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FUZZY α-TRANSLATIONS AND FUZZY β-MULTIPLICATIONS OF Z-ALGEBRAS

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ABSTRACT. In this article, fuzzy α - translations, fuzzy extensions and fuzzy β -multiplications of fuzzy Z-Subalgebras (fuzzy Z-ideals) of Z-algebras are initiated and some interesting results are proved.

1. INTRODUCTION

A new class of algebra that arise from the propositional calculi is the Z-algebra introduced by Chandramouleeswaran et al. [1] in the year 2017. This algebra differs from the BCK, BCI, BF-algebras [2–5] and so on.

Zadeh [9] in the year 1965, introduced the notion of fuzzy sets as the generalization of set theory to deal with the problems of uncertainty under real physical world. Since then many authors fuzzified different algebraic structures. The idea of fuzzy translations and fuzzy multiplications have been discussed by Lee et al. [6]. Similar concept have been discussed in BF-algebras by Chandramouleeswaran et al. [2]. In [7, 8] we have launched the notion of fuzzy Z-Subalgebras and fuzzy Z-ideals respectively. In this paper, we examine fuzzy α -translations and fuzzy β - multiplications of fuzzy Z-subalgebras and fuzzy Z-ideals in Z-algebras.

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2. Preliminaries

Now we collect the necessary definitions from the articles([1], [7], [8], [9]).

Definition 2.1. *[1] A Z-algebra* **(Z-algr)** $(J, *, 0)$ *is a nonempty set J with constant 0 and a binary operation* $*$ *satisfying the following conditions:*

(Z1) $u * 0 = 0$
(Z2) $0 * u = u$
(Z3) $u * u = u$
(Z4) $u * \omega = \omega * u$ *when* $u \neq 0$ *and* $\omega \neq 0$ $\forall\ u, \omega \in J$.

Definition 2.2. *[9] A fuzzy set* **(fy set)** *A in a set J is defined by a membership function* **(msfn)** $\mu_A : J \to [0, 1]$.

Definition 2.3. *[7] Let* $(J, *, 0)$ *be a Z-algr. A fy set A in J with msfn* μ_A *is said to be a fy Z-Subalgebra***(fy Z-Salgr)** *of a Z-algr J if* $\mu_A(u * \omega) \geq min\{\mu_A(u), \mu_A(\omega)\}$ $\forall\ u, \omega \in J$.

Definition 2.4. *[8] Let* $(J, *, 0)$ *be a Z-algr. A fy set A in J with msfn* μ_A *is said to be a fy Z-ideal***(fy Z-idl)** *of a Z-algr J if :*

(i) $\mu_A(0) \geq \mu_A(u)$;
(ii) $\mu_A(u) \geq min\{\mu_A(u * \omega), \mu_A(\omega)\}$ $\forall$ *u,* ω *in J.*

3. Fuzzy α-Translations and Fuzzy β-Multiplications of Fuzzy Z-Subalgebras (Fuzzy Z-Ideals)

Hereafter, $(J, *, 0)$ denotes a Z-algr; and $1 - sup\{\mu_A(u) | u \in J\}$ is denoted by T.

Definition 3.1. *Let A be a fy set of a Z-algr J and let* $\alpha \in [0, T]$. *A fy* α*- translation* **(fy** α**-tlt)** A^T_α *of A with msfn* $\mu_{A^T_\alpha} : J \to [0, 1]$ *is defined by* $\mu_{A^T_\alpha}(u) = \mu_A(u) + \alpha$ *,* $\forall$ $u \in J$.

Example 1.

TABLE 1

$*$	0	s	p	g
0	0	s	p	g
s	0	s	g	p
p	0	g	p	s
g	0	p	s	g

TABLE 2

J	0	s	p	g
μ_A	0.8	0.6	0.5	0.5

From Table 1, $J = \{0, s, p, g\}$ is a Z-algr. If a fy set A of J is given in Table 2, then $A^T_{0.1}$ is a fy 0.1- tlt of A.

Theorem 3.1. *Let A be a fy set of Z-algr J and $\alpha \in [0, T]$. Then the fy α- tlt A^T_α of A is a fy Z-Salgr of J $\Longleftrightarrow$ A is a fy Z-Salgr of J.*

Definition 3.2. *When A_1 and A_2 are fy sets of Z-algr J, A_2 is called a fy Z-Salgr extension* **(fy Z-Salgr ext)** *of A_1 if:*

(S_1) A_2 is a fy ext of A_1 ($\mu_{A_1}(u) \leq \mu_{A_2}(u)$ $\forall$ $u \in J$) .

(S_1) If A_1 is a fy Z-Salgr of J, then A_2 is a fy Z-Salgr of J.

It follows from the definition of fy α-tlt, $\mu_{A^T_\alpha}(u) \geq \mu_A(u)$ $\forall u \in J$. This proves the following propositions.

Proposition 3.1. *Let A be a fy Z-Salgr of a Z-algr J and $\alpha \in [0, T]$.Then the fy α-tlt A^T_α of A is a fy Z-Salgr ext of A.*

Proposition 3.2. *Arbitrary intersection of fy Z-Salgr exts of a fy set A of a Z-algr J is a fy Z-Salgr ext of A.*

Definition 3.3. *For a fy set A of a Z-algr J, $\alpha \in [0, T]$ and $t \in [0, 1]$ with $t \geq \alpha$, we define the upper level subset of A^T_α as $U_\alpha(\mu_A; t) = \{u \in J | \mu_A(u) \geq t - \alpha\}$.*

Proposition 3.3. *Let A be a fy set of a Z-algr J and $\alpha \in [0, T]$. Then the fy α-tlt A^T_α of A is a fy Z-Salgr of J $\Longleftrightarrow$$U_\alpha(\mu_A; t)$ is a Z-Salgr of J, $\forall t \in Im(A)$ with $t \geq \alpha$.*

Proposition 3.4. *Let A be a fy Z-Salgr of a Z-algr J and $\alpha, \lambda \in [0, T]$. If $\alpha \geq \lambda$, then the fy α-tlt A^T_α of A is a fy Z-Salgr ext of the fy λ-tlt A^T_λ of A.*

Proposition 3.5. *Let A be a fy Z-Salgr of a Z-algr J and $\lambda \in [0, T]$. For every fy Z-Salgr ext B of the fy λ-tlt A^T_λ of A, $\exists\ \alpha \in [0, T] \ni \alpha \geq \lambda$ and B is a fy Z-Salgr ext of the fy α-tlt A^T_α of A.*

Definition 3.4. *Let A be a fy set of a Z-algr J and $\beta \in (0, 1]$. A fy β-multiplication* **(fy β-mlc)** *A^M_β of A with msfn $\mu_{A^M_\beta} : J \to [0, 1]$ is defined by $\mu_{A^M_\beta}(u) = \beta \cdot \mu_A(u)\ \forall$ $u \in J$.*

Example 2. *Consider a Z-algr J={0,s,p,g} and a fy Z-Salgr A of J as in Example 3.2. Then $A^M_{0.1}$ is a fy 0.1-mlc of A.*

Proposition 3.6. *If A is a fy Z-Salgr of a Z-algr J, then the fy β-mlc A^M_β of A is a fy Z-Salgr of J for all $\beta \in [0, 1]$.*

Proposition 3.7. *For any fy set A of Z-algr J, A is a fy Z-Salgr of J iff A^M_β is a fy Z-Salgr of J, $\forall\beta \in (0, 1]$.*

Proposition 3.8. *Let A be a fy set of a Z-algr J, $\alpha \in [0, T]$ and $\beta \in (0, 1]$. Then every fy α-tlt A^T_α of A is a fy Z-Salgr ext of the fy β-mlc A^M_β of A.*

Proof. A^T_α is a fy ext of A^M_β, since

$$\mu_{A^T_\alpha}(u) = \mu_A(u) + \alpha \geq \mu_A(u) \geq \beta \cdot \mu_A(u) = \mu_{A^M_\beta}(u) \forall u \in J.$$

If A^M_β is a fy Z-Salgr of J. Then A is a fy Z-Salgr of J by Proposition 3.14. It follows from Theorem 3.3 that A^T_α is a fy Z-Salgr of J $\forall\alpha \in [0, T]$. ☐

Theorem 3.2. *Let A be a fy set of Z-algr J and $\alpha \in [0, T]$. Then the fy α-tlt A^T_α of A is a fy Z-idl of J $\Longleftrightarrow$ A is a fy Z-idl of J.*

Definition 3.5. *When A_1 , A_2 are fy sets of Z-algr J, A_2 is called a* **fy Z-idl ext** *of A_1 if :*

(I_1) *A_2 is a fy ext of A_1.*

(I_1) *If A_1 is a fy Z-idl of J, then A_2 is a fy Z-idl of J.*

Proposition 3.9. *Let A be a fy Z-idl of a Z-algr J and $\alpha, \gamma \in [0, T]$. If $\alpha \geq \gamma$, then the fy α-tlt A^T_α of A is a fy Z-idl ext of the fy γ-tlt A^T_γ of A.*

Proposition 3.10. *Let A be a fy Z-idl of a Z-algr J and $\gamma \in [0, T]$. For every fy Z-idl ext B of the fy γ-tlt A^T_γ of A, $\exists\alpha \in [0, T] \ni \alpha \geq \gamma$ and B is a fy Z-idl ext of the fy α-tlt A^T_α of A.*

Proposition 3.11. *Let A be a fy Z-idl of a Z-algr J and $\alpha \in [0, T]$. Then the fy α-tlt A_α^T of A is a fy Z-idl ext of A.*

Proposition 3.12. *Arbitrary intersection of fy Z-idl ext of a fy Z-idl A of a Z-algr J is also a fy Z-idl ext of A.*

Theorem 3.3. *For $\alpha \in [0, T]$,let A_α^T be the fy α-tlt of a fy set A of a Z-algr J. Then A_α^T is a fy Z-idl of J $\Longleftrightarrow \forall t \in Im(A)$, $t > \alpha \Rightarrow U_\alpha(\mu_A; t)$ is an Z-idl of J.*

Proposition 3.13. *For any fy set A of a Z-algr J, A is a fy Z-idl of J $\Longleftrightarrow \forall \beta \in (0, 1]$, the fy β-mlc A_β^M of A is a fy Z-idl of J.*

Proposition 3.14. *Let A be a fy set of a Z-algr J, $\alpha \in [0, T]$ and $\beta \in (0, 1]$. Then every fy α-tlc A_α^T of A is a fy Z-idl ext of the fy β-mlc A_β^M of A.*

4. Conclusion

In this article, we have introduced fy α-tlts and fy β-mlcs of Z-algrs and discussed their properties.We extend this concept in our research work.

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