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OPERATIONS ON INTUITIONISTIC FUZZY L-RING IDEALS

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ABSTRACT. In this paper, some notions meet in intuitionistic fuzzy L-ring ideals are introduced and their propositionerties are discussed.

1. INTRODUCTION

Intuitionistic fuzzy set is an buliding of fuzzy set, done by K. T. Atanassov, [2] a bulgarian engineer. He introduce a new element which determine the degree of non-membership in essential intutionistic fuzzy subset. The lattice of fuzzy ideals of a ring is discussed in [1]. Fuzzy set to Lattice ordered ring is developed in [4]. Now operations on Intuitionistic fuzzy L-ring ideals are established.

2. PRELIMINARIES

Definition 2.1. *Let μ be any fuzzy subset of a set X and t $\in$ Imμ. After that the set* μ_t= $\{ x \in X/ \mu(x) \geq t\}$ *is called the level subset of* μ.

Obviously $\mu_t \subseteq \mu_s$ whenever t>s.

Definition 2.2. *Let μ be any fuzzy subset of a set X and μ= (x_i, t_i)/i=1 to n and $t_i \in$ [0, 1]. Then $\{t_i$ = i/ i = 1 to n$\}$ is called the image set of μ and is denoted by Im μ.*

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Definition 2.3. *[5] An intuitionistic fuzzy subset A of an L-ring R is called a intuitionistic fuzzy L-ring ideal or intuitionistic fuzzy L-ideal of R if for all x, y $\in$ R.*

(i) $\mu(x \vee y) \geq min\ \{\mu(x)\ ,\ \mu(y)\}$.
(ii) $\mu(x \wedge y) \geq max\ \{\mu(x)\ ,\ \mu(y)\}$.
(iii) $\mu(x - y) \geq min\ \{\mu(x),\ \mu(y)\}$.
(iv) $\mu(xy) \geq max\ \{\mu(x), \mu(y)\}$.
(v) $\nu(x \vee y) \leq max\ \{\nu(x)\ ,\ \nu(y)\}$.
(vi) $\nu(x \wedge y) \leq min\ \{\nu(x)\ ,\ \nu(y)\}$.
(vii) $\nu(x - y) \leq max\ \{\nu(x)\ ,\ \nu(y)\}$.)
(viii) $\nu(xy) \leq min\ \{\nu(x), \nu(y)\}$.

Definition 2.4. *Let A be an Intuitionistic fuzzy set on a set X.*
The (α, β)-cut of A is a crisp subset. $A_{\alpha,\beta} = \{\ x \in X/\ \mu_A(x) \geq \alpha$ *and* $\nu_A(x) \leq \beta\ \}$ *where* α, $\beta \in [0, 1]$ *with* $\alpha + \beta \leq 1$.

Definition 2.5. *Let A be an intuitionistic fuzzy set on a set X. The Support of A is the crisp subset on X given by:*

$$Supp(A) = \{x \in X/\mu_A(x) \text{ and } \nu_A(x) \text{ is less between } 0 \text{ and } 1\}\,.$$

3. Operations on Intuitionistic fuzzy L-ring ideals

Definition 3.1. *Let M, N be any two intuitionistic fuzzy L-ideals of an L-ring R.*
$M=\{\ r,\ \mu_M(r), \nu_M(r)/r \in R\}$ *and* $N=\{\ r,\ \mu_N(r), \nu_N(r)/r \in R\}$.

(i) *Sum*$(\mu_M + \mu_N)$ *of* μ_M, μ_N *and* ν_M, ν_N *is defined by*
$(\mu_M + \mu_N)(r) = \sup_{r=p+q}\{min\{\mu_M(p), \mu_N(q)\}\}$
$(\nu_M + \nu_N)(r) = \inf_{r=p+q}\{max\{\nu_M(p), \nu_N(q)\}\}$, *where* $p,q,r \in R$.
(ii) *Product*$(\mu_M\ .\ \mu_N)$ *of* μ_M, μ_N *and* ν_M, ν_N *is defined by*
$(\mu_M\ .\ \mu_N)(r) = \sup_{r=pq}\{min\{\mu_A(p), \mu_B(q)\}\}$
$(\nu_M\ .\ \nu_N)(r) = \inf_{r=pq}\{max\{\nu_A(p), \nu_B(q)\}\}$, *where* $p,q,r \in R$.
(iii) *Join*$(\mu_M \vee \mu_N)$ *of* μ_M, μ_N *and* ν_M, ν_N *is defined by*
$(\mu_M \vee \mu_N)(r) = \sup_{r=p\vee q}\{min\{\mu_M(p), \mu_N(q)\}\}$
$(\nu_M \vee \nu_N)(r) = \inf_{r=p\vee q}\{max\{\nu_M(p), \nu_N(q)\}\}$, *where* $p,q,r \in R$.

(iv) *Meet*($\mu_M \wedge \mu_N$) *of* μ_M, μ_N *and* ν_M,ν_N *is defined by*
$(\mu_M \wedge \mu_N)(x) = \sup_{r=p\wedge q} \{min\{\mu_M(p),\mu_N(q)\}\}$
$(\nu_M \wedge \nu_N)(r) = \inf_{r=p\wedge q}\{max\{\nu_M(p),\nu_N(q)\}\}$, *where p,q,r* $\in R$.

Theorem 3.1. *If M and N are any two Intuitionistic fuzzy L-ideals of an L-ring R, then M + N is also an Intuitionistic fuzzy L-ideal of R.*

Proof. Assume that M and N are any two Intuitionistic fuzzy L-ideals of an L-ring R. Now consider the Intuitionistic fuzzy subset $(\mu_M + \mu_N)$ of R defined by
$(\mu_M + \mu_N)(x) = \sup_{r=p+q}\{\min\{\mu_M(p),\mu_N(r)\}\}$
$(\nu_M + \nu_N)(r) = \inf_{r=p+q}\{\max\{\nu_M(p),\nu_N(q)\}\}$, where p,q,r $\in R$.
We have to prove that $M + N$ is an Intuitionistic fuzzy L-ideal of R. This means that we have to prove: $(\mu_M + \mu_N)$, $(\nu_M + \nu_N)$ is an Intuitionistic fuzzy L-ideal of R.

According to [3], it is enough to prove that:
$(\mu_M + \mu_N)_t$ is an L-ideal of R,for all t $\in$ Im$(\mu_M + \mu_N)$
$(\nu_M + \nu_N)_n$ is an L-ideal of R,for all n $\in$ Im$(\nu_M + \nu_N$.)
Let t $\in$ Im$(\mu_M + \mu_N)$ be arbitrary. Then either t $\in$ Im μ_M or t $\in$ Im μ_N or both. Suppose t $\in$ Im μ_M. Then, μ_M and μ_N are Intuitionistic fuzzy L-ideals of R, μ_{M_t} be an L-ideal of R then there exists s $\in$ Im μ_N such that t $<$ s and μ_{N_s} is an L-ideal of R such $\mu_{N_s} = \mu_{M_t}$. $\mu_M(0) \geq$ t and $\mu_N(0) \geq$ s $>$ t for all t.

Similarly, we can prove min$\{\mu_M(0), \mu_N(0)\} \geq$ t for t $\in$ Im μ_N and for both. Now consider level subset $(\mu_M + \mu_N)_t$ of R. Clearly,

$$\begin{aligned}(\mu_M + \mu_N)(0) &= \sup_{0=u+v}\{min\{\mu_M(u), \mu_N(v)\}\} \ \ where \ \ u, v \ \in R. \\ &\geq t.\end{aligned}$$

for $0 \in (\mu_M + \mu_N)_t$.

Let x,y $\in (\mu_M + \mu_N)_t$ be arbitrary. Now, it is enough to prove

(i) r,p $\in (\mu_M + \mu_N)_t \Longrightarrow$ r $\vee$ p, r - p $\in (\mu_M + \mu_N)_t$.
(ii) r $\in (\mu_M + \mu_N)_t$, r$\in$ R such that x $\leq$ r $\Longrightarrow$ x $\in (\mu_M + \mu_N)_t$
(iii) r $\in (\mu_M + \mu_N)_t$, r$\in$ R $\Longrightarrow$ rx, xr $\in (\mu_M + \mu_N)_t$.

Here r, p $\in (\mu_M + \mu_N)_t$, and now

$$\begin{aligned}(\mu_M + \mu_N)(r) &= \sup_{r=u+v}\{min\{\mu_M(u), \mu_N(v)\}\} \ \ where \ \ u, v \ \in R. \\ &\geq t. \\ &\Longrightarrow \mu_M(r_1) \geq t \ \ and \ \ \mu_N(r_2) \geq t.\end{aligned}$$

Also,

$$\begin{aligned}(\mu_M + \mu_N)(p) &= \sup_{p=a+b}\{min\{\mu_M(a), \mu_N(b)\}\} \ \ where \ \ a, b \ \ \in R.\\ &\geq \ t.\end{aligned}$$

Let n $\in$ Im(ν_M + ν_N) be arbitrary. Suppose n $\in$ Im ν_M. As ν_M and ν_N are Intuitionistic fuzzy L-ideals of R, ν_{M_n} is an L-ideal of R then there exsits m $\in$ Im ν_N such that m $<$ n and ν_{N_m} is an L-ideal of R such that $\nu_{N_m} = \nu_{M_n}$.
$\Longrightarrow \nu_M(1) \leq$ n and $\nu_N(1) \leq$ m $<$ n for all n.
Similarly, we can prove max{ $\nu_M(1)$, $\nu_N(1)$ } $<$ n for n $\in$ Imν_N and for both.
Now consider the level subset (ν_M + ν_N)$_n$ of R. Clearly,

$$1 \in (\nu_M + \nu_N)_n.$$

Let r,p $\in$ (ν_M + ν_N)$_n$ be arbitrary. Now, to prove:

(i) r,p $\in$ (ν_M + ν_N)$_n$ $\Longrightarrow$ r $\vee$ p, r - p $\in$ (ν_M + ν_N)$_n$.
(ii) r $\in$ (ν_M + ν_N)$_n$, x $\in$ R such that x $\leq$ r $\Longrightarrow$ x $\in$ (ν_M + ν_N)$_n$
(iii) r $\in$ (ν_M + ν_N)$_n$, x $\in$ R $\Longrightarrow$ rx, xr $\in$ (ν_M + ν_N)$_n$.

Here r, p $\in$ (ν_M + ν_N)$_n$, hence $\nu_M(p_1) \ \leq \ n \ \ and \ \ \nu_N(p_2) \ \leq \ n.$

(i) Here r$_1$,p$_1$ $\in$ μ_{M_t} and μ_M is an intuitionistic fuzzy L-ideal of R.
$\Longrightarrow$ r$_1$ - p$_1$, r$_1$ $\vee$ p$_1$ $\in$ μ_{M_t}.
$\Longrightarrow$ μ_M(r$_1$ - p$_1$) $\geq$ t and μ_M(r$_1$ $\vee$ p$_1$) $\geq$ t. And

$$\begin{aligned}\mu_M(r_1 \vee p_2) &\geq t\\ \mu_M(r_1 \vee p) &= \mu_M(r_1 \vee (p_1 + p_2))\\ &\geq t\,.\end{aligned}$$

Again r$_2$,p$_2$ $\in$ μ_{N_t} and μ_N is an intuitionistic fuzzy L-ideal of R.
$\Longrightarrow$ r$_2$ - p$_2$, r$_2$ $\vee$ p$_2$ $\in$ μ_{N_t}.
$\Longrightarrow$ μ_N(r$_2$ - p$_2$) $\geq$ t and μ_N(r$_2$ $\vee$ p$_2$) $\geq$ t.
Hence $\mu_N(r_2 \vee p) \geq t$. Now,
r - p = (r$_1$ + r$_2$) - (p$_1$ + p$_2$) = (r$_1$ - p$_1$) + (r$_2$ - p$_2$).
r $\vee$ p = (r$_1$ + r$_2$) $\vee$ p = (r$_1$ $\vee$ p) + (r$_2$ $\vee$ p).
Then:

$$\begin{aligned}R_1 - P_1 &\in (\mu_M + \mu_N)_t\\ R_1 \vee P_1 &\in (\mu_M + \mu_N)_t\,.\end{aligned}$$

Here $r_1,p_1 \in \nu_{M_n}$ and ν_M is an intuitionistic fuzzy L-ideal of R.
Again $r_2,p_2 \in \nu_{N_n}$ and ν_N is an intuitionistic fuzzy L-ideal of R.

$$\begin{aligned} \nu_M(r_1 \vee p) &< n \\ \nu_N(r_2 \vee p) &< n \\ Finally\;\; R_1 - P_1 &\in (\nu_M + \nu_N)_n \\ R_1 \vee P_1 &\in (\nu_M + \nu_N)_n\,. \end{aligned}$$

(ii) Assume that x $\in$ R such that x $\leq$ r. Here x = r $\wedge$ x. Then,

$$\begin{aligned} (\mu_M + \mu_N)(x) &= {}_{x=c+d}^{sup}\{min\{\mu_M(c), \mu_N(d)\}\} \;\; where \;\; c, d \in R \\ &\geq t, \\ \text{Since } \; \mu_M(r_1 \wedge x) &\geq t. \\ \text{and } \; \mu_N(r_2 \wedge x) &\geq t. \\ \text{Hence} \qquad & x \in (\mu_M + \mu_N)_t. \end{aligned}$$

Also, assume that x $\in$ R such that x $\leq$ r. Here x = r $\wedge$ x. Then,

$$\begin{aligned} (\nu_M + \nu_N)(x) &\leq n, \\ \text{Since } \; \nu_M(r_1 \wedge x) &\leq n. \\ \text{Hence} \qquad & x \in (\nu_M + \nu_N)_n. \end{aligned}$$

(iii) Assume that r $\in$ R.
Here $xr = x(r_1 + r_2) = xr_1 + xr_2$ and then: $(\mu_M + \mu_N)(xr) \geq t$. Since $\mu_M(xr_1) \geq t$, and $\mu_N(xr_2) \geq t$, $xr \in (\mu_M + \mu_N)_t$.
Similarly, $rx \in (\mu_M + \mu_N)_t$. Also, assume that x $\in$ R. Here $xr = x(r_1 + r_2) = xr_1 + xr_2$ and then $(\nu_M + \nu_N)(xr) < n$. Since $\nu_M(xr_1) \leq n$, and $\nu_N(xr_2) < n$. $\Longrightarrow xr \in (\nu_M + \nu_N)_n$. Also $rx \in (\nu_M + \nu_N)_n$.
Hence $(\nu_M + \nu_N)_n$ is an L-ideal of R. Thus $(\mu_M + \mu_N)$, $(\nu_M + \nu_N)$ are an intuitionistic fuzzy L- ideal of R.

Hence M + N is an intuitionistic fuzzy L-ideal of R. □

Proposition 3.1. *If M is an intuitionistic fuzzy L-ideal of an L-ring R, then* $\mu_M + \mu_M = \mu_M$ *and* $\nu_M + \nu_M = \nu_M$.

Proof. Let r $\in$ R be arbitrary. Now

$$(\mu_M + \mu_M)(r) = \sup_{r=p+q}\{min\{\mu_M(p), \mu_M(q)\} \quad where \ p,q,r \in R$$

$$= \mu_M(r). \tag{3.1}$$

On the other hand if $r = c + d, \ c,d \in R,$ then

$$\mu_M(r) = \mu_M(c+d)$$

$$= (\mu_M + \mu_N)(r). \tag{3.2}$$

From (3.1) and (3.2) combined it follows $\mu_M + mu_M = \mu_M$.

Let r $\in$ R be arbitrary.

$$(\nu_M + \nu_M)(r) = \inf_{r=p+q}\{max\{\nu_M(p), \nu_M(q)\} \quad where \ p,q,r \in R$$

$$= \nu_M(r). \tag{3.3}$$

On the other hand, if $r = c + d, \ c,d \in R,$ then,

$$\nu_M(r) = \nu_M(c+d)$$

$$= (\nu_M + \nu_M)(r). \tag{3.4}$$

From (3.3) and (3.4) combined it follows $\nu_M + \nu_M = \nu_M$. □

Proposition 3.2. *Let M and N be any two Intuitionistic fuzzy L-ideals of an L-ring R. If $\mu_M \subseteq \mu_N$, $\nu_M \supseteq \nu_N$, then*

(i) $\mu_M \subseteq \mu_M + \mu_N$,
(ii) $\mu_M \subseteq \mu_M \vee \mu_N$,
(iii) $\mu_M \subseteq \mu_N \wedge \mu_N$.

Proof. Let r $\in$ R be arbitrary. Assume that $\mu_M \subseteq \mu_N$. Then, $\Longrightarrow \mu_M(r) \leq \mu_N(r),$ for all $r \in R.$ By proposition 3.4 in [5], $\mu_M(1) \leq \mu_M(r) \leq \mu_M(0)$. We have $\mu_N(r) \leq \mu_N(0)$.

(i) Now

$$(\mu_M + \mu_M)(r) = \sup_{r=p+q}\{min\{\mu_M(p), \mu_n(q)\} \quad where \ p,q,r \in R$$

$$\mu_M \subseteq (\mu_M + \mu_N).$$

Let r $\in$ R be arbitrary. Assume that $\nu_M \supseteq \mu_N$. $\Longrightarrow \nu_M(r) \leq \nu_N(r)$, for all $r \in R$. By proposition 3.4 in [3], $\nu_M(1) \geq \nu_M(r) \geq \nu_M(0)$. We have $\nu_N(r) \geq \nu_N(0)$. Now

$$\begin{aligned} (\nu_M + \nu_N)(x) &= \inf_{r=p+q}\{max\{\nu_M(p), \nu_N(q)\} \\ &\quad where \ \ p, q, r \ \in \ R \\ \nu_M &\subseteq (\nu_M + \nu_N). \end{aligned}$$

(ii) Now

$$\begin{aligned} (\mu_M \vee \mu_N)(r) &= \sup_{r=p\vee q}\{min\{\mu_M(p), \mu_N(q)\} \\ &\quad where \ \ p, q \ \in \ R \\ \mu_M &\subseteq (\mu_M \vee \mu_N). \end{aligned}$$

Again,

$$\begin{aligned} (\nu_M \vee \nu_N)(r) &= \inf_{r=p\vee q}\{max\{\nu_M(p), \nu_N(q)\} \\ &\quad where \ \ p, q \ \in \ R \\ (\nu_M \vee \nu_N) &\subseteq \nu_M. \end{aligned}$$

(iii) Now

$$\begin{aligned} (\mu_M \wedge \mu_N)(x) &= \sup_{r=p\wedge q}\{min\{\mu_M(p), \mu_N(q)\} \\ &\quad where \ \ p, q \ \in \ R \\ \mu_M &\subseteq (\mu_M \vee \mu_N). \end{aligned}$$

Again,

$$\begin{aligned} (\nu_M \wedge \nu_N)(r) &= \inf_{r=p\wedge q}\{max\{\nu_M(p), \nu_N(q)\} \\ &\quad where \ \ p, q \ \in \ R \\ (\nu_M \wedge \nu_N) &\subseteq \nu_M. \end{aligned}$$

□

Proposition 3.3. *Let M and N be any two Intuitionistic fuzzy L-ideals of an L-ring R. Then $\mu_M\mu_N \subseteq \mu_M \cap \mu_N$ and $\nu_M\nu_N \subseteq \nu_M \cap \nu_N$.*

Proof. Let M and N be any two Intuitionistic fuzzy L-ideals of R.
Let r $\in$ R be arbitrary.Then

$$\begin{aligned}(\mu_M\mu_N)(r) &= \sup_{r=kl}\{min\{\mu_M(k),\mu_N(l)\}\}\\ &\quad where\ \ k,l\in R\\ &= min\{\mu_M(p),\mu_N(q)\}\ for\ some\ r=pq\\ &\quad where\ p,q\in R.\end{aligned}$$

Now

$$\begin{aligned}\mu_N(r) &= \mu_N(pq)\geq max\{\mu_N(p),\mu_N(q)\}\\ &\geq \mu_N(q). \quad (3.5)\\ \mu_M(r) &= \mu_M(pq)\geq max\{\mu_M(p),\mu_M(q)\}\\ &\geq \mu_M(p). \quad (3.6)\end{aligned}$$

From (3.5) and (3.6), we have

$$\implies \mu_M\mu_N \ \subseteq\ \mu_M\cap\mu_N.$$

Also

$$\begin{aligned}(\nu_M\nu_N)(r) &= \inf_{r=kl}\{max\{\nu_M(k),\nu_N(l)\}\}\\ &\quad where\ \ k,l\in R\\ &= max\{\nu_M(p),\nu_N(q)\}\ for\ some\ r=pq\\ &\quad where\ p,q\in R.\end{aligned}$$

Now

$$\begin{aligned}\nu_N(r) &= \nu_N(pq)\leq min\{\nu_N(p),\nu_N(q)\}\\ &\leq \nu_N(q). \quad (3.7)\\ Again\ \ \nu_M(r) &= \nu_M(pq)\leq min\{\nu_M(p),\nu_M(q)\}\\ &\leq \nu_M(p). \quad (3.8)\end{aligned}$$

From (3.7) and (3.8) combined, we have

$$\implies (\nu_M\nu_N)\ \supseteq\ \mu_M\cap\mu_N.$$

□

Remark 3.1. *Let M be an intuitionistic L-ideal of R, then* $\mu_M\mu_M\subseteq\mu_M$ *and* $\nu_M\nu_M\supseteq\nu_M$.

4. Conclusion

It is probable to apply this manner of extension to further types of lattices.

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