NO IDLE SCHEDULING WITH FUZZY APPROACH

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ABSTRACT. In this paper we consider two machines flow shop scheduling which minimizes the total elapsed time and hence reduces the rental cost. We have used no idle scheduling where idle time is not allowed on machines. Here the processing time of jobs are uncertain, that is not known exactly and only estimated values are given. This leads to the use of fuzzy numbers for representing these imprecise values.

1. INTRODUCTION

In flow shop, many situations exist when it becomes necessary that some components perform uninterruptedly. These are real situations which arises in practice when costly machines are used or when specific machines cannot be started and stopped easily due to technological constraints. In this case we use no-idle scheduling where machines are required to run with no inserted idle time. To fulfill the restriction,we delay the operation of first job on second machine. In flow shop problems, processing time of jobs are usually given exactly.But as we see in our real life, we have many situations where exact operating time is not given in advance, we are given uncertain time.To handle such uncertainities fuzzy numbers play an important role. To solve scheduling problems with uncertainities many authors have used different fuzzy numbers. It was Zadeh who first introduced fuzzy sets to represent impreciseness by inducing fuzzy set

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theory [1]. Machon and Lee [2] solved problems of fuzzy processing time using mean values. Deepak Gupta et al. [3] worked on two machine flow shop problem with processing and setup times given in fuzzy. Gupta D. , Goel R. [5]. worked on minimizing hiring cost under no idle constraints in two machines problem. In this paper the work done by Gupta D., Goel R. is extended by using the concept of uncertainity in processing time with no idle constraints.

2. PRACTICAL SITUATION

Various practical situations occur in real life when one has got the assignment but does not have one's own machine or does not have enough money or does not want to take risk of investing huge amount of money to purchase machine. Under such circumstances, the machine has to be taken on rent in order to complete the assignments. In his starting career, we find a medical practitioner does not buy expensive machines say X-ray machine, the Ultra Sound Machine etc., but instead takes on rent. Rental of medical equipment is an affordable and quick solution for hospitals, nursing homes etc., which are presently constrained by the availability of limited funds due to the recent global economic recession.

3. BASIC FUZZY THEORY

A fuzzy set $A = (p, q, r)$ which we define on real numbers U, where U is universal set, is a fuzzy number if its characteristic function $\eta(x)$ satisfies the following properties:

(i) $\eta(x) : U \to [0, 1]$ is continuous.
(ii) $\eta(x) = 0$ for all x $\epsilon(-\infty, p) \cup (r, \infty)$.
(iii) $\eta(x)$ is strictly increasing on [p, q] and strictly decreasing on [q, r].
(iv) $\eta(x) = 1$ for $x = q$.

4. MEMBERSHIP FUNCTION

In fuzzy set membership functions describe all informations. The fuzzy processing times are represented by triangular membership functions. The membership value of x denoted by $\eta(x)$, $x \epsilon U^+$, can be calculated using the formula.

$$\eta(x) = \begin{cases} 0; & x \leq p \\ \frac{x-p}{q-p}; & p \leq x \leq q \\ \frac{r-x}{r-q}; & q \prec x \prec r \\ \eta(x) = 0; & x \geq r \end{cases}.$$

4.1. **Average high ranking $\prec$A.H.R.$\succ$.** Yager's [4] average high ranking formula is used to calculate job processing times by

$$H(a) = \frac{3q + r - p}{3}.$$

4.2. **Arithmetic operations in fuzzy.** If $A_1 = (m_{A1}, \alpha_{A1}, \beta_{A1})$ and $A_2 = (m_{A2}, \alpha_{A2}, \beta_{A2})$ are two triangular fuzzy numbers, then

(1) $A_1 + A_2 = (m_{A1}, \alpha_{A1}, \beta_{A1}) + (m_{A2}, \alpha_{A2}, \beta_{A2}) = (m_{A1} + m_{A2}, \alpha_{A1} + \alpha_{A2}, \beta_{A1} + \beta_{A2})$

(2) $A_1 - A_2 = (m_{A1}, \alpha_{A1}, \beta_{A1}) - (m_{A2}, \alpha_{A2}, \beta_{A2}) = (m_{A1} - m_{A2}, \alpha_{A1} + \beta_{A2}, \alpha_{A2} - \beta_{A1})$

(3) $kA_1 = k(m_{A1}, \alpha_{A1}, \beta_{A1}) = (km_{A1}, k\alpha_{A1}, k\beta_{A1}); if k \succ 0$

(4) $kA_1 = k(m_{A1}, \alpha_{A1}, \beta_{A1}) = (k\beta_{A1}, k\alpha_{A1}, km_{A1}); if k \prec 0.$

5. NOTATIONS

- S : Job sequence $1, 2, 3, \ldots, n$
- S_1 : Optimum sequence which is obtained using technique of Johnson's.
- M_j : Machines, j= 1,2
- t_{ij} : uncer tain processing time of ith job on jth machine $i = 1, 2, 3, \ldots n$; $j = 1, 2$
- A_{ij} : A.H.R or crisp value of processing time of ith job on jth machine.
- $U_j(S_1)$: Utilization time of machines; j=1,2.
- $R(S_1)$: Total hiring cost of machines for the sequence S_1
- C_i : Hiring cost of ith machine.

6. Rental policy P

In the present study, the rental policy is used that first machine will be leased at the beginning of processing of first job and returned after the completion of last job on it and machine two shall only be hired for a period equal to amount of processing time of all jobs there on and is returned when the last job is done on it.

7. Formulation of the problem

Let some jobs $k = 1, 2, 3, \ldots, n$ be processed via machines M_1 and M_2 in a given order i.e. first on machine M_1 and next on machine M_2. Let a job k (k = 1, 2, 3,..., n) has fuzzy dispensation time u_{k1} and u_{k2} on machines M_1 and M_2 accordingly. Let C_1 and C_2 be the rent charges for per unit time for machine M_1 and M_2 respectively. Our objective is to minimize the rental cost.
The matrix form of the paper can be shown as:

Jobs	MachineM_1	MachineM_2
K	μ_{k1}	μ_{k2}
1	μ_{11}	μ_{12}
2	μ_{21}	μ_{22}
3	μ_{31}	μ_{32}
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n	μ_{n1}	μ_{n2}

7.1. **Assumptions.**

(i) jobs are operated via two available machines M_1 and M_2 in the order M_1 M_2 i.e. no passing is permitted.
(ii) Jobs are independent of each other.
(iii) The job sequence on the two machines must be identical.
(iv) Two operations of same job may not be parallel processed.
(v) During scheduling cycle machines never breakdown.

8. RESEARCH METHODOLOGY

To minimize the rental cost of two machines in which no idle constraint is used and processing times are uncertain and represented by triangular fuzzy numbers, the following algorithm is proposed.

Step 1: Compute average high ranking of the processing times for all the jobs on two machines M_1 and M_2 using Yager's formula.

Step 2: Apply Johnson's technique to attain optimum sequence S_1.

Step 3: Obtain the total elapsed time by preparing the in out table for optimum sequence S_1 obtained in step 2 .

Step 4: Find the latest time H_2 to hire second machine by H_2=total elapsed time minus sum of processing time of all jobs on second machine M_2.

Step 5: Take H_2 as starting time to start processing on machine M_2 and get the elapsed time T_2.

Step 6: Calculate utilization time $U_1 (S_1)$ and $U_2 (S_1)$ of machines by

$$U_1(S_1) = \sum_{k=1}^{n} u_{k1},$$
$$U_2(S_1) = T_2 - H_2.$$

Step 7: Obtain
$$R(S_1) = U_1(S_1) * C_1 + U_2(S_1) * C_2.$$

9. NUMERICAL EXAMPLE

Optimize rental cost for 4 jobs and 2 machine problem where the processing times are calculated by triangular fuzzy numbers. The rental costs for per unit time of machines M_1 and M_2 are 6 units and 5 units respectively.

TABLE 1. Fuzzy processing times of jobs

Jobs	Machine M_1	Machine M_2
K	μ_{k1}	μ_{k2}
1	$(14, 15, 20)$	$(2, 4, 14)$
2	$(15, 17, 20)$	$(7, 8, 12)$
3	$(12, 14, 17)$	$(4, 6, 13)$
4	$(7, 9, 29)$	$(6, 9, 16)$

Solution: The A.H.R of the jobs with fuzzy processing times is as follows:

TABLE 2. Jobs with average high ranking

Jobs	Machine M_1	Machine M_2
K	A_{k1}	A_{k2}
1	17	8
2	18.67	9.67
3	15.67	9
4	16.33	12.33

Using the Johnson's technique, we have the optimal sequence $S_1 - 4, 2, 3, 1$:

TABLE 3. Flow in-out table for the optimal sequence

Jobs	Machine M_1	Machine M_2
K	In - Out	In - Out
4	$(0, 0, 0) - (7, 9, 29)$	$(7, 9, 29) - (13, 18, 45)$
2	$(7, 9, 29) - (22, 26, 49)$	$(22, 26, 49) - (29, 34, 61)$
3	$(22, 26, 49) - (34, 40, 66)$	$(34, 40, 66) - (38, 46, 79)$
1	$(34, 40, 66) - (48, 55, 86)$	$(48, 55, 86) - (50, 59, 100)$

The total elapsed time, C T (S_1) = (50, 59, 100).

Latest time to hire second machine H_2 = (50, 59,100) - (19, 27, 55) = (31, 32, 45).

Prepare the in-out table with H_2 as time of starting for second machine and we get the idle time equal to zero.

TABLE 4. Flow table for M_2 with H_2 as starting time

Jobs	Machine M_1	Machine M_2
K	In - Out	In - Out
4	$(0,0,0) - (7,9,29)$	$(7,9,29) - (37,41,61)$
2	$(7,9,29) - (22,26,49)$	$(37,41,61) - (44,49,73)$
3	$(22,26,49) - (34,40,66)$	$(44,49,73) - (48,55,86)$
1	$(34,40,66) - (48,55,86)$	$(48,55,86) - (50,59,100)$

The total elapsed time $T_2 = (50.59, 100)$.

Utilization time for $M_2, U_2(S_1) = (50,59,100) - (31,32,45) = (19,27,55)$.

Finally calculate R (S_1), total hiring cost for the optimal sequence S_1 as given in step 7:

$$R(S_1) = 6(48,55,86) + 5(19,27,55) = (383,465,791) = 601 \text{ units}.$$

10. CONCLUSION

The methodolgy applied in this study for no idle two stage flow shop scheduling problem is less time consuming. It helps to find an optimal sequence which makes idle time zero on second machine and hence minimizes the hiring cost.

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