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LAPLACE DECOMPOSITION METHOD FOR NONLINEAR BURGER'S-FISHER'S EQUATION

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ABSTRACT. This paper is devoted to obtaining the solution of nonlinear Burger's-Fisher's equation by using Laplace Decomposition Method. It is mainly focussing on the combined effect of initial condition and nonlinear source term on the wave profile of the approximations. The analytical /approximate solutions and relative errors are computed and graphically presented by using MATLAB software. The approach and the results obtained in this work are not only helpful in understanding the wave profile of a wide class of nonlinear inhomogeneous dissipative phenomena but also for the investigation of efficient approximate method. As the paper deals with quadratic source term, the results can be readily extended to inhomogeneous dissipative partial differential equations with different types of source terms.

1. INTRODUCTION

The partial differential equations (PDEs) play very important role in the field of applied sciences such as mathematical biology, chemistry, physics, engineering and technology [14]. Particularly, the nonlinear PDEs are having the applications in industries, fluid mechanics etc. The fundamental equation of fluid mechanics is a Burger's equation. Moving on the path of fluid mechanics,

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the Burger's-Fisher's (B-F) equation is a classic example of convection-reaction-diffusion equation and appears in a number of physical applications such as shock waves, turbulence, gas dynamics, traffic flow, fluid dynamics, sound waves in viscous medium, etc. The B-F equation is consider as a nonlinear inhomogeneous PDE given by,

$$u_t - u_{xx} + \alpha u^\gamma u_x + \beta u(u^\gamma - 1) = 0\,, \tag{1.1}$$

where $\alpha,\ \beta,\ \gamma$ are nonzero parameters. This equation is the balance between nonlinearity, time evolution, reaction and dissipation. The nonlinear source term $\beta u(u^\gamma - 1)$ of (1.1) with different values of γ will induce extra nonlinearity and influence the solution. When $\beta = 0$, (1.1) reduces to Burgers' equation and when $\alpha = 0,$ (1.1) reduces to nonlinear Fisher's equation. Therefore, B-F equation has the characteristics of convective phenomenon of Burgers' equation and dissipative and reaction properties from nonlinear Fisher's equation.

Consider the B-F equation with initial condition

$$u_t - u_{xx} + \alpha u^\gamma u_x + \beta u(u^\gamma - 1) = 0, x \in (0,1), t > 0\,, \tag{1.2}$$

$$\text{with initial condition} \quad u\,(x, 0) = h(x)\,. \tag{1.3}$$

equation (1.2) together with initial condition (1.3) is called initial value problem for B-F equation. Here $h(x)$ represents an arbitrary function of single variable. It represents the physical state at particular time and is responsible to single out physically relevant solution.

The B-F equation has been studied by many authors both analytically and numerically for different values of parameters via different approaches. One of the popular methods is Adomian Decomposition Method (ADM) which was introduced by George Adomian for solving nonlinear differential equations. Subsequently, many modified techniques have been derived from ADM with various forms and combinations such as Modified Adomian-Rach Decomposition Method [1], multi-stage modified Adomian decomposition method [2, 9], Discrete ADM [3, 8] and so on. Among these modifications, Laplace Decomposition Method (LDM), a semi analytical method is the appropriate technique for solving initial value problems for nonlinear B-F equation. Even though LDM was introduced by Khuri [15], the paper [1] was the first published research paper on combining the Laplace Transform and the ADM to solve differential equations. LDM was subsequently developed by many researchers and successfully tested for solution of diverse nonlinear PDEs [7, 10–13].

The B-F equation (1.1) has been studied by many researchers by using various analytical and numerical methods viz. ADM [1, 2, 6, 20], by tanh method [19], standard tanh method [20].The hybrid approach of Exp-function method and evolutionary algorithm [16], by authors [4] and [17] attempted CAS wavelet quasi-linearization technique [18]. In all the above literature mostly following two initial conditions were used, $u(x,0) = \left(\frac{1}{2} - \frac{1}{2}\tanh\left(\frac{\alpha\gamma}{2(1+\gamma)}x\right)\right)^{\frac{1}{\gamma}}$ and $u(x,0) = \frac{1}{2} + \frac{1}{2}\tanh\left(\frac{x}{4}\right)$.

The LDM uses Adomian polynomials to decompose nonlinear terms and the iterative procedure to obtain the solution which is expressed as an infinite series which helps in solving the nonlinear PDEs without any transformation such as linearization, perturbation and discretization. As the convergence of the method highly depends upon the initial wave profile which is the combination of both source term and initial condition.

In this paper, we implemented LDM to determine approximate analytical solution to initial value problem (1.2)-(1.3) for B-F equation. The main focus of this paper is to study the effect of initial condition on wave profile of various approximations. Also, an error estimation is done for specific values of time t and $0 \leq x \leq 1$. The MATLAB software is used for computation of Adomian polynomials [5] and its graphical output to compare various approximations of the solution.

The plan of the paper is as follow. A brief description of implementation of LDM on B-F equation is given in section 2. The numerical and graphical results are discussed in Section 3 and the conclusion is given in Section 4.

2. Laplace Decomposition Method

In this section, we developed LDM for the B-F equation. Consider equation (2.1) with indicated initial condition given by

$$u_t - u_{xx} + \alpha u^\gamma u_x + \beta u(u^\gamma - 1) = 0, x \in (0,1), t > 0 \tag{2.1}$$

with initial condition $\quad u\left(x,0\right) = h(x)\,.$

As per LDM, operating Laplace transform L with respect to t on both sides of equation (2.1), and by using differential property of Laplace transform we obtain,

$$su\left(x,s\right) - u\left(x,0\right) \ - L(u_{xx}) + \alpha L\left(u^\gamma u_x\right) + \beta L\left(u u^\gamma\right) - \beta u\left(x,s\right) = 0\,.$$

By using initial condition $u\left(x,0\right) = h(x)$ and taking inverse Laplace transform we get,

$$\begin{aligned} u\left(x,t\right) \ &= \ L^{-1}\left(\frac{h\left(x\right)}{\left(s-\beta\right)}\right) + \ L^{-1}\left(\frac{1}{\left(s-\beta\right)}L\left(u_{xx}\right)\right) \\ &-L^{-1}\left(\frac{1}{\left(s-\beta\right)}L\left(\alpha u^\gamma u_x + \beta u u^\gamma\right)\right)\,. \end{aligned}$$

In LDM, the solution $u\left(x,t\right)$ is decomposed as an infinite series given by,

$$u\left(x,t\right) = \sum_{n=0}^{\infty} u_n\left(x,t\right)\ . \tag{2.2}$$

We obtain the components of $u\left(x,t\right)$ recursively as given below,

$$u_0\left(x,t\right) = L^{-1}\left(\frac{h(x)}{\left(s-\beta\right)}\right) = h(x)e^{\beta t} \tag{2.3}$$

$$\begin{aligned} u_{n+1}\left(x,t\right) &= L^{-1}\left(\frac{1}{\left(s-\beta\right)}L\left(\frac{\partial^2}{\partial x^2}(u_n\left(x,t\right)\right)\right) \\ &- L^{-1}\left(\frac{1}{\left(s-\beta\right)}L(\sum_{n=0}^{\infty}A_n + B_n)\right), n = 0 \end{aligned} \tag{2.4}$$

where the Adomian polynomials A_n and B_n of degree n are given by,

$$\sum_{n=0}^{\infty} A_n\left(u\right) = \alpha u^\gamma u_x, \ \ \sum_{n=0}^{\infty} B_n\left(u\right) = \beta u u^\gamma \tag{2.5}$$

and can be computed by using the formula [5] given by,

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[N\left(\sum_{i=0}^{\infty}\lambda^i u_i\right)\right]_{\lambda=0}, n = 0, 1, 2, 3, \ldots$$

Adomian polynomials can be generated for any order and for any types of nonlinearity considered in this paper are computed and by using the equations (2.1), (2.3) and equation (2.4) we obtained the components, $u_0(x,t), u_1(x,t)$, $u_2(x,t)\ldots$ substituting in equation (2.2), we get,

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t) = u_0(x,t) + u_1(x,t) + u_2(x,t) + u_3(x,t) + \ldots$$

3. Numerical and Graphical Results

In this section we are solved the Burgers-Fisher's equation with different initial conditions.

3.1. **Burgers-Fisher's equation with linear initial condition.** Consider B-F equation with linear initial condition

(3.1) $$u_t - u_{xx} + \alpha u^\gamma u_x + \beta u(u^\gamma - 1) = 0,$$

with initial condition $\quad u(x,0) = h(x) = x$

Let us choose the parameters, $\alpha = \beta = \gamma = 1$ for computation. By applying LDM on equation (3.1) and by using the procedure as explained in section 2, we obtain,

$$u_0(x,t) = L^{-1}\left(\frac{x}{s-1}\right) = xe^t$$

$$\sum_{n=0}^{\infty} A_n(u) = uu_x \quad \sum_{n=0}^{\infty} B_n(u) = u^2$$

$$A_0(u) = u_0 u_{0x} = xe^{2t} \quad B_0(u) = {u_0}^2 = x^2 e^{2t}$$

$$u_1(x,t) = L^{-1}\left[\frac{1}{s-1}\left(L\left(\frac{\partial^2}{\partial x^2}u_0(x,t)\right)\right)\right] - L^{-1}\left[\frac{1}{s-1}L(A_0(u) + B_0(u))\right]$$

$$u_1(x,t) = \left(x^2 + x\right)\left(e^t - e^{2t}\right)$$

$$A_1(u) = u_0 u_{1x} + u_1 u_{0x} \qquad B_1(u) = 2u_0 u_1 .$$

By calculating first few components $A_2(u)$, $A_3(u)\ldots$, $B_2(u)$, $B_3(u)$ and $u_2(x,t), u_3(x,t), u_4(x,t)$, etc, the six-term approximation denoted by $u_{app}(x,t)$ is given by,

$$u_{app}(x,t) = \frac{7}{2}e^{3t} - 16e^{2t} + \frac{25}{2}e^{t} - \left(4x + 7x^2\right)e^{2t} + \left(2x + \frac{7}{2}x^2\right)e^{3t} + \left(33x + 6x^2 + \frac{37}{2}\right)e^{3t} - \left(\frac{22x}{3} + \frac{4x^2}{3} + \frac{37}{9}\right)e^{4t} - \ldots$$

The graphs of initial approximation, two-term and six-term LDM for linear initial condition is depicted in Figure 1. The wave propagation from initial approximation to higher- term approximations of the equation for different values of x, t can be clearly seen in this graph.

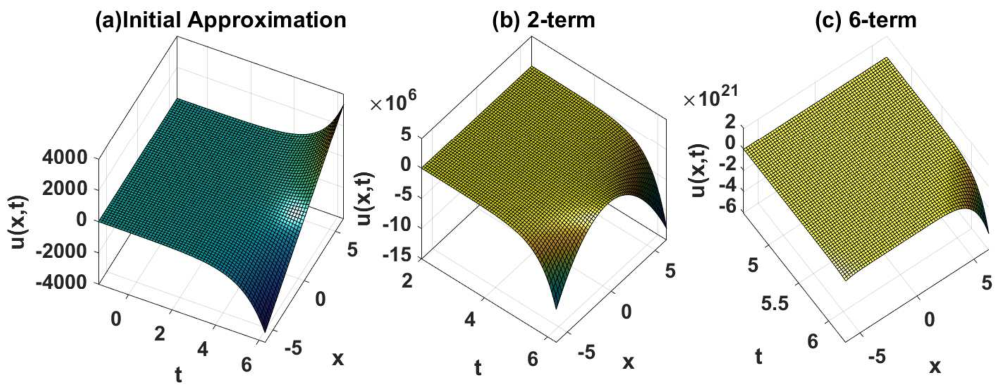

FIGURE 1. Surface plot of LDM approximation of the B-F equation for $u(x,0) = x$ and the parameters $\alpha = \beta = \gamma = 1$ (a) Initial approximation (b) Two-term LDM (c) Six-term LDM.

Table 1 presents the numerical results of n-term LDM and the relative errors for the specific values of x and t. It is observed that, $u(x,t)$ decreases when t increases and x decreases. It is noted that, the four-term approximations are same as six-term approximations correct to four decimal places and the error is negligible for smaller values of t and more for greater values. However, there is decrease in the error when x increases from 0.1 to 0.9 and is smaller for higher number of terms ensuring the rapid convergence of LDM. The table shows the rapid convergence as only few terms are required to get desired accuracy. It is also observed that, the accuracy of approximate solution is more for $x = 0.9$.

TABLE 1. Numerical results of LDM approximations and relative errors of the B-F equation for $u(x, 0) = x$ and the parameters $\alpha = 1 = \beta = \gamma$

	Solution for $t = 0.01$			**6-term LDM for t**			**Relative errors**	
x	$n = 2$	$n = 4$	$n = 6$	0.02	0.03	0.04	0.01	0.03
0.1	0.0999	0.0998	0.0998	0.0994	0.0988	0.0981	4.3986e-09	1.1467e-06
0.2	0.1996	0.1995	0.1995	0.1988	0.1980	0.1969	3.5332e-09	9.1620e-07
0.3	0.2991	0.2990	0.2990	0.2979	0.2965	0.2951	3.2406e-09	8.3879e-07
0.4	0.3983	0.3983	0.3983	0.3965	0.3945	0.3924	3.0158e-09	7.7937e-07
0.5	0.4974	0.4974	0.4974	0.4948	0.4920	0.4891	2.7372e-09	7.0582e-07
0.6	0.5963	0.5964	0.5964	0.5926	0.5889	0.5850	2.3465e-09	6.0244e-07
0.7	0.6950	0.6951	0.6951	0.6901	0.6852	0.6802	1.8003e-09	4.5759e-07
0.8	0.7934	0.7936	0.7936	0.7873	0.7810	0.7747	1.0597e-09	2.6059e-07
0.9	0.8917	0.8920	0.8920	0.8840	0.8762	0.8685	8.4198e-11	6.5656e-10

The numerical results presented in Table1 have been depicted through graphical illustration in Figure 2. It is clear from all these figures that, successive LDM are approximately equal for different time levels and the accuracy is higher for lower time levels.

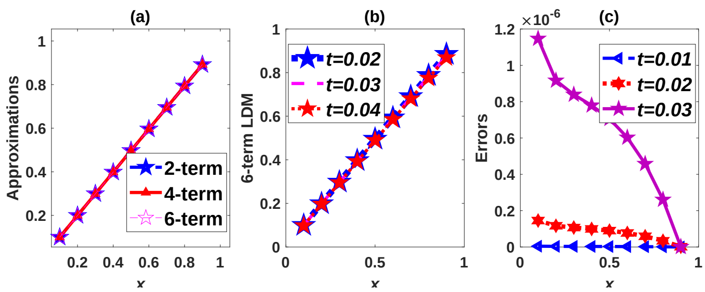

FIGURE 2. Graph of B-F equation for $u(x, 0) = x$ and $\alpha = 1 = \beta = \gamma$ (a) LDM for $\mathrm{t} = 0.01$ (b) Six-term LDM for various t (c)Relative errors for $\mathrm{t} = 0.01, 0.02, 0.03$.

3.2. B-F equation with exponential initial condition. Consider B-F equation

$$u_t - u_{xx} + \alpha u^\gamma u_x + \beta u(u^\gamma - 1) = 0, \tag{3.2}$$

with initial condition $\quad u(x,0) = e^{ax}$.

Let $\alpha = 1, \beta = 1, \gamma = 1$. By implementation of LDM on equation (3.2) as explained in section 2 and by (2.3), (2.4), (2.5), we can obtain the successive approximations for series solution.

Case 1: $\boldsymbol{a = -1}$ The first few components of approximations of equation (3.2) are given by,

$$u_0(x,t) = L^{-1}\left(\frac{e^{-x}}{s-1}\right) = e^{t-x},$$

$$A_0(u) = u_0 u_{0x} = -e^{2t-2x}, B_0(u) = {u_0}^2 = e^{2t-2x}$$

$$A_0(u) + B_0(u) = -e^{2t-2x} + e^{2t-2x} = 0\ ,\ \frac{\partial^2}{\partial x^2} u_0(x,t) = e^{t-x}$$

$$u_1(x,t) = te^{t-x},$$

$$A_1(u) = u_0 u_{1x} + u_1 u_{0x} = -2te^{2t-2x}, B_1(u) = 2u_0 u_1 = 2te^{2t-2x}$$

$$A_1(u) + B_1(u) = -2te^{2t-2x} + 2te^{2t-2x} = 0$$

$$u_2(x,t) = \frac{t^2}{2} e^{t-x}.$$

By calculating first few components, we can obtain the series solution as,

$$u(x,t) = e^{t-x} + te^{t-x} + \frac{t^2}{2} e^{t-x} + \frac{t^3}{6} e^{t-x} + \ldots \tag{3.3}$$

By Taylor's series expansion of equation (3.3), the exact solution (u_{exact}) can be calculated as, $u_{exact} = e^{2t-x}$.

Case 2: $\boldsymbol{a = +1}$ The first few components of approximations of (3.2) are given by:

$$u_0(x,t) = L^{-1}\left(\frac{e^{x}}{s-1}\right) = e^{t+x},$$

$$u_1(x,t) = 2e^{t+2x} - 2e^{2t+2x} + te^{t+x},$$

$$u_2(x,t) = e^{2t}\left(4e^{2x} - 10e^{3x}\right) + 8e^{2x+t} - e^{t}\left(4e^{2x} - 5e^{3x}\right) - 8e^{2t+2x} + 5e^{3t+3x} - 4te^{2t+2x} + 8te^{2x+t} + \frac{t^2 e^{t+x}}{2}.$$

Here we have calculated five-term approximation of $u(x,t)$, $u_{app}(x,t)$ given by:

$$u_{app}(x,t) = e^{t+x} - e^{t}\left(16e^{2x} - \frac{695e^{3x}}{4}\right) + \frac{578e^{4x}}{3} - 42e^{5x} + e^{2t}\left(4e^{2x} - 10e^{3x}\right) + e^{3t}\left(\frac{315e^{3x}}{4} - 336e^{4x}\right) + \cdots + 8te^{2x+t} + \frac{t^2e^{t+x}}{2} + \frac{t^3e^{t+x}}{6} + \frac{t^4e^{t+x}}{24}.$$

Figure 3 and Figure 4 depict the wave profiles of successive approximations of equation (3.2) for case 1 and case 2 respectively. It is observed that, $u(x,t)$ increases when the terms increase. The convergence faster for case 2 and also there is sharp peak for $x = t = 6.283$. It is observed from the figure that, there is a complete agreement between the computed results of successive approximations and the exact solution.

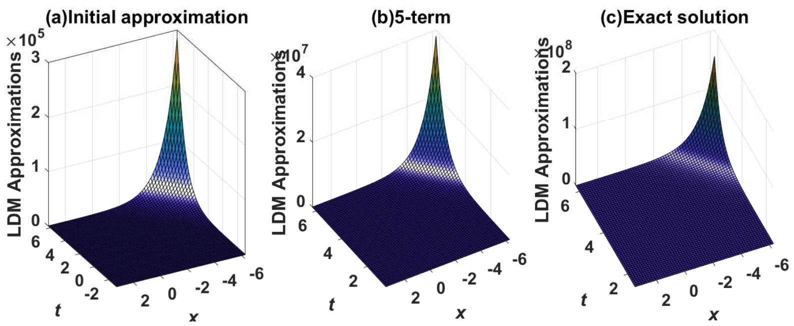

FIGURE 3. Surface plot of LDM of B-F equation for $u(x,0) = e^{-x}$ and the parameters $\alpha = 1, \beta = 1, \gamma = 1$ (a) Initial (b) Five-term (c) Exact solution

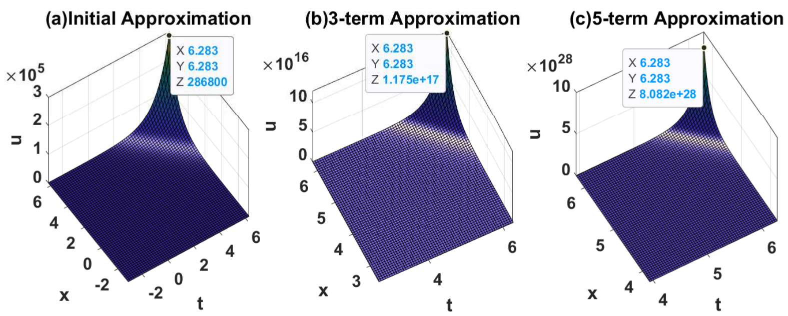

FIGURE 4. Plot of LDM for $u(x,0) = e^{x}, \alpha = \beta = \gamma = 1$ (a) Initial (b) 3-term (c) 5-term

Table 2 which presents the results of case 1, indicates the accuracy of the solution correct to four decimals in only very few terms. It is also noted from the table that, $u(x,t)$ decreases when x increases and it increases when t increases. Table 3 presents the n -term series LDM approximations of $u(x,t)$ of B-F equation for case 2 where four-term and five-term approximations for $t=0.01$ and five-term and six-term approximations for $t=0.02$ correct to four decimal places are equal. $u(x,t)$ decreases when t increases except the initial approximation and $u(x,t)$ increases when x increases.

TABLE 2. Numerical results of B-F equation for $u\left(x,0\right)=e^{-x},\alpha=1=\beta=\gamma$.

x	$u(x,t)$ for $t=0.01$				$u(x,t)$ for $t=0.02$			
	1-term	**2-term**	**3-term**	u_{exact}	**1-term**	**2-term**	**3-term**	u_{exact}
0.1	0.9139	0.9231	0.9231	0.9231	0.9231	0.9416	0.9418	0.9418
0.5	0.6126	0.6188	0.6188	0.6188	0.6188	0.6312	0.6313	0.6313
0.9	0.4107	0.4148	0.4148	0.4148	0.4148	0.4231	0.4232	0.4232

TABLE 3. LDM of B-F equation for $u\left(x,0\right)=e^{x},\ \alpha=1,\beta=1,\gamma=1$

x	$u(x,t)$ for $t=0.01$			$u(x,t)$ for $t=0.02$			
	1-term	**4-term**	**5-term**	**1-term**	**3-term**	**5-term**	**6-term**
0.1	1.1163	1.1027	1.1027	1.1275	1.0997	1.0998	1.0998
0.5	1.6653	1.6275	1.6275	1.6820	1.6066	1.6065	1.6065
0.9	2.4843	2.3902	2.3902	2.5093	2.3266	2.3255	2.3255

The numerical results of Table 2 and Table 3 have been depicted graphically in Figure 5 and Figure 6 and the graphs of six-term LDM for different time are depicted in Figure 6 (c).

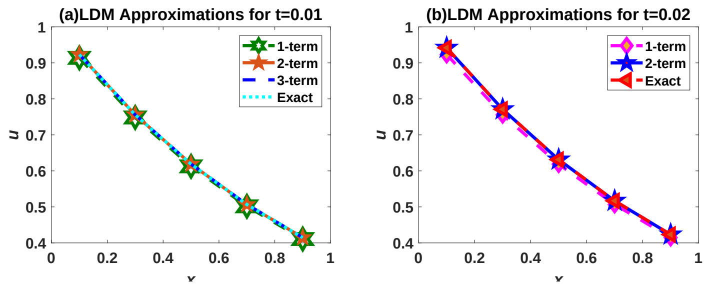

FIGURE 5. LDM and exact solution for $u\,(x, 0) = e^{-x}$ (a) $t = 0.01$ (b) $t = 0.02$

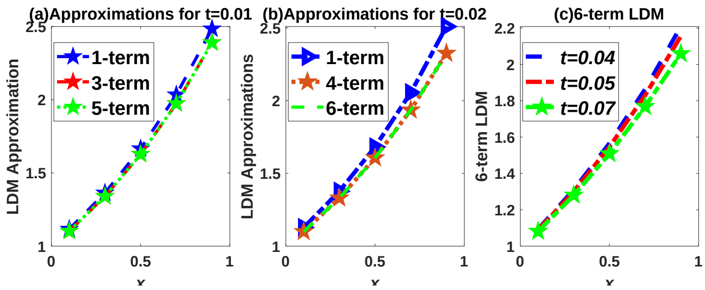

FIGURE 6. Graph of LDM for $u\,(x, 0) = e^{x}$ (a) $t = 0.01$ (b) $t = 0.02$ (c) 6-term for different t

Table 4 exhibits the relative errors of B-F equation for case 1. It is observed that, the relative error increases when t increases and the accuracy decreases for greater values of time. However, there is not much difference in the errors for different values of x. It is obvious that, the errors are negligible and convergence is achieved for $n = 3$ only. The table also presents the relative errors between exact solution and n-term approximation. In this case also the relative error is smaller for higher number of terms.

Figure 7 depicts the relative errors of five-term approximations for $t = 0.01$ and $t = 0.02$.

Table 5 exhibits the relative error of B-F equation for case 2. It is observed that, the accuracy is higher for smaller values of x, t and higher number of terms. It is obvious that the errors can be made negligible by including more number of terms for the approximations and hence the convergence. Figure 8. depicts the graph of relative errors taken from six-term approximation to various approximations of B-F equation for case 2 and different values of time levels. A deviation of error is observed at $x = 0.5$.

TABLE 4. Errors of B-F equation for $u(x,0) = e^{-x}, \alpha = 1 = \beta = \gamma$

x/n	Relative errors $t = 0.01$			Relative errors $t = 0.02$		
	1-term	3-term	5-term	1-term	3-term	5-term
0.1	0.0100	1.6542e-07	8.2651e-13	0.0198	1.3135e-06	2.6226e-11
0.3	0.0100	1.6542e-07	8.2651e-13	0.0198	1.3135e-06	2.6226e-11
0.5	0.0100	1.6542e-07	8.2651e-13	0.0198	1.3135e-06	2.6226e-11
0.7	0.0100	1.6542e-07	8.2651e-13	0.0198	1.3135e-06	2.6226e-11
0.9	0.0100	1.6542e-07	8.2651e-13	0.0198	1.3135e-06	2.6226e-11

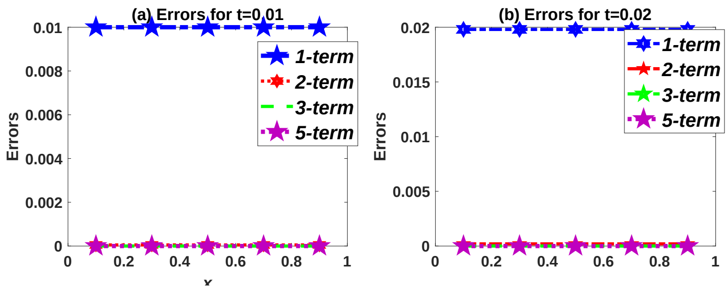

FIGURE 7. Errors of B-F equation for $u(x,0) = e^{-x}, \alpha = 1 = \beta = \gamma, \quad (a) t = 001, \quad (b) t = 002$

TABLE 5. Relative errors B-F equation for $u(x,0) = e^{x}$ and $\alpha = 1 = \beta = \gamma$

x/n	Relative errors $t = 0.01$		Relative errors $t = 0.02$		Relative errors	
	$n = 2$	$n = 4$	$n = 2$	$n = 4$	$t = 0.03$	$t = 0.04$
0.1	5.4847e-06	1.2169e-09	4.3195e-05	3.6930e-08	2.6353e-07	1.0349e-06
0.5	1.6004e-06	1.3937e-08	1.8041e-05	4.6441e-07	3.6724e-06	1.6102e-05
0.9	6.1883e-05	3.1740e-08	5.1190e-04	1.2066e-06	1.0751e-05	5.2672e-05

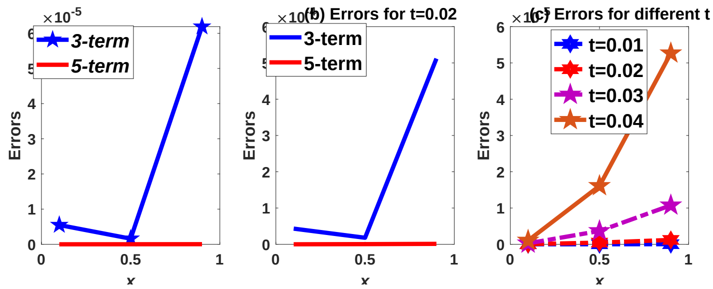

FIGURE 8. Relative errors of B-F equation for $u(x,0) = e^x, \alpha = 1 = \beta = \gamma$ (a)$t = 001$ (b)$t = 002$ (c)For different t

3.3. B-F equation for Gaussian initial condition.

Consider B-F equation

$$u_t - u_{xx} + \alpha u^\gamma u_x + \beta u(u^\gamma - 1) = 0, \tag{3.4}$$

with initial condition $u(x,0) = e^{-x^2}$

Let $\alpha = \beta = \gamma = 1$. By implementation of LDM on equation (3.4) as explained in section 2 and by using equations (2.3), (2.4) and (2.5), we obtained the initial approximations, successive approximations and hence the solution.

$$u_0(x,t) = L^{-1}\left(\frac{e^{-x^2}}{s-1}\right) = e^{t-x^2}$$

$$A_0(u) = u_0 u_{0x} = -2xe^{2t-2x^2}, B_0(u) = {u_0}^2 = e^{2t-2x^2}$$

$$u_1(x,t) = \left(e^t - e^{2t}\right)\left(e^{-2x^2}\right)(1-2x) - e^{-2x^2}\left(e^{2t} - 2x\right) - 2e^{-2x^2}\left(te^t - 2x^2\right)$$

In this case, we have calculated six-term approximation of $u(x,t)$. So, approximate series solution $u(x,t)$ of equation (3.4) denoted by u_{app} is given by,

$$u_{app} = e^{2t}\left(4e^{-2x^2} - 24xe^{-2x^2} - 16x^2e^{-2x^2} + 32x^3e^{-2x^2}\right) - e^{2t}\left(e^{-2x^2} - 2xe^{-2x^2}\right) - 3010x^3e^{-5x^2} + \frac{5950x^4e^{-5x^2}}{3} + \frac{7000x^5e^{-5x^2}}{3} - \frac{5000x^6e^{-5x^2}}{3}$$

The wave profile of various approximations of graphs of $u(x,t)$ of equation (3.4) is depicted graphically in Fig 9. The graphical observations indicate the rapid increase in the wave amplitude as per the time which is uniform for all approximations. Figure 9 (a) representing the initial wave profile of the equation (3.4) shows the rapid increase in the wave amplitude with time whereas in Figure 9.(c) representing five-term approximate series solution of equation (3.4), the wave amplitude remains steady for a long time and then undergoes distortion. This may be because the influence of parameters and the initial condition in the initial stage and the effect gradually decreases as the time increases.

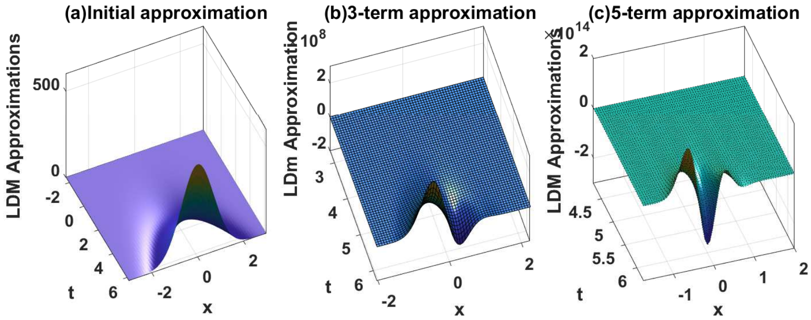

FIGURE 9. Evolution results for the B-F equation B-F equation for $u(x,0) = e^{-x^2}$ and the parameters $\alpha = 1, \beta = 1, \gamma = 1$ (a) Initial (b) Three- term (c) Five-term

Table 6 presents the numerical results of n -term LDM approximations of $u(x,t)$ of equation(3.4) where approximations upto four decimal places are same from $n = 3$ only, indicating the rapid convergence of the solution. It is observed that, $u(x,t)$ decreases when x increases from 0.1 to 0.9 and for both the values of t.The numerical results presented in Table 6 have been depicted graphically in Figure 10.

TABLE 6. Approximations $\sum_{n=1}^{n} u_{n-1}$ of B-F equation for $u(x,0) = e^{-x^2}$ and $\alpha = \beta = \gamma = 1$

x	t	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$
0.1	0.01	1.0000	0.9724	0.9732	0.9732	0.9732
0.5		0.7866	0.7788	0.7784	0.7784	0.7784
0.9		0.4493	0.4565	0.4560	0.4560	0.4560
0.1	0.02	1.0101	0.9543	0.9572	0.9572	0.9572
0.5		0.7945	0.7786	0.7771	0.7774	0.7773
0.9		0.4538	0.4684	0.4664	0.4664	0.4664

Table 7 presenting the relative errors of equation (3.4) indicates that, the relative error is small for smaller values of t and for higher number of terms of approximations.

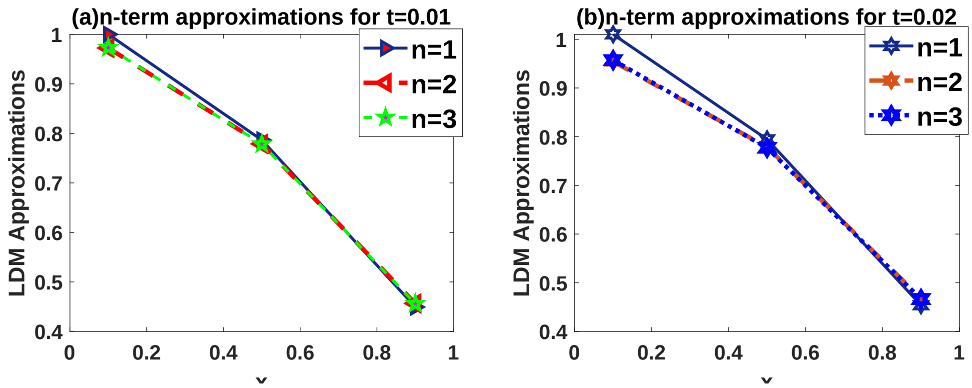

FIGURE 10. Evolution results for the B-F equation B-F equation for $u(x,0) = e^{-x^2}$ and the parameters $\alpha = 1, \beta = 1, \gamma = 1$ (a) Initial (b) Three- term (c) Five-term

TABLE 7. Relative errors of B-F equation for $u(x,0) = e^{-x^2}$ and $\alpha = 1 = \beta = \gamma$

		Relative errors			
x	t	$n=1$	$n=2$	$n=3$	$n=4$
0.1	0.01	0.0276	7.3470e-04	7.2628e-06	1.6169e-06
0.5		0.0106	4.4591e-04	3.9787e-05	1.3551e-06
0.9		0.0147	0.0011	1.4866e-05	1.1019e-06
0.1	0.02	0.0553	0.0030	7.2123e-05	2.6900e-05
0.5		0.0221	0.0017	3.1230e-04	2.1922e-05
0.9		0.0270	0.0041	1.2558e-04	1.7535e-05

The relative errors of equation (3.4) for specific values of x and t has been depicted in Figure 11.

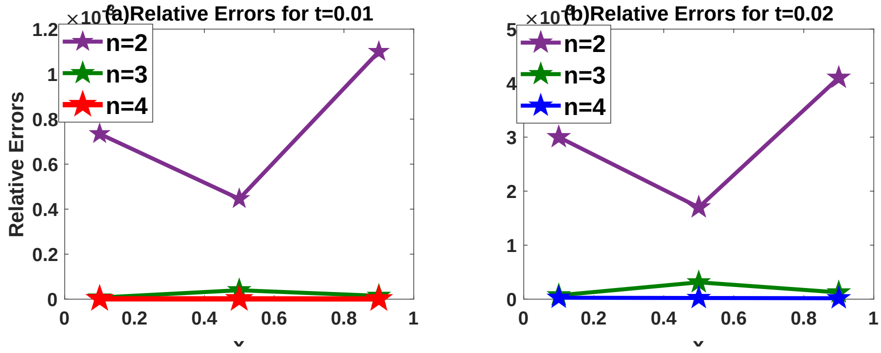

FIGURE 11. Relative errors of equation B-F equation for $u\,(x, 0) = e^{-x^2}$, $\alpha = \beta = \gamma = 1$

4. CONCLUSION

In this paper, the combined effect of initial condition and nonlinear source term on the solution of the B-F equation is studied by employing LDM. Linear and exponential initial conditions are taken to investigate the exact solution of the B-F equation. It is shown that, the accuracy of the approximation at higher levels of time is relatively lower because of first order time derivative and the accuracy is more for larger values of x and for higher number of terms ensuring the rapid convergence of LDM. The numerical results and graphical representations indicate the changes in the wave amplitude and solitary behavior for different initial conditions. As different PDEs represent different physical processes and the initial condition represents the physical state of the equation, the procedure can be implemented to different initial conditions. The approach can also be extended by taking higher order nonlinear source term of B-F equation. As the B-F equation has very important interdisciplinary applications, the method can also be implemented to solve other important nonlinear inhomogeneous PDEs with dissipative effects arising in the fields of engineering and mathematical sciences.

REFERENCES

[1] G. ADOMIAN, R. RACH: *On the solution of nonlinear differential equations with convolution product nonlinearities,* J. Matht. Anal. Appl., **114** (1986), 171–175.

[2] G. ADOMIAN, R. RACH: *Transformation of series* , Appl. Math. Lett., (1991), 69–71.

[3] G. A. BIRAJDAR: *Numerical solution of time fractional Navier-Stokes equation by discrete Adomian decomposition method,* Nonlinear Engineering, **3**(1)(2014), 21–26.

[4] S. S. BEHZADI : *Convergence of Iterative Methods Applied to Generalized Fisher Equation*, Int. J.Diff. Eq., 2010, 1–16.

[5] J. BIAZAR, S. M. SHAFIOF: *A Simple Algorithm for Calculating Adomian Polynomials*, Int. J. Contemp. Math. Sci., **2** (2007), 975 – 982.

[6] V. CHANDRAKER, A. AWASTHI, S. JAYARAJ: *Numerical Treatment of Burger-Fisher equation*, Global Colloquium in Recent Advancement and Effectual Researches in Engineering, Science and Technology (RAEREST 2016), Elsevier,(2016), 1217 – 1225.

[7] H. CHEN, H. ZHANG : *New multiple soliton solutions to the general Burgers–Fisher equation and the Kuramoto–Sivashinsky equation*, Chaos Solitons and Fractals, **19** (2004), 71–76.

[8] D. B. DHAIGUDE, G. A. BIRAJDAR: *Numerical solutions of fractional partial differential equations by discrete Adomian decomposition method*, Adv. Appl. Math. Mech., **6** (1) (2014), 107–119.

[9] J. S. DUAN, R. RACH, A. M. WAZWAZ: *Higher order numeric solutions of the Lane-Emden-type equations derived from the multi-stage modified Adomian decomposition method* , Int. J. Computer Math, **94** (2017), 197–215.

[10] J. FADAEI: *Application of Laplace-Adomian decomposition method on linear and nonlinear system of PDEs*, Appl. Math. Sci., **5** (2011), 1307–1315.

[11] H. GÜNDOĞDU, G. F. ÖMER: *Solving nonlinear partial differential equations by using Adomian decomposition method, modified decomposition method and Laplace decomposition method*, MANAS J. Engg., **5** (2017), 1–13.

[12] M. A. HUSSAIN: *Modified Laplace decomposition method*, Appl. Math. Sci., **4** (2010), 1769–1783.

[13] H. N. ISMAIL, K. RASLAN, A. A. ABD RABBOH: *Adomian decomposition method for Burger's Huxley and Burger-Fisher equations*, Appl. Math. Comput., **159** (2004), 291–301.

[14] W. JIANHONG: *Theory and Application of Partial Differential Equations*, Springer Science and Business Media, 2012.

[15] S. A. KHURI: *A Laplace decomposition algorithm applied to a class of nonlinear differential equations*, J. Appl. Math., **1** (2001), 141–155.

[16] S. A. MALIK, I. M. QURESHI, M. AMIR, A. N. MALIK, I. HAQ : *Numerical solution to generalized Burgers'-Fisher equation using exp-function method hybridized with heuristic computation,* PLoS One, **10** (2015), 25811858.

[17] Y. R. MOHAMMAD, Z. ALI: *Application of Laplace decomposition method for Burgers-Huxley and Burgers-Fisher equations,* J. Math. Modeling, **1** (2013), 41–67.

[18] U. SAEED, K. GILANI: *CAS wavelet quasi-linearization technique for the generalized Burger–Fisher equation,* Mathematical Sciences, **12** (2018), 61–69.

[19] A. W. WAZWAZ: *The tanh method for generalized forms of nonlinear heat conduction and Burgers–Fisher Equations,* Appl. Math. Comput., **169** (2005), 321–338.

[20] A. W. WAZWAZ: *Analytic study on Burgers, Fisher, Huxley equations and combined forms of these equations,* Appl. Math. Comput., **195** (2008), 754–761.

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