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δ^*gα-CLOSED SETS IN TOPOLOGICAL SPACES

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ABSTRACT. In this paper, the authors introduce a new class of sets called $\delta^* g\alpha$-closed set in Topological spaces. Some of their properties and characterizations are investigated. Also we introduce and study a new class of space namely ${}_{\alpha\delta}T^{**}_{\frac{3}{4}}g\alpha$-space, ${}_{\delta}T^{**}_{c}$-space, ${}^{**}_{\delta\alpha}T_{\frac{1}{2}}$-space and ${}_{\delta\alpha}T^{**}_{c}$-space.

1. INTRODUCTION

Levine [13], Mashhour et al. [2], Njastad [15] and Velicko [14] introduced semi - open sets, pre-open sets, α-open sets and δ-closed sets respectively. Levine [12] introduced generalized closed (briefly g-closed) sets and studied their basic properties. Bhattacharya and Lahiri [16], Arya and Nour [17], Maki et a [6,7], Dontchev and Ganster [8] introduced generalized semi-closed (briefly gs-closed) sets, α-generalized closed (briefly αg-closed) sets and δ-generalized closed (briefly δg-closed) sets respectively. M.Vigneshwaran and R.Devi [10] introduced *generalized α-closed (briefly ${}^*g\alpha$-closed) sets. The purpose of this paper is to define a new class of closed sets called $\delta^* g\alpha$-closed sets and also we obtain some basic properties of $\delta^* g\alpha$ closed sets in topological spaces. Applying this set, we obtain some new spaces such as ${}_{\alpha\delta}T^{**}_{\frac{3}{4}}g\alpha$-space, ${}_{\delta}T^{**}_{c}$-space, ${}^{**}_{\delta\alpha}T_{\frac{1}{2}}$-space and ${}_{\delta\alpha}T^{**}_{c}$-space.

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2. PRELIMINARIES

Throughout this paper (X, τ) (or simply X) represent topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of X, cl(A), int(A) and X- A denote the closure of A, the interior of A and the complement of A respectively. Let us recall the following definitions, which are useful in the sequel.

Definition 2.1. *A subset A of (X, τ) is said to be*

(i) *semi-open set [13] if A⊆cl(int(A)).*
(ii) *pre-open set [2] if A⊆int(cl(A)).*
(iii) *semi-preopenset [1] if A⊆cl(int(cl(A))).*
(iv) *α-open set [15] if A⊆int(cl(int(A))).*
(v) *regular open set [11] if A=int(cl(A)).*

The complement of a semi-open (resp. pre-open, α-open, regular open)set is called semi-closed (resp. semi-closed, α-closed, regular closed).

Definition 2.2. *The δ-interior [14] of a subset A of X is the union of all regular open set of X contained in A and is denoted by Int_δ(A). The subset A is called δ-open [14] if A =Int_δ(A), i.e. a set is δ-open if it is the union of regular open sets. The complement of a δ-open is called δ-closed. Alternatively, a set A⊆ (X, τ) is called δ-closed [14] if $A = cl_\delta$(A), where*
$cl_\delta(A) = \{x \in X : int\,(cl\,(U)) \neq \phi, U \in \tau$ *and* $x \in U\}$.

Definition 2.3. *A subset A of (X, τ) is called*

(i) *a generalized closed (briefly g-closed) set [12] if cl(A)⊆U whenever A⊆U and U is open set in (X, τ).*
(ii) *a generalized semi-closed (briefly gs-closed) set [17] if scl(A)⊆U whenever A⊆U and U is open set in (X, τ).*
(iii) *a α-generalized closed (briefly αg-closed) set [6] if αcl(A)⊆U whenever A⊆U and U is open set in (X, τ).*
(iv) *a δ-generalized closed (briefly δg-closed) set [8] if cl_δ(A)⊆U whenever A⊆U and U is open set in (X, τ).*
(v) *a generalized preclosed (briefly gp-closed) set [5] if pcl(A)⊆U whenever A⊆U and U is open set in (X, τ).*
(vi) *a generalized semi-preclosed (briefly gsp-closed) set [3] if spcl(A)⊆U whenever A⊆U and U is open set in (X, τ).*

(vii) *a *generalized α-closed (briefly *gα-closed) set [10] if cl(A)$\subseteq$U whenever A$\subseteq$U and U is gα-open set in (X, τ).*

(viii) *a generalized-δ closed (briefly gδ-closed) set [4] if cl(A)$\subseteq$U whenever A$\subseteq$U and U is δ-open set in (X, τ).*

(ix) *a δgeneralized*-closed (briefly δg*-closed) set [21] if cl$_\delta$(A)$\subseteq$U whenever A$\subseteq$U and U is δ-open set in (X, τ).*

(x) *a generalized-δ semi closed (briefly gδs-closed) set [9] if scl(A)$\subseteq$U whenever A$\subseteq$U and U is δ-open set in (X, τ).*

(xi) *a δ-generalized b-closed (briefly δgb-closed) set [19] if bcl(A)$\subseteq$U whenever A$\subseteq$U and U is δ-open set in (X, τ).*

The complement of a g-closed (resp. gs-closed, αg-closed, δg-closed, gp-closed, gsp-closed, gδ-closed, gδ-closed, gδs-closed and δgb-closed) set is called g-open (resp. gs-open, αg-open, δg-open, gp-open, gsp-open, gδ-open, gδ*-open, gδs-open and δgb-open).*

Definition 2.4. *A space (X, τ) is called a*

(i) *$T_{1/2}$-space [12] if every g-closed set in it is closed.*

(ii) *$T_{3/4}$-space [8] if every δg-closed set in it is δ-closed.*

(iii) *${}_\delta T_{3/4}$-space [18] if every gδs-closed set in it is δ-closed.*

(iv) *${}_\delta T_{\delta gb}$-space [20] if every δgb-closed set in it is δ-closed.*

(v) *${}_\alpha T_d$-space [7] if every αg-closed set in it is g-closed.*

3. Properties of δ^*gα-closed sets in Topological Spaces

Definition 3.1. *A subset A of a space (X, τ) is called $\delta^* g\alpha$-closed if cl$_\delta$(A)$\subseteq$U whenever A$\subseteq$U and U is a *gα-open set in (X, τ).*

Theorem 3.1. *Every δ-closed set is *gα-closed.*

Proof. Let A be δ-closed and U be any gα-open set containing A. Since A is δ-closed, cl$_\delta$(A) = A. Therefore $cl_\delta(A) \subseteq A \subseteq U$. We know that $cl(A) \subseteq cl_\delta\,(A)$ $\subseteq$U. Hence A is *gα-closed. □

Theorem 3.2. *Every δ-closed set is $\delta^* g\alpha$-closed set. Converse is not true is showed through an example.*

Proof. Let A $\subseteq$ U and U is $^*g\alpha$-open set. Since A is δ-closed cl$_\delta$(A) = A, then cl$_\delta$(A)$\subseteq$U therefore A is $\delta^*g\alpha$-closed set. □

Example 1. *Let* $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}\}$; *Here* $\{a, c\}$ *is* $\delta^*g\alpha$*-closed but not* δ*-closed in* (X, τ).

Theorem 3.3. *Every* $\delta^*g\alpha$ *-closed set is gs-closed. Converse is not true is showed through an example.*

Proof. Let A$\subseteq$U and U is open set. Since every open set is $^*g\alpha$-open[9], then U is $^*g\alpha$-open set. Since A is $\delta^*g\alpha$-closed, then cl$_\delta$(A)$\subseteq$U. But scl(A)$\subseteq$cl$_\delta$(A), then scl(A)$\subseteq$U, Therefore A is gs-closed set. □

Example 2. *Let* $X = \{a, b, c\}$, $\tau = \{\phi, X, \{c\}, \{a, c\}\}$; *Here* $\{a\}$ *is gs-closed but not* $\delta^*g\alpha$ *-closed in* (X, τ).

Theorem 3.4. *Every* $\delta^*g\alpha$ *-closed set is* α*g-closed. Converse is not true is showed through an example.*

Proof. Let A$\subseteq$U and U is open set. Since every open set is $^*g\alpha$-open, then U is $^*g\alpha$-open set. Since A is $\delta^*g\alpha$-closed, then cl$_\delta$(A)$\subseteq$U. But αcl(A)$\subseteq$cl$_\delta$(A), then αcl(A)$\subseteq$U, Therefore A is αg-closed set. □

Example 3. *Let* $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}, \{a, b\}\}$; *Here* $\{b\}$ *is* α*g-closed but not* $\delta^*g\alpha$*-closed in* (X, τ).

Theorem 3.5. *Every* $\delta^*g\alpha$ *-closed set is gsp-closed. Converse is not true is showed through an example.*

Proof. Let A$\subseteq$U and U is open set. Since every open set is $^*g\alpha$-open, then U is $^*g\alpha$-open set. Since A is $\delta^*g\alpha$-closed, then cl$_\delta$(A)$\subseteq$U. But spcl(A)$\subseteq$cl$_\delta$(A), then spcl(A)$\subseteq$U, Therefore A is gsp-closed set. □

Example 4. *Let* $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$; *Here* $\{a\}$ *and* $\{b\}$ *are gsp-closed but not* $\delta^*g\alpha$*-closed in* (X, τ).

Theorem 3.6. *Every* $\delta^*g\alpha$ *-closed set is gp-closed. Converse is not true is showed through an example.*

Proof. Let A$\subseteq$U and U is open set. Since every open set is $^*g\alpha$-open, then U is $^*g\alpha$-open set. Since A is $\delta^*g\alpha$-closed, then cl$_\delta$(A)$\subseteq$U. But pcl(A)$\subseteq$cl$_\delta$(A), then pcl(A)$\subseteq$U, Therefore A is gp-closed. □

Example 5. *Let* $X = \{a, b, c\}$, $\tau = \{\phi, X, \{b, c\}\}$*; Here* $\{b\}$ *and* $\{c\}$ *are gp-closed but not* $\delta^* g\alpha$*-closed in* (X, τ).

Theorem 3.7. *Every* $\delta^* g\alpha$ *-closed set is* δ*gp-closed. Converse is not true is showed through an example.*

Proof. Let A⊆U and U is δ-open set. Since every δ-open set is $^*g\alpha$-open, then U is $^*g\alpha$-open set. Since A is $\delta^* g\alpha$-closed, then cl_δ(A)⊆U. But pcl(A)⊆cl_δ(A), then pcl(A)⊆U, Therefore A is δgp-closed. □

Example 6. *Let* $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a, b\}\}$*; Here* $\{a\}$, $\{b\}$ *and* $\{a, b\}$*is* δ*gp-closed but not* $\delta^* g\alpha$*-closed in* (X, τ).

Theorem 3.8. *Every* $\delta^* g\alpha$ *-closed set is g*δ*-closed. Converse is not true is showed through an example.*

Proof. Let A⊆U and U is δ-open set. Since every δ-open set is $^*g\alpha$-open, then U is $^*g\alpha$-open set. Since A is $\delta^* g\alpha$-closed, then cl_δ(A)⊆U. But cl(A)⊆cl_δ(A), then cl(A)⊆U, Therefore A is gδ-closed. □

Example 7. *Let* $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}\}$*; Here* $\{a\}$*is g*δ*-closed but not* $\delta^* g\alpha$*-closed in* (X, τ).

Theorem 3.9. *Every* $\delta^* g\alpha$*-closed set is* δg^**-closed. Converse is not true is showed through an example.*

Proof. Let A⊆U and U is δ-open set. Since δ-every open set is $^*g\alpha$-open, then U is $^*g\alpha$-open set. Since A is $\delta^* g\alpha$-closed, then cl_δ(A)⊆U. hence A is gδ^*-closed. □

Example 8. *Let* $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a, b\}\}$*; Here* $\{a\}$,$\{b\}$ *and* $\{a, b\}$ *are g*δ^**-closed but not* $\delta^* g\alpha$*-closed in* (X, τ).

Theorem 3.10. *Every* $\delta^* g\alpha$ *-closed set is g*δ*s-closed. Converse is not true is showed through an example.*

Proof. Let A⊆U and U is δ-open set. Since every δ-open set is $^*g\alpha$-open, then U is $^*g\alpha$-open set. Since A is $\delta^* g\alpha$-closed, then cl_δ(A)⊆U. But scl(A)⊆cl_δ(A), then scl(A)⊆U, Therefore A is gδs-closed. □

Example 9. *Let* $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}, \{a, b\}\}$*; Here* $\{a\}$,$\{b\}$ *and* $\{a, b\}$ *are g*δ*s-closed but not* $\delta^* g\alpha$*-closed in* (X, τ).

Theorem 3.11. *Every $\delta^* g\alpha$ -closed set is δgb-closed. Converse is not true is showed through an example.*

Proof. Let A⊆U and U is δ-open set. Since every δ-open set is $^*g\alpha$-open, then U is $^*g\alpha$-open set. Since A is $\delta^* g\alpha$-closed, then cl$_\delta$(A)⊆U. But bcl(A)⊆cl$_\delta$(A), then bcl(A)⊆U, Therefore A is δgb-closed. □

Example 10. *Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{c\}, \{a, c\}\}$; Here $\{a\}$,$\{c\}$ and $\{a, c\}$ are δgb-closed but not $\delta^* g\alpha$-closed in (X, τ).*

Theorem 3.12. *The finite union of $\delta^* g\alpha$-closed sets is $\delta^* g\alpha$-closed.*

Proof. Let $\{A_i/i = 1, 2, ...n\}$ be a finite class of $\delta^* g\alpha$-closed subsets of a space (X, τ). Then for each $^*g\alpha$-open set U_i in X containing $A_i, cl_\delta(A_i) \subseteq ?U_i, i \in \{1, 2, ...n\}$. Hence $\bigcup_i A_i \subseteq \bigcup_i U_i = V$. Since arbitrary union of $^*g\alpha$-open sets in (X, τ) is also $^*g\alpha$-open set in (X, τ), V is $^*g\alpha$-open in (X, τ). Also $\bigcup_i cl_\delta(A_i) = cl_\delta(\bigcup_i A_i) \subseteq V$. Therefore $U_i A_i$ is $\delta^* g\alpha$-closed in (X, τ). □

Remark 3.1. *Intersection of any two $\delta^* g\alpha$-closed sets in (X, τ) need not be $\delta^* g\alpha$-closed in (X, τ), it can be seen by the following example.*

Example 11. *Let $X = \{a, b, c, d\}, \tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}; \{b, c\}$ and $\{b, d\}$ are $\delta^* g\alpha$-closed sets but their intersection $\{b\}$ is not $\delta^* g\alpha$-closed.*

Theorem 3.13. *Let A be a $\delta^* g\alpha$-closed set of (X, τ), then cl$_\delta$(A)-A does not contain a non-empty $*g\alpha$-closed set.*

Proof. Suppose that A is $\delta^* g\alpha$-closed, let F be a $^*g\alpha$-closed set contained in cl$_\delta$(A)-A. Now F^c is $^*g\alpha$-open set of (X, τ) such that A⊆ F^c. Since A is $\delta^* g\alpha$-closed set of (X, τ), then cl$_\delta(A) \subseteq F^c$. Thus F⊆ $(cl_\delta((A))^c$. Also F $\subseteq cl_\delta(A) - A$. Therefore F⊆ $(cl_\delta(A)) \subset \bigcap(cl_\delta(A)) = \phi$. Hence F=$\phi$. □

Theorem 3.14. *If A is $^*g\alpha$-open and $\delta^* g\alpha$-closed subset of (X, τ) then A is an δ-closed subset of (X, τ).*

Proof. Since A is $g\alpha$-open and $\delta^* g\alpha$-closed, $cl_\delta(A) \subseteq A$. Hence A is δ-closed. □

Theorem 3.15. *The intersection of a $\delta^* g\alpha$-closed set and a δ-closed set is always $\delta^* g\alpha$-Closed.*

Proof. Let A be $\delta^{*}g\alpha$-Closed and let F be δ-closed. If U is an $^{*}g\alpha$-open set with $A \cap F \subseteq U$, then $A \subseteq U \cap F^{c}$ and so $\mathrm{cl}_{\delta}(A) \subseteq U \cap F^{c}$. Now $\mathrm{cl}_{\delta}(A \cap F) \subseteq cl_{\delta}(A) \cap F \subseteq U$. Hence $A \cap F$ is $\delta^{*}g\alpha$-closed. □

Theorem 3.16. *In a $T_{3/4}$-space every $\delta^{*}g\alpha$-closed set is δ-closed.*

Proof. Let X be $T_{3/4}$-space. Let A be $\delta^{*}g\alpha$-closed set of X. We know that every $\delta^{*}g\alpha$-closed set is δg-closed. Since X is $T_{3/4}$-space, A is δ-closed. □

Theorem 3.17. *If A is a $\delta^{*}g\alpha$-closed set in a space (X,τ) and $A \subseteq B \subseteq cl_{\delta}(A)$,then B is also a $\delta^{*}g\alpha$-closed set.*

Proof. Let U be a $^{*}g\alpha$-open set of (X,τ) such that $B \subseteq cl_{\delta}(A)$, Then $A \subseteq U$. Since A is $\delta^{*}g\alpha$-closed set, $cl_{\delta}(A) \subseteq U$. Also since $B \subseteq cl_{\delta}(A)$, $cl_{\delta}(B) \subseteq cl_{\delta}(cl_{\delta}(A)) = cl_{\delta}(A) \subseteq U$. Implies cl?(B)⊆U. Therefore B is also a $\delta^{*}g\alpha$-closed set. □

Theorem 3.18. *Let A be $\delta^{*}g\alpha$-closed of (X,τ), then A is δ-closed iff $cl_{\delta}(A) - A$ is $^{*}g\alpha$-closed.*

Proof. Necessity. Let A be a δ-closed subset of X. Then $cl_{\delta}(A) = A$ and so $cl_{\delta}(A) - A = \phi$ which is $^{*}g\alpha$-closed.
Sufficiency. Since A is $\delta^{*}g\alpha$-closed, by proposition, $cl_{\delta}(A) - A$ does not contain a non-empty $^{*}g\alpha$-closed set. But $cl_{\delta}(A) - A = \phi$. That is $cl_{\delta}(A) = A$. Hence A is δ-closed. □

4. Some spaces using $\delta^{*}g\alpha$-closed sets

We introduce the following definition.

Definition 4.1. *A space (X,τ) is called $_{\alpha\delta}T^{**}_{\frac{3}{4}}g\alpha$-space if every $\delta^{*}g\alpha$-closed set is an δ-closed.*

Theorem 4.1. *For a topological space (X,τ), the following conditions are equivalent.*

(i) *(X,τ) is a $_{\alpha\delta}T^{**}_{\frac{3}{4}}g\alpha$-space.*
(ii) *Every singleton $\{x\}$ is either $^{*}g\alpha$-closed or δ-open.*

Proof. $(i) \Rightarrow (ii)$ Let $x \in X$. Suppose $\{x\}$ is not a $^*g\alpha$-closed set of (X, τ). Then $X - \{x\}$ is not a $^*g\alpha$-open set. Thus $X - \{x\}$ is an $\delta^*g\alpha$-closed set of (X, τ). Since (X, τ) is $_{\alpha\delta}T^{**}_{\frac{3}{4}}g\alpha$, $X - \{x\}$ is an δ-closed set of (X, τ), i.e. $\{x\}$ is δ-open set of (X, τ).
$(ii) \Rightarrow (i)$ Let A be an $\delta^*g\alpha$-closed set of (X, τ). Let $x \in cl_\delta(A)$. By (ii), $\{x\}$ is either $^*g\alpha$-closed or δ-open.
Case(i). Let $\{x\}$ be $^*g\alpha$-closed. If we assume that $x \notin A$, then we would have $x \in cl_\delta(A) - A$, which cannot happen according to proposition Hence $x \in A$.
Case(ii) Let $\{x\}$ be δ-open. Since $x \in cl_\delta(A)$, then $\{x\} \bigcap A = \phi$. This shows that $x \in A$. So in both cases we have $cl_\delta(A) \subseteq A$. Trivially $A \subseteq cl_\delta(A)$. Therefore $A = cl_\delta(A)$ or equivalently A is δ-closed. Hence (X, τ) is a $_{\alpha\delta}T^{**}_{\frac{3}{4}}g\alpha$-space. □

Theorem 4.2. *Every $_\delta T_{\frac{3}{4}}$-space is a $_{\alpha\delta}T^{**}_{\frac{3}{4}}g\alpha$-space. Converse is not true is showed through an example.*

Proof. Let A be a $\delta^*g\alpha$-closed set of (X, τ). Since every $\delta^*g\alpha$-closed set is gδ-closed, then A is gδsŒčlosed. Since (X, τ) is $_\delta T_{\frac{3}{4}}$-space, then A is δ-closed. Therefore (X, τ) is $_{\alpha\delta}T^{**}_{\frac{3}{4}}g\alpha$-space. □

Example 12. *Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}, \{c\}, \{a, c\}\}$; Here it is $_{\alpha\delta}T^{**}_{\frac{3}{4}}g\alpha$-space but not $_\delta T_{\frac{3}{4}}$-space, Since $\{a\}$ is $g\delta$-closed set but not δ-closed set.*

Theorem 4.3. *Every $_\delta T_{\delta gb}$-space is a $_{\alpha\delta}T^{**}_{\frac{3}{4}}g\alpha$-space. Converse is not true is showed through an example.*

Proof. Let A be a $\delta^*g\alpha$-Closed set of (X, τ). Since every $\delta^*g\alpha$-Closed set is δgb-closed, then A is δgb-closed. Since (X, τ) is $_\delta T_{\delta gb}$-space, then A is δ-closed. Therefore (X, τ) is $_{\alpha\delta}T^{**}_{\frac{3}{4}}g\alpha$-space. □

Example 13. *Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$; Here it is $_{\alpha\delta}T^{**}_{\frac{3}{4}}g\alpha$-space but not $_\delta T_{\delta gb}$-space, Since $\{a\}$ is δgb-closed set but not δ-closed set.*

Definition 4.2. *A space (X, τ) is called $_\delta T^{**}_c$-space if every gs-Closed set in it is an $\delta^*g\alpha$-closed.*

Theorem 4.4. *Every $_\delta T^{**}_c$-space is a $_\alpha T_d$-space. Converse is not true is showed through an example.*

Proof. Let A be a αg-Closed set of (X, τ). Since αg-Closed set is gs-closed, then A is gs-closed. Since (X, τ) is a $_\delta T^{**}_c$-space in (X, τ), then A is $\delta^*g\alpha$-closed.

Since every $\delta^{*}g\alpha$-closed set is g-closed, then A is g-closed. Therefore (X,τ) is ${}_{\alpha}T_{d}$-space. □

Example 14. *Let* $X = \{a,b,c\}$, $\tau = \{\phi, X, \{a\}, \{b\}, \{a,b\}\}$*; Here it is* ${}_{\alpha}T_{d}$*-space but not* ${}_{\delta}T_{c}^{**}$*-space.*

Definition 4.3. *A space* (X,τ) *is called* ${}_{\delta\alpha}T_{c}^{**}$*-space if every* αg*-Closed set in it is an* $\delta^{*}g\alpha$*-closed.*

Theorem 4.5. *Every* ${}_{\delta}T_{c}^{**}$*-space is a* ${}_{\delta\alpha}T_{c}^{**}$*-space. Converse is not true is showed through an example.*

Proof. Let A be a αg-Closed set of (X,τ). Since every αg-Closed set is gs Œčclosed, then A is gs-closed. Since (X,τ) is ${}_{\delta}T_{c}^{**}$-space, then A is $\delta^{*}g\alpha$-closed. Therefore (X,τ) is an ${}_{\delta\alpha}T_{c}^{**}$-space. □

Example 15. *Let* $X = \{a,b,c,d\}$, $\tau = \{\phi, X, \{a\}, \{b\}, \{a,b\}\}$*; Here it is* ${}_{\delta\alpha}T_{c}^{**}$*-space but not* ${}_{\delta}T_{c}^{**}$*-space, Since* $\{a\}$ *is gs-closed set but not* $\delta^{*}g\alpha$*-closed set.*

Definition 4.4. *A space* (X,τ) *is called* ${}^{**}_{\delta\alpha}T_{\frac{1}{2}}$*-space if every* g*-Closed set in it is an* $\delta^{*}g\alpha$*-closed.*

Theorem 4.6. *Every* ${}_{\delta}T_{c}^{**}$*-space is a* ${}^{**}_{\delta\alpha}T_{\frac{1}{2}}$*-space. Converse is not true is showed through an example.*

Proof. Let A be a g-Closed set of (X,τ). Since every g-Closed set is gs-closed, then A is gs-closed. Since (X,τ) is ${}_{\delta}T_{c}^{**}$-space, then A is $\delta^{*}g\alpha$-closed. Therefore (X,τ) is an ${}^{**}_{\delta\alpha}T_{\frac{1}{2}}$-space. □

Example 16. *Let* $X = \{a,b,c,d\}$, $\tau = \{\phi, X, \{a\}, \{a,b\}, \{a,b,c\}\}$*; Here it is* ${}^{**}_{\delta\alpha}T_{\frac{1}{2}}$*-space but not* ${}_{\delta}T_{c}^{**}$*-space, Since* $\{b\}$ *is gs-closed set but not* $\delta^{*}g\alpha$*-closed set.*

Theorem 4.7. *Every* ${}_{\delta\alpha}T_{c}^{**}$*-space is a* ${}^{**}_{\delta\alpha}T_{\frac{1}{2}}$*-space. Converse is not true is showed through an example.*

Proof. Let A be a g-Closed set of (X,τ). Since every g-Closed set is αg-closed, then A is αg-closed. Since (X,τ) is ${}_{\delta\alpha}T_{c}^{**}$-space, then A is $\delta^{*}g\alpha$-closed. Therefore (X,τ) is an ${}^{**}_{\delta\alpha}T_{\frac{1}{2}}$-space. □

Example 17. *Let* $X = \{a,b,c,d\}$, $\tau = \{\phi, X, \{a\}, \{a,b\}, \{a,b,c\}\}$*; Here it is* ${}^{**}_{\delta\alpha}T_{\frac{1}{2}}$*-space but not* ${}_{\delta\alpha}T_{c}^{**}$*-space, Since* $\{b\}$ *is* αg*-closed set but not* $\delta^{*}g\alpha$*-closed set.*

5. Conclusion

This article defined $\delta^* g\alpha$-closed set in Topological Spaces and relation with other exciting sets in topology were studied. Along with that some of there properties were discussed. Also ${}_{\alpha\delta}T^{**}_{\frac{3}{4}}g\alpha$-space, ${}_{\delta}T^{**}_{c}$-space, ${}^{**}_{\delta\alpha}T_{\frac{1}{2}}$-space and ${}_{\delta\alpha}T^{**}_{c}$-space of a set were introduced and discussed their properties. This set can be used to derive few more functions such as $\delta^* g\alpha$-continuous and $\delta^* g\alpha$-irresolute functions. In addition to that it can be extended to homeomorphsims of topological spaces.

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