + M'_{\alpha_1} + \ldots + (p-1)times) + (x_{\alpha_1} + x_{\alpha_1} + \ldots + (p-1)times)$$
$$+ (x_{\alpha_2} + x_{\alpha_2} + \ldots + (p-2)times)$$
$$+ \ldots + (x_{\alpha_{p-1}} - (M'_{\alpha_2} + M'_{\alpha_3} + \ldots + M'_{\alpha_p}))$$
$$T_w = (p-1)M'_{\alpha_1} + (p-1)(x_{\alpha_1} + (p-2)x_{\alpha_2} + \ldots + x_{\alpha_{p-1}} - (\sum_{i=1}^{p} M'_{\alpha_i} - M'_{\alpha_1})$$
$$= pM'_{\alpha_1} + \sum_{r=1}^{p-1}(p-r)x_{\alpha_r} - \sum_{i=1}^{p} M'_{\alpha_i}$$

□

Equivalent Job Block Theorem: In processing a schedule s=(1,2,3,...,p) of p jobs on two machines M and N in the order MN with no passing allowed. A job i (i=1,2,3,...,p) has processing time M_i and N_i on each machine respectively. The job block(k,m) is equivalent to the single job α. Now the processing times of job α on the machine M and N are denoted respectively by M_α, N_α are given by

$$M_\alpha = M_k - M_m - \min(M_m, N_k)$$
$$N_\alpha = N_k - N_m - \min(M_m, N_k)$$

The proof of the theorem is given by Maggu P.L. and Das G [13].

4. Problem formulation

Let n jobs are to be processed through two machines A and B in order AB. Let a_k and b_k denotes the processing time for k^{th} job on these machines. Let t_k be the transportation time of k^{th} job from machine A to machine B. Let two job blocks be α and βsuch that block α consists of i jobs out of n jobs in which the order of jobs is fixed and α consists of r jobs out of n in which order of jobs is arbitrary such that $i + r = n$. Let $\alpha \cap \beta = \phi$ i.e. the two job blocks α and β form a disjoint set in the sense that the two job blocks have no job in common. Also we consider the structural relationship i.e. $Max a_k \leq Min b_k$ holds good.

TABLE 1. Matrix form of the problem

Jobs	Machine A	Transportation Time	Machine B
J	a_k	t_k	b_k
1	a_1	t_1	b_1
2	a_2	t_2	b_2
3	a_3	t_3	b_3
⋮	⋮	⋮	⋮
n	a_n	t_n	b_n

ASSUMPTIONS

In the given flowshop scheduling the following assumptions are made

1) There are n number of jobs (J) and two machines(A and B)

2) Time intervals for processing time are independent of the order in which operations are performed.

3) A job is an entity i.e. even though the job represents a lot of individual part, no job may be processed by more than one machine at a time.

4) Each operation once started must performed till completion.

ALGORITHM

Step 1: Define the fictitious machines X and Y with processing times X_k and Y_k as follows:

$$X_k = a_k + t_k \text{ and } Y_k = b_k + t_k$$

and verify the structural relationship $Max X_k \leq Min Y_k$.

Step 2: Take equivalent job $\alpha = (r, m)$ and and calculate the processing times A_{α_1} and A_{α_2} on the guidelines of Maggu and Das [13] as follows:

$$X_{\alpha_1} = X_{r_1} + X_{m_1} - \min(X_{m_1}, X_{r_2})$$

$$Y_{\alpha_2} = Y_{r_2} + Y_{m_2} - \min(X_{m_1}, X_{r_2}).$$

If a job block has three or more than three jobs then to find the expected flow times we use the property that the equivalent job for job-block is associative i.e. $((i_1, i_2), i_3) = (i_1, (i_2, i_3))$.

Step 3: Obtain the new job block from the job block (disjoint from job block) by the proposed algorithm. Obtain the processing times and as defined in step 2.

Step 4: Now, reduce the given problem to a new problem by replacing s-jobs by job block α with the processing times X_{α_1} and X_{α_2} and remaining $r{=}(n-s)$ jobs by a disjoint job block β_k with processing times and as defined in step 2. Form the table in the following format.

TABLE 2

Jobs (J)	Machine X (X_k)	Machine Y (Y_k)	$x_k = Y_k - X_k$
α	X_{α_1}	Y_{α_1}	x_1
β_k	$X_{\beta_{k_2}}$	$Y_{\beta_{k_2}}$	x_2

Step 5: Arrange the jobs in increasing order of x_k. Let the sequence be

$$(\mu_1, \mu_2, \ldots, \mu_n).$$

Step 6: Find MinX_k. Now two cases arise:
a) If $X_{\mu_1} = MinX_k$, then schedule according to step 5 is required optimal sequence.
b) If $X_{\mu_1} \neq MinX_k$, then go to next step.

Step 7: Consider the different sequences of jobs $S_1, S_2, \ldots, S_r$ where S_1 is the sequence obtained in step 5 , sequence $S_k (k = 2, 3, \ldots, r)$ can be obtained by placing $k^t h$ job in the sequence S_1 to the first position and rest of the sequence remaining same.

Step 8: Form the table in the following format:

Step 8: Calculate the waiting time T_w for all the sequences $S_1, S_2, \ldots, S_r$ using the formula:

$$T_w = nX_{\mu_1} + \sum_{r=1}^{n-1} Z_{a_r} - \sum_{k=1}^{n} X_k,$$

where

X_{μ_1} =Equivalent processing time of first job on machine X in sequence S_k,

TABLE 3

Jobs	Machine X	Machine Y	$z_{k_r} = (n-r)x_k$				
(J)	(X_k)	(Y_k)	$x_k = Y_k - X_k$	$r=1$	$r=2$	$\ldots$	$r=(n-1)$
1	X_1	Y_1	x_1	z_{11}	z_{12}	$\ldots$	$z_{1(n-1)}$
2	X_2	Y_2	x_2	z_{21}	z_{22}	$\ldots$	$z_{2(n-1)}$
3	X_3	Y_3	x_3	z_{31}	z_{32}	$\ldots$	$z_{3(n-1)}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
n	X_n	Y_n	x_n	z_{n_1}	z_{n_2}	$\ldots$	$z_{n(n-1)}$

$Z_{a_r} = (n-r)x_{a_r}$;$a = \mu_1, \mu_2, \ldots, \mu_n$.
The sequence with minimum waiting time is required optimal sequence.

5. NUMERICAL ILLUSTRATION

Assume 5 jobs 1,2,3,4,5 are to be processed on two machines A and B with processing times a_k, b_k and t_k is the transportation time of k^th job from machine A to machine B.

TABLE 4

Jobs	Machine A	Transportation Time	Machine B
J	a_k	t_k	b_k
1	6	5	16
2	9	4	22
3	13	3	25
4	12	5	20
5	15	3	24

Our propose is to achieve a most favourable schedule,minimizing the total waiting time for the jobs.

As per step 1- Defining the fictitious machines X and Y with processing times $X_k = a_k + t_k$ and $Y_k = b_k + t_k$ respectively.

TABLE 5

Jobs J	Machine X X_k	Machine Y Y_k
1	11	21
2	13	26
3	16	28
4	17	25
5	18	27

Max$X_k = 18$ ≤Min$Y_k = 21$

As per step 3: Take equivalent job $\alpha = (2,5)$. Then processing times are defined as follows
$X_\alpha = X_2 + X_5 - Min(X_5 - Y_2) = 13$ and $Y_\alpha = Y_2 + Y_5 - Min(X_5 - Y_2) = 35$
As per step 4: Taking new job block β= (1,3,4) or (γ,4) where γ = (1,3). Then processing times are defined as described in step 2. And forming a table in following format-

TABLE 6

Jobs J	Machine X X_k	Machine Y Y_k	$x_k = Y_k - X_k$
β	11	41	30
α	13	35	22

As per step 5- Arrange the jobs in increasing order of x_k i.e. the sequence found to be 4,1,α,3

TABLE 7

Jobs (J)	Machine X (X_k)	Machine Y (Y_k)	$x_k = Y_k - X_k$
α	13	35	22
β	11	41	30

As per step 6- Min$X_k = 11 \neq 13$

As per step 7- The sequences obtained are

$$S_1 = (\alpha, \beta) = (2, 5, 1, 3, 4)$$
$$S_2 = (\beta, \alpha) = (1, 3, 4, 2, 5)$$

As per step 8- Fill the values in the following table

TABLE 8

Jobs (J)	Machine X (X_k)	Machine Y (Y_k)	$z_{kr} = (n-r)x_k$				
			$x_k = Y_k - X_k$	$r=1$	$r=2$	$r=3$	$r=4$
1	11	21	10	40	30	20	10
2	13	26	13	52	39	26	13
3	16	28	12	48	36	24	12
4	17	25	8	32	24	16	8
5	18	27	9	36	27	18	9

As per step 9- Calculate the total waiting time for the sequences S_1, S_2.

$$\sum_{k=1}^{n} X_k = 75$$

For the sequence $S_1 = (\alpha, \beta) = (2, 5, 1, 3, 4)$

Total waiting time

$$T_w = 5 \times 13 + 52 + 27 + 20 + 12 - 75 = 101.$$

For the sequence $S_2 = (\beta, \alpha) = (1, 3, 4, 2, 5)$

Total waiting time $T_w = 5 \times 11 + 40 + 36 + 16 + 13 - 75 = 85.$

Hence the sequence $S_2 = (1, 3, 4, 2, 5)$ is the required sequence with minimum total waiting time.

6. CONCLUSION

The present study deals with the flowshop scheduling model with the main idea to optimize the total waiting time of jobs. However, it may increase the other costs like machine idle cost or penality cost of the jobs, yet the idea of minimizing the waiting time is a matter that cannot be avoided in the cases when there is a minimum time contract with the customers. The study can be extended by introducing various parameters like weightage of jobs , setup time of machines, breakdown interval of machines etc.

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