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EVEN EDGE MAGIC TOTAL LABELING OF SOME 2-REGULAR GRAPHS

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ABSTRACT. An edge magic total labeling (EMTL) of a graph G is $1-1$ function $f(V \cup E) = \{1, 2, 3, \ldots, \nu + \mu\}$ provided that the integer h is $f(x) + f(xy) + f(y) = h \; \forall \; xy \in E$. If $f(V) = \{2, 4, 6, ..., 2\nu\}$ then EMTL is called even EMTL. In this paper, we exhibit even EMTL of some $2-$ regular graphs.

1. INTRODUCTION

This article deals with no loops and multiple edges of finite graphs without direction. The vertices and the edges of a graph G can be indicated by V and E. In this article the domain will regularly be the set of vertices and edges, labels such as those referred to as total labeling, some labelings use the vertex set alone or the edge set alone, and we shall call the vertex labelings and the edge labels correspondingly. The recent survey of graph labelings is [2].

The labeling f of G is edge magic if every edge has the same weight and G is called an edge magic graph if an EMTL of G exists. Specifically EMTL of G to be a super EMTL with the additional that $f(V) = \{1, 2, \ldots, \nu\}$.In 1998, Enomoto etal [1] presented a special type of EMTL, namely super EMTL. All cycles are independenting show to be edge magic by Godbold and slater [3].

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2. Main results

Theorem 2.1. *The graph* $c_3 \cup c_{4s+2}, s \geq 1$ *is an even EMG.*

Proof. For $s = 1$ and $s = 2$ the labelings are

$$[4, 17, 8, 11, 10, 15], \quad [2, 13, 14, 3, 12, 1, 16, 7, 65, 18, 9]$$

and

$$[6, 25, 10, 19, 12, 23], \quad [2, 21, 18, 7, 16, 5, 20, 17, 15, 22, 11, 8, 9, 24, 3, 14, 1, 26, 13].$$

For $s \geq 3$, the labeling of c_3 is $[2s+2, 8s+9, 2s+6, 8s+3, 2s+8, 8s+7]$. The vertex label of c_{4s+2} is as follows.

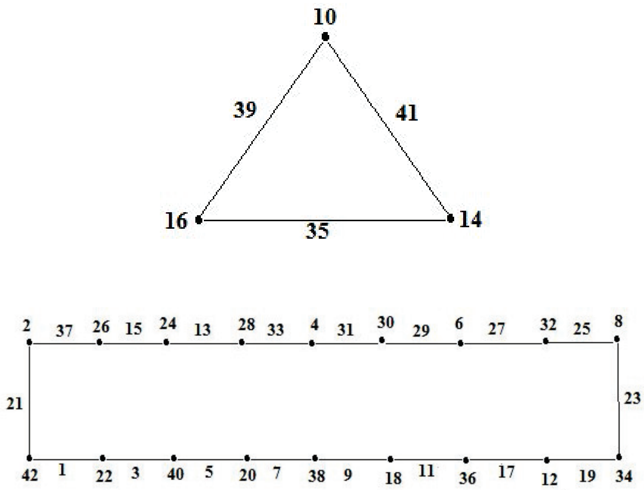
$$f(v_k) = \begin{cases} 2 & if \;\; k = 1 \\ 4s+8 & if \;\; k = 3 \\ k-1 & if \;\; k \quad odd, \;\; 5 \leq k \leq 2s+1 \\ 2s+4 & if \;\; k = 2s+3 \\ k+5 & if \;\; k \quad odd, \;\; 2s+5 \leq k \leq 4s+1 \\ k+4s+8 & if \;\; k \;\; even \end{cases}$$

The edge label of c_{4s+2} is as follows

$$f(e_k) = \begin{cases} 8s+5 & if \;\; k = 1 \\ 4s-1 & if \;\; k = 2 \\ 4s-3 & if \;\; k = 3 \\ 8s+9-2k & if \;\; 4 \leq k \leq 2s+1 \\ 4s+3 & if \;\; k = 2s+2 \\ 4s+1 & if \;\; k = 2s+3 \\ 8s+3-2k & if \;\; 2s+4 \leq k \leq 4s+1 \\ 4s+5 & if \;\; k = 4s+2 \end{cases}$$

Therefore f is an Even EMTL with magic constant $h = 12s + 17$. □

Example 1.



$c_3 \cup c_{18}, h = 65$

Theorem 2.2. *The graph $c_3 \cup c_{4s}, s \geq 1$ is an even EMG.*

Proof. The labeling of c_3 is $[2s+6, 8s+3, 2s+2, 8s+5, 2s+4, 8s+1]$. The vertex label of c_{4t} is

$$f(v_\kappa) = \begin{cases} \kappa + 3 & if \quad \kappa \quad \text{odd}, \quad 1 \leq \kappa \leq 2s-3 \\ \kappa + 9 & if \quad \kappa \quad \text{odd}, \quad 2s-1 \leq \kappa \leq 4s-3 \\ 2 & if \quad \kappa = 4s-1 \\ 4s + 6 + \kappa & if \quad \kappa \quad even \end{cases}$$

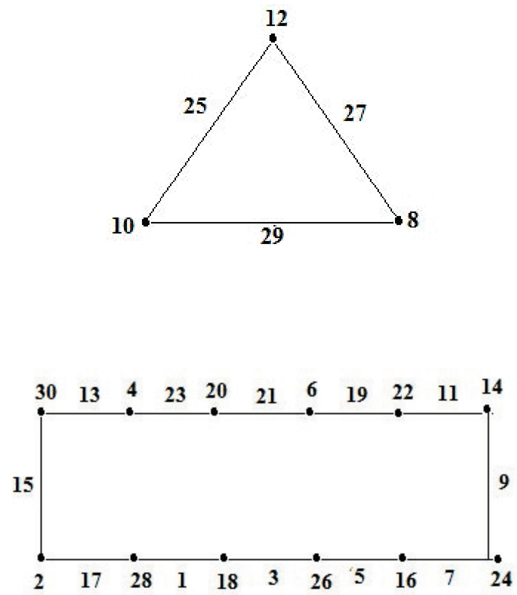
The edge label of c_{4s} is

$$f(e_\kappa) = \begin{cases} 8s + 1 - 2\kappa & if \quad 1 \leq \kappa \leq 2s-3 \\ 8s - 5 - 2\kappa & if \quad 2s-2 \leq \kappa \leq 4s-3 \\ 4s + 5 & if \quad \kappa = 4s-2 \\ 4s + 3 & if \quad \kappa = 4s-1 \\ 4s + 1 & if \quad \kappa = 4s \end{cases}$$

Therefore f is an even EMTL with magic constant $h = 12s + 11$.

□

Example 2.



$c_3 \cup c_{12}, h = 47$

Theorem 2.3. *The graph $c_4 \cup c_{4s-1}$ for $s \geq 1$ is an even EMG.*

Proof. The labeling of c_4 is $[4s+4, 3, 8s+4, 7, 4s, 5, 8s+6]$
The vertex label of c_{4s-1} is

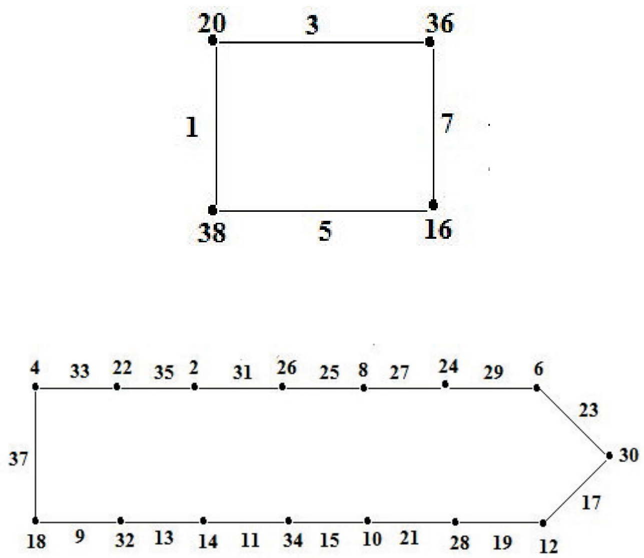
$$f(v_k) = \begin{cases} k+3 & if \;\; k \equiv 1 \;\; mod \;\; 4, \;\; k < 4s-3 \\ 4s+6 & if \;\; k = 2 \\ k+4s+2 & if \;\; k \equiv 2 \;\; mod \;\; 4, \;\; k > 2 \\ k-1 & if \;\; k \equiv 3 \;\; mod \;\; 4, \;\; k < 4s-1 \\ k+4s+6 & if \;\; k \equiv 0 \;\; mod \;\; 4 \\ 4s-2 & if \;\; k = 4s-3 \\ 4s+2 & if \;\; k = 4s-1 \end{cases}$$

The edge label of c_{4s} is

$$f(e_k) = \begin{cases} 8s+1 & if \;\; k=1 \\ 8s+3 & if \;\; k=2 \\ 8s-2k+5 & if \;\; k \equiv 3 \quad mod \quad 4 \\ 8s-5 & if \;\; k=4, \\ 8s-2k+1 & if \;\; k \equiv 0 \quad mod \quad 4, \;\; k \le 4s-7 \\ 8s-2k+5 & if \;\; k \equiv 5 \quad mod \quad 4 \\ 8s-2k+9 & if \;\; 6 \le k \le 4s-6, \;\; k \equiv 2 \quad mod \quad 4 \\ 11 & if \;\; k=4s-4 \\ 13 & if \;\; k=4s-3 \\ 9 & if \;\; k=4s-2 \\ 8s-5 & if \;\; k=4s-1 \end{cases}$$

Therefore f is an even EMTL with the magic constant $h = 12s+11$. □

Example 3.



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