

CARTESIAN PRODUCT OF INTUITIONISTIC FUZZY PMS-IDEALS OF PMS-ALGEBRAS

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ABSTRACT. In this paper, the notion of intuitionistic fuzzy PMS-ideal of PMS-algebras is introduced. Also, the notions of an intuitionistic fuzzy translation and intuitionistic fuzzy multiplications of PMS-algebras are introduced. Furthermore, Cartesian product of intuitionistic fuzzy PMS-ideals of PMS-algebras is introduced and investigated several properties. Finally, the Cartesian product of intuitionistic fuzzy translation and multiplication are introduced.

1. INTRODUCTION

Present-day science and technology is featured with advanced processes and phenomena for which complete data isn't continuously offered. In such cases, mathematical models are developed to handle numerous forms of systems containing components of uncertainty. An outsized a part of these models square measure supported a recent extension of the standard pure mathematics, namely, the alleged fuzzy sets. Fuzzy sets (FSs, for short) were introduced by L.A. Zadeh [5] in 1965.

The other basic field of analytic interest for outlining by Lotfi Zadeh, that he considerably extended Atanassov [1] by launching the conception of "Intuitionistic Fuzzy Sets" and investigated their basis properties. Sithar Selvam

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2010 *Mathematics Subject Classification.* 03E72, 03G25, 06F35, 08A72.

Key words and phrases. Intuitionistic fuzzy PMS-ideal, Intuitionistic fuzzy γ -multiplication, Intuitionistic fuzzy γ -translation, Cartesian product of intuitionistic fuzzy set, Cartesian product of intuitionistic fuzzy translation and multiplication.

and Nagalakshmi [2] introduced the concept of homomorphism and Cartesian product of fuzzy PMS-algebras and established some of its properties in detail.

In this paper, the notions of an intuitionistic fuzzy translation and intuitionistic fuzzy multiplications are introduced. Furthermore, Cartesian product of intuitionistic fuzzy PMS-ideals of PMS- algebras are introduced and investigates several properties. Finally, the Cartesian product of intuitionistic fuzzy translation and multiplication are introduced.

2. PRELIMINARIES

Definition 2.1. A BCK-algebra is an algebra $(P; *, 0)$ of type $(2, 0)$ fulfilling the subsequent conditions:

- (i) $(p * q) * (p * r) \leq (r * q)$,
- (ii) $p * (p * q) \leq q$,
- (iii) $p \leq p$,
- (iv) $p \leq q$ and $q \leq p \Rightarrow p = q$,
- (v) $0 \leq p \Rightarrow p = 0$,

where $p \leq q$ is defined by $p * q = 0$, for all $p, q, r \in P$.

Definition 2.2. A non-empty set P with a continuing zero and a binary arithmetic operation $'*'$ is known as PMS-algebra if it satisfies the subsequent axioms.

- 1. $0 * p = p$;
- 2. $(q * p) * (r * p) = r * q$, for all $p, q, r \in P$.

For brevity, P denotes a PMS-algebra. In P , we define a binary relation $\leq$ by $p \leq q$ if and only if $p * q = 0$.

Definition 2.3. A fuzzy set ζ in a P is referred to as a fuzzy PMS-ideal of P if $\zeta(q * p) \geq m\{\zeta(r * q), \zeta(r * p)\}$, for all $p, q \in P$.

Definition 2.4. Let $f : P \rightarrow P$ be an associate endomorphism and ζ be a fuzzy set in P . We tend to outline a replacement fuzzy set in P as $\zeta^f(p) = \zeta(f(p))$ for all $p \in P$.

3. INTUITIONISTIC FUZZY TRANSLATION AND INTUITIONISTIC FUZZY MULTIPLICATION

Definition 3.1. Let $f : P \rightarrow P$ be associate endomorphism and (ζ_ψ, ξ_ψ) be an intuitionistic fuzzy set in P . We tend to outline a replacement intuitionistic fuzzy set (IFS) in P as $\zeta_\psi^f(p) = \zeta_\psi(f(p))$ and $\xi_\psi^f(p) = \xi_\psi(f(p))$ for all $p \in P$.

Definition 3.2. [3] An IFS (ζ_ψ, ξ_ψ) in P is known as an IF PMS-ideal of P if

- (i) $\zeta_\psi(0) \geq \zeta_\psi(p)$ and $\xi_\psi(0) \geq \xi_\psi(p)$,
- (ii) $\zeta_\psi(q * p) \geq m\{\zeta_\psi(r * q), \zeta_\psi(r * p)\}$,
- (iii) $\xi_\psi(q * p) \leq M\{\xi_\psi(r * q), \xi_\psi(r * p)\}$,

for all $p, q, r \in P$.

Definition 3.3. Let $f : P \rightarrow P$ be an endomorphism and $((\zeta_\psi)_\gamma^T, (\xi_\psi)_\gamma^T)$ be an IF γ -translation of ζ_ψ and ξ_ψ in P . We define a new IFS in P by $((\zeta_\psi)_\gamma^T, (\xi_\psi)_\gamma^T)$ in P as $((\zeta_\psi)_\gamma^T)f(p) = (\zeta_\psi)_\gamma^T(f(p)) = \zeta_\psi[f(p)] + \gamma$ and $((\xi_\psi)_\gamma^T)f(p) = (\xi_\psi)_\gamma^T(f(p)) = \xi_\psi[f(p)] - \gamma$, for any $p \in P$.

Theorem 3.1. If (ζ_ψ, ξ_ψ) is an IF ideal of P , then $((\zeta_\psi)_\gamma^T, (\xi_\psi)_\gamma^T)^f$ is an IF ideal of P .

Proof. Let (ζ_ψ, ξ_ψ) be an IF ideal of P . Now,

$$((\zeta_\psi)_\gamma^T)f(0) = (\zeta_\psi)_\gamma^T[f(0)] = \zeta_\psi[f(0)] + \gamma \geq \zeta_\psi[f(p)] + \gamma = (\zeta_\psi)_\gamma^T[f(p)]$$

and

$$((\xi_\psi)_\gamma^T)f(0) = (\xi_\psi)_\gamma^T[f(0)] = \xi_\psi[f(0)] - \gamma \leq \xi_\psi[f(p)] - \gamma = (\xi_\psi)_\gamma^T[f(p)].$$

Let $p, q \in P$. Then

$$\begin{aligned} ((\zeta_\psi)_\gamma^T)f(p) &= (\zeta_\psi)_\gamma^T[f(p)] = \zeta_\psi[f(p)] + \gamma \geq m\{\zeta_\psi(f(q) * f(p)), \zeta_\psi(f(q))\} + \gamma = \\ &= m\{\zeta_\psi(f(q * p)), \zeta_\psi(f(q))\} + \gamma = m\{\zeta_\psi(f(q * p)) + \gamma, \zeta_\psi(f(q)) + \gamma\} = m\{(\zeta_\psi)_\gamma^T[f(q * p)], (\zeta_\psi)_\gamma^T[f(q)]\} \end{aligned}$$

and

$$\begin{aligned} ((\xi_\psi)_\gamma^T)f(p) &= (\xi_\psi)_\gamma^T[f(p)] = \xi_\psi[f(p)] - \gamma \leq M\{\xi_\psi(f(q) * f(p)), \xi_\psi(f(q))\} - \gamma = \\ &= M\{\xi_\psi(f(q * p)), \xi_\psi(f(q))\} - \gamma = M\{\xi_\psi(f(q * p)) - \gamma, \xi_\psi(f(q)) - \gamma\} = M\{(\xi_\psi)_\gamma^T[f(q * p)], (\xi_\psi)_\gamma^T[f(q)]\}. \end{aligned}$$

Hence $((\zeta_\psi)_\gamma^T, (\xi_\psi)_\gamma^T)^f$ is an IF ideal of P . □

Definition 3.4. Let $f : P \rightarrow P$ be an endomorphic and $((\zeta_\psi)_\gamma^M, (\xi_\psi)_\gamma^M)$ be an IF- γ -multiplication of (ζ_ψ, ξ_ψ) in P . We define new IFS in p by $((\zeta_\psi)_\gamma^M)_f(p) = (\zeta_\psi)_\gamma^M(f(p)) = \gamma\zeta_\psi[f(p)]$ and $((\xi_\psi)_\gamma^M)_f(p) = (\xi_\psi)_\gamma^M(f(p)) = \gamma\xi_\psi[f(p)]$, for any $p \in P$.

Theorem 3.2. If (ζ_ψ, ξ_ψ) is an IF ideal of P , then therefore is $((\zeta_\psi)_\gamma^M, (\xi_\psi)_\gamma^M)_f$.

Proof. Let (ζ_ψ, ξ_ψ) is an IF ideal of Q and let $p, q \in P$. Then

$$((\zeta_\psi)_\gamma^M)_f(0) = (\zeta_\psi)_\gamma^M[f(0)] = \gamma\zeta_\psi(f(0)) \geq \gamma\zeta_\psi(f(p)) = (\zeta_\psi)_\gamma^M[f(p)]$$

and

$$((\xi_\psi)_\gamma^M)_f(0) = (\xi_\psi)_\gamma^M[f(0)] = \gamma\xi_\psi(f(0)) \leq \gamma\xi_\psi(f(p)) = (\xi_\psi)_\gamma^M[f(p)].$$

Also

$$\begin{aligned} ((\zeta_\psi)_\gamma^M)_f(p) &= (\zeta_\psi)_\gamma^M[f(p)] = \gamma\zeta_\psi(f(p)) \geq \gamma m\{\zeta_\psi(f(q) * f(p)), \zeta_\psi(f(q))\} = \\ &= \gamma m\{\zeta_\psi(f(q * p)), \zeta_\psi(f(q))\} = m\{\gamma\zeta_\psi(f(q * p)), \gamma\zeta_\psi(f(q))\} = m\{(\zeta_\psi)_\gamma^M[f(q * p)], \\ &(\zeta_\psi)_\gamma^M[f(q)]\} \end{aligned}$$

and

$$\begin{aligned} ((\xi_\psi)_\gamma^M)_f(p) &= (\xi_\psi)_\gamma^M[f(p)] = \gamma\xi_\psi(f(p)) \leq \gamma M\{\xi_\psi(f(q) * f(p)), \xi_\psi(f(q))\} = \\ &= \gamma M\{\xi_\psi(f(q * p)), \xi_\psi(f(q))\} = M\{\gamma\xi_\psi(f(q * p)), \gamma\xi_\psi(f(q))\} = M\{(\xi_\psi)_\gamma^M[f(q * p)], \\ &(\xi_\psi)_\gamma^M[f(q)]\}. \end{aligned}$$

Hence $((\zeta_\psi)_\gamma^M, (\xi_\psi)_\gamma^M)_f$ is an IF ideal of P . □

4. CARTESIAN PRODUCT OF INTUITIONISTIC FUZZY PMS-IDEALS

Definition 4.1. [4] Let (ζ_ψ, ξ_ψ) and $(\zeta_\varpi, \xi_\varpi)$ are IF sets in P . Then the Cartesian product $\zeta_\psi \times \zeta_\varpi : P \times P \rightarrow [0, 1]$ and $\xi_\psi \times \xi_\varpi : P \times P \rightarrow [0, 1]$ are defined by $(\zeta_\psi \times \zeta_\varpi)(p, q) = m\{\zeta_\psi(p), \zeta_\varpi(q)\}$ and $(\xi_\psi \times \xi_\varpi)(p, q) = M\{\xi_\psi(p), \xi_\varpi(q)\}$ for all $p, q \in P$.

Theorem 4.1. If (ζ_ψ, ξ_ψ) and $(\zeta_\varpi, \xi_\varpi)$ are IF ideals in P , then $\zeta_\psi \times \zeta_\varpi$ and $\xi_\psi \times \xi_\varpi$ are an IF ideal in $P \times P$.

Proof. Let $(p_1, p_2) \in P \times P$. Then

$$(\zeta_\psi \times \zeta_\varpi)(0, 0) = m\{\zeta_\psi(0), \zeta_\varpi(0)\} \geq m\{\zeta_\psi(p_1), \zeta_\varpi(p_2)\} = (\zeta_\psi \times \zeta_\varpi)(p_1, p_2)$$

and

$$(\xi_\psi \times \xi_\varpi)(0, 0) = M\{\xi_\psi(0), \xi_\varpi(0)\} \leq M\{\xi_\psi(p_1), \xi_\varpi(p_2)\} = (\xi_\psi \times \xi_\varpi)(p_1, p_2).$$

Let $(p_1, p_2), (q_1, q_2), (r_1, r_2) \in P \times P$. Then

$$\begin{aligned} (\zeta_\psi \times \zeta_\varpi)[(q_1, q_2) * (p_1, p_2)] &= (\zeta_\psi \times \zeta_\varpi)[q_1 * p_1, q_2 * p_2] = m\{\zeta_\psi(q_1 * p_1), \zeta_\varpi(q_2 * p_2)\} \geq \\ &m\{m\{\zeta_\psi(r_1 * q_1), \zeta_\psi(r_1 * p_1)\}, m\{\zeta_\varpi(r_2 * q_2), \zeta_\varpi(r_2 * p_2)\}\} = m\{m\{\zeta_\psi(r_1 * q_1), \zeta_\varpi(r_2 * \\ &q_2)\}, m\{\zeta_\psi(r_1 * p_1), \zeta_\varpi(r_2 * p_2)\}\} = m\{(\zeta_\psi \times \zeta_\varpi)((r_1 * q_1), (r_2 * q_2)), (\zeta_\psi \times \zeta_\varpi)((r_1 * \\ &p_1), (r_2 * p_2))\} \end{aligned}$$

and

$$\begin{aligned} (\xi_\psi \times \xi_\varpi)[(q_1, q_2) * (p_1, p_2)] &= (\xi_\psi \times \xi_\varpi)[q_1 * p_1, q_2 * p_2] \\ &= M\{\xi_\psi(q_1 * p_1), \xi_\varpi(q_2 * p_2)\} \leq M\{M\{\xi_\psi(r_1 * q_1), \xi_\psi(r_1 * p_1)\}, M\{\xi_\varpi(r_2 * q_2), \xi_\varpi(r_2 * \\ &p_2)\}\} = M\{M\{\xi_\psi(r_1 * q_1), \xi_\varpi(r_2 * q_2)\}, M\{\xi_\psi(r_1 * p_1), \xi_\varpi(r_2 * p_2)\}\} = M\{(\xi_\psi \times \xi_\varpi)((r_1 * q_1), (r_2 * q_2)), (\xi_\psi \times \xi_\varpi)((r_1 * \\ &p_1), (r_2 * p_2))\}. \end{aligned}$$

Hence, $\zeta_\psi \times \zeta_\varpi$ and $\xi_\psi \times \xi_\varpi$ are an IF ideal in $P \times P$. $\square$

Definition 4.2. Let γ be an IF subset of P . The strongest IF γ -relation on P is the IF subset $(\zeta_\psi)_\gamma$ and $(\xi_\psi)_\gamma$ of $P \times P$ given by $(\zeta_\psi)_\gamma(p, q) = m\{\gamma(p), \gamma(q)\}$ and $(\xi_\psi)_\gamma(p, q) = M\{\gamma(p), \gamma(q)\}$, for all $p, q \in P$.

Theorem 4.2. If γ is an IF ideal of P , then $((\zeta_\psi)_\gamma, (\xi_\psi)_\gamma)$ is an IF ideal of $P \times P$.

Proof. Let γ be an IF ideal of P . Let $(p_1, p_2) \in P \times P$. Then

$$(\zeta_\psi)_\gamma(0, 0) = m\{(\zeta_\psi)_\gamma(0), (\zeta_\psi)_\gamma(0)\} \geq m\{(\zeta_\psi)_\gamma(p_1), (\zeta_\psi)_\gamma(p_2)\}$$

and

$$(\xi_\psi)_\gamma(0, 0) = M\{(\xi_\psi)_\gamma(0), (\xi_\psi)_\gamma(0)\} \leq M\{(\xi_\psi)_\gamma(p_1), (\xi_\psi)_\gamma(p_2)\}.$$

Let $(p_1, p_2), (q_1, q_2), (r_1, r_2) \in P \times P$. Then

$$\begin{aligned} (\zeta_\psi)_\gamma[(q_1, q_2) * (p_1, p_2)] &= (\zeta_\psi)_\gamma[q_1 * p_1, q_2 * p_2] = m\{\gamma(q_1 * p_1), \gamma(q_2 * p_2)\} \geq \\ &m\{m\{\gamma(r_1 * q_1), \gamma(r_1 * p_1)\}, m\{\gamma(r_2 * q_2), \gamma(r_2 * p_2)\}\} = m\{m\{\gamma(r_1 * q_1), \gamma(r_2 * \\ &q_2)\}, m\{\gamma(r_1 * p_1), \gamma(r_2 * p_2)\}\} = m\{(\zeta_\psi)_\gamma[(r_1 * q_1), (r_2 * q_2)], (\zeta_\psi)_\gamma[(r_1 * p_1), (r_2 * p_2)]\} \end{aligned}$$

and

$$\begin{aligned} (\xi_\psi)_\gamma[(q_1, q_2) * (p_1, p_2)] &= (\xi_\psi)_\gamma[q_1 * p_1, q_2 * p_2] = M\{\gamma(q_1 * p_1), \gamma(q_2 * p_2)\} \leq \\ &M\{M\{\gamma(r_1 * q_1), \gamma(r_1 * p_1)\}, M\{\gamma(r_2 * q_2), \gamma(r_2 * p_2)\}\} = M\{M\{\gamma(r_1 * q_1), \gamma(r_2 * \\ &q_2)\}, M\{\gamma(r_1 * p_1), \gamma(r_2 * p_2)\}\} = M\{(\xi_\psi)_\gamma[(r_1 * q_1), (r_2 * q_2)], (\xi_\psi)_\gamma[(r_1 * p_1), (r_2 * \\ &p_2)]\}. \end{aligned}$$

Therefore, $(\zeta_\psi)_\gamma$ and $(\xi_\psi)_\gamma$ are an IF ideal of $P \times P$. $\square$

Theorem 4.3. *If $((\zeta_\psi)_\gamma, (\xi_\psi)_\gamma)$ is an IF ideal of $P \times P$, then γ is an IF ideal of P .*

Proof. Let $(\zeta_\psi)_\gamma$ and $(\xi_\psi)_\gamma$ are an IF ideal of $P \times P$. Then for all $(p_1, p_2), (q_1, q_2), (r_1, r_2) \in P \times P$. $m\{\gamma(0), \gamma(0)\} = (\zeta_\psi)_\gamma(0, 0) \geq (\zeta_\psi)_\gamma(p_1, p_2) \Rightarrow m\{\gamma(0), \gamma(0)\} \geq m\{\gamma(p_1), \gamma(p_2)\} \Rightarrow \gamma(0) \geq \gamma(p_1)$

or $\gamma(0) \geq \gamma(p_2)$ and

$M\{\gamma(0), \gamma(0)\} = (\xi_\psi)_\gamma(0, 0) \leq (\xi_\psi)_\gamma(p_1, p_2) \Rightarrow M\{\gamma(0), \gamma(0)\} \leq M\{\gamma(p_1), \gamma(p_2)\} \Rightarrow \gamma(0) \leq \gamma(p_1)$ or $\gamma(0) \leq \gamma(p_2)$.

Also,

$m\{\gamma(q_1 * p_1), \gamma(q_2 * p_2)\} = (\zeta_\psi)_\gamma[q_1 * p_1, q_2 * p_2] = (\zeta_\psi)_\gamma[(q_1, q_2) * (p_1, p_2)] \geq m\{(\zeta_\psi)_\gamma[(r_1, r_2) * (q_1, q_2)], (\zeta_\psi)_\gamma[(r_1, r_2) * (p_1, p_2)]\} = m\{(\zeta_\psi)_\gamma[(r_1 * q_1), (r_2 * q_2)], (\zeta_\psi)_\gamma[(r_1 * p_1), (r_2 * p_2)]\} = m\{m\{\gamma(r_1 * q_1), \gamma(r_2 * q_2)\}, m\{\gamma(r_1 * p_1), \gamma(r_2 * p_2)\}\} = m\{m\{\gamma(r_1 * q_1), \gamma(r_1 * p_1)\}, m\{\gamma(r_2 * q_2), \gamma(r_2 * p_2)\}\}.$

Put $p_2 = q_2 = r_2 = 0$, we get

$$\gamma(q_1 * p_1) \geq m\{\gamma(r_1 * q_1), \gamma(r_1 * p_1)\}$$

and

$M\{\gamma(q_1 * p_1), \gamma(q_2 * p_2)\} = (\xi_\psi)_\gamma[q_1 * p_1, q_2 * p_2] = (\xi_\psi)_\gamma[(q_1, q_2) * (p_1, p_2)] \leq M\{(\xi_\psi)_\gamma[(r_1, r_2) * (q_1, q_2)], (\xi_\psi)_\gamma[(r_1, r_2) * (p_1, p_2)]\} = M\{(\xi_\psi)_\gamma[(r_1 * q_1), (r_2 * q_2)], (\xi_\psi)_\gamma[(r_1 * p_1), (r_2 * p_2)]\} = M\{M\{\gamma(r_1 * q_1), \gamma(r_2 * q_2)\}, M\{\gamma(r_1 * p_1), \gamma(r_2 * p_2)\}\} = M\{M\{\gamma(r_1 * q_1), \gamma(r_1 * p_1)\}, M\{\gamma(r_2 * q_2), \gamma(r_2 * p_2)\}\}.$

Put $p_2 = q_2 = r_2 = 0$, we get $\gamma(q_1 * p_1) \leq M\{\gamma(r_1 * q_1), \gamma(r_1 * p_1)\}.$

Hence, γ is an IF ideal of P . □

5. CARTESIAN PRODUCT OF INTUITIONISTIC FUZZY TRANSLATION AND FUZZY MULTIPLICATION

Definition 5.1. *Let $((\zeta_\psi)_\gamma^T, (\xi_\psi)_\gamma^T)$ and $((\zeta_\varpi)_\gamma^T, (\xi_\varpi)_\gamma^T)$ are IF sets in P . Then the Cartesian product $(\zeta_\psi)_\gamma^T \times (\zeta_\varpi)_\gamma^T : P \times P \rightarrow [0, 1]$ and $(\xi_\psi)_\gamma^T \times (\xi_\varpi)_\gamma^T : P \times P \rightarrow [0, 1]$ are defined by $((\zeta_\psi)_\gamma^T \times (\zeta_\varpi)_\gamma^T)(p, q) = m\{(\zeta_\psi)_\gamma^T(p), (\zeta_\varpi)_\gamma^T(q)\}$ and $((\xi_\psi)_\gamma^T \times (\xi_\varpi)_\gamma^T)(p, q) = M\{(\xi_\psi)_\gamma^T(p), (\xi_\varpi)_\gamma^T(q)\}$ for all $p, q \in P$.*

Theorem 5.1. *If (ζ_ψ, ξ_ψ) and $(\zeta_\varpi, \xi_\varpi)$ are IF ideals in P , then $((\zeta_\psi)_\gamma^T \times (\zeta_\varpi)_\gamma^T)$ and $((\xi_\psi)_\gamma^T \times (\xi_\varpi)_\gamma^T)$ are an IF ideal in $P \times P$.*

Proof. Let $(p_1, p_2) \in P \times P$. Then

$$((\zeta_\psi)_\gamma^T \times (\zeta_\varpi)_\gamma^T)(0, 0) = m\{(\zeta_\psi)_\gamma^T(0), (\zeta_\varpi)_\gamma^T(0)\} \geq m\{\zeta_\psi(0) + \gamma, \zeta_\varpi(0) + \gamma\} = m\{\zeta_\psi(0), \zeta_\varpi(0)\} + \gamma \geq m\{\zeta_\psi(p_1) + \gamma, \zeta_\varpi(p_2) + \gamma\} = m\{\zeta_\psi(p_1) + \gamma, \zeta_\varpi(p_1) + \gamma\} = m\{(\zeta_\psi)_\gamma^T(p_1), (\zeta_\varpi)_\gamma^T(p_1)\} = ((\zeta_\psi)_\gamma^T \times (\zeta_\varpi)_\gamma^T)(p_1, p_2)$$

and

$$((\xi_\psi)_\gamma^T \times (\xi_\varpi)_\gamma^T)(0, 0) = M\{(\xi_\psi)_\gamma^T(0), (\xi_\varpi)_\gamma^T(0)\} \leq M\{\xi_\psi(0) - \gamma, \xi_\varpi(0) - \gamma\} = M\{\xi_\psi(0), \xi_\varpi(0)\} - \gamma \leq M\{\xi_\psi(p_1) - \gamma, \xi_\varpi(p_2) - \gamma\} = M\{\xi_\psi(p_1) - \gamma, \xi_\varpi(p_1) - \gamma\} = M\{(\xi_\psi)_\gamma^T(p_1), (\xi_\varpi)_\gamma^T(p_1)\} = ((\xi_\psi)_\gamma^T \times (\xi_\varpi)_\gamma^T)(p_1, p_2).$$

Let $(p_1, p_2), (q_1, q_2), (r_1, r_2) \in P \times P$. Then

$$\begin{aligned} ((\zeta_\psi)_\gamma^T \times (\zeta_\varpi)_\gamma^T)(p_1, p_2) &= m\{(\zeta_\psi)_\gamma^T(p_1), (\zeta_\varpi)_\gamma^T(p_2)\} = m\{\zeta_\psi(p_1) + \gamma, \zeta_\varpi(p_2) + \gamma\} = m\{\zeta_\psi(p_1), \zeta_\varpi(p_2)\} + \gamma \geq m\{m\{\zeta_\psi(q_1 * p_1), \zeta_\psi(y_1)\}, m\{\zeta_\varpi(q_2 * p_2), \zeta_\varpi(q_2)\}\} + \gamma \\ &= m\{m\{\zeta_\psi(q_1 * p_1), \zeta_\psi(y_1 - 1)\}, m\{\zeta_\varpi(q_2 * p_2), \zeta_\varpi(q_2)\} + \gamma\} = m\{m\{\zeta_\psi(q_1 * p_1) + \gamma, \zeta_\psi(y_1) + \gamma\}, m\{\zeta_\varpi(q_2 * p_2) + \gamma, \zeta_\varpi(q_2) + \gamma\}\} = m\{m\{(\zeta_\psi)_\gamma^T(q_1 * p_1), (\zeta_\psi)_\gamma^T(y_1)\}, m\{(\zeta_\varpi)_\gamma^T(q_2 * p_2), (\zeta_\varpi)_\gamma^T(y_2)\}\} \\ &= m\{m\{(\zeta_\psi)_\gamma^T(q_1 * p_1), (\zeta_\varpi)_\gamma^T(q_2 * p_2)\}, m\{(\zeta_\psi)_\gamma^T(y_1), (\zeta_\varpi)_\gamma^T(y_2)\}\} = m\{m\{((\zeta_\psi)_\gamma^T \times (\zeta_\varpi)_\gamma^T)((q_1 * p_1), (q_2 * p_2))\}, m\{((\zeta_\psi)_\gamma^T \times (\zeta_\varpi)_\gamma^T)(y_1, y_2)\}\} \\ &= m\{m\{((\zeta_\psi)_\gamma^T \times (\zeta_\varpi)_\gamma^T)((q_1, q_2) * (p_1, p_2))\}, m\{((\zeta_\psi)_\gamma^T \times (\zeta_\varpi)_\gamma^T)(y_1, y_2)\}\} \quad \text{and} \quad ((\xi_\psi)_\gamma^T \times (\xi_\varpi)_\gamma^T)(p_1, p_2) = M\{(\xi_\psi)_\gamma^T(p_1), (\xi_\varpi)_\gamma^T(p_2)\} \\ &= M\{\xi_\psi(p_1) - \gamma, \xi_\varpi(p_2) - \gamma\} = M\{\xi_\psi(p_1), \xi_\varpi(p_2)\} - \gamma \leq M\{M\{\xi_\psi(q_1 * p_1), \xi_\psi(y_1)\}, M\{\xi_\varpi(q_2 * p_2), \xi_\varpi(q_2)\}\} - \gamma = M\{M\{\xi_\psi(q_1 * p_1), \xi_\psi(y_1)\}, M\{\xi_\varpi(q_2 * p_2), \xi_\varpi(q_2)\} - \gamma\} \\ &= M\{M\{\xi_\psi(q_1 * p_1) - \gamma, \xi_\psi(y_1) - \gamma\}, M\{\xi_\varpi(q_2 * p_2) - \gamma, \xi_\varpi(q_2) - \gamma\}\} = M\{M\{(\xi_\psi)_\gamma^T(q_1 * p_1), (\xi_\psi)_\gamma^T(y_1)\}, M\{(\xi_\varpi)_\gamma^T(q_2 * p_2), (\xi_\varpi)_\gamma^T(y_2)\}\} \\ &= M\{M\{(\xi_\psi)_\gamma^T(q_1 * p_1), (\xi_\varpi)_\gamma^T(q_2 * p_2)\}, M\{(\xi_\psi)_\gamma^T(y_1), (\xi_\varpi)_\gamma^T(y_2)\}\} = M\{M\{((\xi_\psi)_\gamma^T \times (\xi_\varpi)_\gamma^T)((q_1 * p_1), (q_2 * p_2))\}, M\{((\xi_\psi)_\gamma^T \times (\xi_\varpi)_\gamma^T)(y_1, y_2)\}\} \\ &= M\{M\{((\xi_\psi)_\gamma^T \times (\xi_\varpi)_\gamma^T)((q_1, q_2) * (p_1, p_2))\}, M\{((\xi_\psi)_\gamma^T \times (\xi_\varpi)_\gamma^T)(y_1, y_2)\}\}. \end{aligned}$$

Hence, $((\zeta_\psi)_\gamma^T \times (\zeta_\varpi)_\gamma^T)$ and $((\xi_\psi)_\gamma^T \times (\xi_\varpi)_\gamma^T)$ are an IF PMS-ideal in $P \times P$. $\square$

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