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GEOMETRIC INTERPRETATIONS OF THE CARTESIAN PRODUCT OVER INTUITIONISTIC FUZZY A - IDEALS OF SUBTRACTION MS-ALGEBRA

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ABSTRACT. The view of intuitionistic fuzzy A-ideals in MS-algebras is familiarized. Settings for an intuitionistic fuzzy ideal to be an intuitionistic fuzzy a-ideal are provided. Using a compilation of a-ideals, intuitionistic fuzzy a-ideals are recognized.

1. INTRODUCTION

In this paper, we apply the idea of an intuitionistic fuzzy set to A-ideals in MS-algebras. We provide Geometric interpretations of the Cartesian product over Intuitionistic fuzzy A-ideals of subtraction MS-Algebra between an intuitionistic fuzzy ideal and an intuitionistic fuzzy A-ideal. Using the compilation of A-ideals, we tend to kind intuitionistic fuzzy A-ideals.

Foe further references see [1–17].

2. PRELIMINARIES

A BM-algebra is a non-empty set X with a endless 0 and a binary operation "-" satisfying the following axioms:

(1) $x - 0 = x$,

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(2) $(z-x)-(z-y)=y-x$, for any $x,y,z\in X$.
Let $(X,-,0)$ be a BM-algebra. Then
(3) $x-x=0$,
(4) $0-(0-x)=x$,
(5) $0-(x-y)=y-x$,
(6) $(x-z)-(y-z)=x-y$,
(7) $x-y=0$ if and only if $y-x=0$, for any $x,y,z\in X$.

Example 1. *Let $(N,-,0)$ be a subtraction MS-algebra, then*
(3 - 4) - (3 - 1) = 1 - 4
-1 - 2 = -3
-3 = -3

Definition 2.1. *An intuitionistic fuzzy set (IFS) A in E is defined as an object of the following form $A=\{\langle x,\zeta_A(x),\Upsilon_A(x)\rangle\,|x\in E\}$, Where the functions $\zeta_A: E\to[0,1]$ and $\Upsilon_A:E\to[0,1]$ define the degree of membership and the degree of non- membership of the element $x\in E$, respectively, and for every $x\in E$. $0\leq\zeta_A(x)+\Upsilon_A(x)\leq 1$Obviously, each ordinary fuzzy set may be written as $\{\langle x,\zeta_A(x),1-\zeta_A(x)\rangle\,|x\in E\}$.*

Definition 2.2. *The value of $\phi_A(x)=1-\zeta_A(x)-\Upsilon_A(x)$ is called the degree of non-determinacy (or uncertainty) of the element $x\in E$ to the intuitionistic fuzzy set A.*

Definition 2.3. *A nonempty subset A of a MS-algebra X is called an ideal of X if it satisfies:*
i) $0\in A$.
ii) $(\forall x,y\in X)(\forall y\in A)(x-y\in A\Rightarrow x\in A)$.

Definition 2.4. *A nonempty subset A of a MS-algebra X is called an A-ideal of X if it satisfies $(\forall x,y\in X)(\forall z\in A)((x-z)\ -(0-y)\in A\Rightarrow y-\ x\in A)$.*

Definition 2.5. *An IFS $A=\langle X,\zeta_A,\Upsilon_A\rangle$ in a MS-algebra X is called an intuitionistic fuzzy sub algebra of X if it satisfies*
i) $(\forall x,y\in X)(\zeta_A(x-y\geq\min\{\zeta_A\left(x\right),\zeta_A(y)\})$,
ii) $(\forall x,y\in X)(\Upsilon_A(x-y)\leq\max\{\Upsilon_A\left(x\right),\Upsilon_A(y)\})$.

Definition 2.6. *An IFS $A=\langle X,\zeta_A,\Upsilon_A\rangle$ in a MS-algebra X is called an intuitionistic fuzzy ideal of X if it satisfies*

i) $(\forall x \in X)(\zeta_A(0) \geq \zeta_A(x), \Upsilon_A(0) \leq \Upsilon_A(x))$*, and*
ii) $(\forall x, y \in X)(\zeta_A(x) \geq \min\{\zeta_A(x - y), \zeta_A(y)\})$*,*
iii) $(\forall x, y \in X)(\Upsilon_A(x) \leq \max\{\Upsilon_A(x - y), \Upsilon_A(y)\})$.

Definition 2.7. *An intuitionistic fuzzy ideal* $A = \langle X, \zeta_A, \Upsilon_A\rangle$ *in a MS-algebra* X *is said to be closed if it satisfies* $(\forall x \in X)(\zeta_A(0 - x) \geq \zeta_A(x), \Upsilon_A(0 - x) \leq \Upsilon_A(x))$.

3. Intuitionistic fuzzy A-ideals

Let X denotes a MS-algebra unless otherwise specified. We first consider the intuitionistic fuzzification of the notion of A-ideals in a MS-algebra as follows

Definition 3.1. *An IFS,* $A = \langle X, \zeta_A, \Upsilon_A\rangle$ *in* X *is called an intuitionistic fuzzy A-ideal of* X *if it satisfies*
i) $(\forall x, y, z \in X)(\zeta_A(y - x) \geq \min\{\zeta_A((x - z) - (0 - y)), \zeta_A(x)\})$*,*
ii) $(\forall x, y, z \in X)(\Upsilon_A(y - x) \leq \max\{\Upsilon_A((x - z) - (0 - y)), \Upsilon_A(x)\})$.

Example 2. *Consider a MS-algebra* $X = \{0, a, b, c\}$ *with the following Cayley table.*

$*$	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

We define an IFS, $A = \langle X, \zeta_A, \Upsilon_A\rangle$ *in* X *by*

$$A = \left\langle X, \left(\frac{0}{0.8}, \frac{a}{0.8}, \frac{b}{0.3}, \frac{c}{0.3}\right), \left(\frac{0}{0.1}, \frac{a}{0.1}, \frac{b}{0.4}, \frac{c}{0.4}\right)\right\rangle.$$

By routine calculations, we know that $A = \langle X, \zeta_A, \Upsilon_A\rangle$ *is an intuitionistic fuzzy A-ideal of* X.

4. Cartesian Products over Intuitionistic Fuzzy Sets

Let E_1 and E_2 be two inverses and let $A = \{\langle x, \zeta_A(x), \Upsilon_A(x)\rangle \,|x \in E_1\}$ and $B = \{\langle y, \zeta_B(y), \Upsilon_B(y)\rangle \,|y \in E_2\}$ be two Intuitionistic Fuzzy Sets 'A' over E_1 and 'B' over E_2.

Definition 4.1. *The Cartesian products of two Intuitionistic Fuzzy Sets A and B are defined as follows.*

(1) *The Cartesian product of* "$\times_1$"
$A \times_1 B = \{\langle\langle x, y\rangle, \zeta_A(x) \cdot \zeta_B(y), \Upsilon_A(x) \cdot \Upsilon_B(y)\rangle \,|x \in E_1 \& y \in E_2\}$.

(2) *The Cartesian product of* "$\times_2$"
$A \times_2 B = \{\langle\langle x, y\rangle, \zeta_A(x) + \zeta_B(y) - \zeta_A(x) \cdot \zeta_B(y), \Upsilon_A(x) \cdot \Upsilon_B(y)\rangle \,|x \in E_1 \& y \in E_2\}$.

(3) *The Cartesian product of* "$\times_3$"
$A \times_3 B = \{\langle\langle x, y\rangle, \zeta_A(x) \cdot \zeta_B(y), \Upsilon_A(x) + \Upsilon_B(y) - \Upsilon_A(x) \cdot \Upsilon_B(y)\rangle \,|x \in E_1 \& y \in E_2\}$.

(4) *The Cartesian product of* "$\times_4$"
$A \times_4 B = \{\langle\langle x, y\rangle, \min(\zeta_A(x), \zeta_B(y)), \max(\Upsilon_A(x), \Upsilon_B(y))\rangle \,|x \in E_1 \& y \in E_2\}$.

(5) *The Cartesian product of* "$\times_5$"
$A \times_5 B = \{\langle\langle x, y\rangle, \max(\zeta_A(x), \zeta_B(y)), \min(\Upsilon_A(x), \Upsilon_B(y))\rangle \,|x \in E_1 \& y \in E_2\}$.

From $0 \leq \zeta_A(x).\zeta_B(y) + \Upsilon_A(x).\ \Upsilon_B(y) \leq \zeta_A(x) + \Upsilon_A(x) \leq 1$. It follows that $A \times_1 B$ is an IFS, but it is an IFS over the inverse $E_1 \times E_2$, where "$\times$" is the classical Cartesian product on ordinary sets E_1 and E_2. For the fourteen other Cartesian products the Computations are analogous.

5. Necessity and Possibility Operators on Intuitionistic Fuzzy A-ideal

Theorem 5.1. *If A and B are Intuitionistic fuzzy A-ideal of subtraction of MS-Algebra, then $\Box(A \times_1 B)$ is also intuitionistic fuzzy A-ideal of subtraction MS-Algebra.*

Proof. If A and B are Intuitionistic fuzzy A-ideal,

i). $\zeta_A(0) \geq \zeta_A(x)$ and $\Upsilon_A(0) \leq \Upsilon_A(x), \forall x \in X$.

ii). $\zeta_A(y-x) \geq \min\{\zeta_A((x-z)-(0-y)), \zeta_A(z)\}$ and $\zeta_B(y-x) \geq \min\{\zeta_B((x-z)-(0-y)), \zeta_B(z)\}$.

iii). $\Upsilon_A(y-x) \leq \max\{\Upsilon_A((x-z)-(0-y)), \Upsilon_A(z)\}$ and $\Upsilon_B(y-x) \leq \max\{B((x-z)-(0-y)), \Upsilon_B(z)\}$.

i) $\zeta_A(0) \cdot \zeta_B(0) \geq \zeta_A(x) \cdot \zeta_B(x) \zeta_{A\ \times_1\ B}(0) \geq \zeta_{A\ \times_1\ B}(x)$.

ii) $A(y-x) \cdot \zeta_B(y-x) \geq \min\{\zeta_A((x-z)-(0-y)), \zeta_A(z)\} \cdot \min\{\zeta_B((x-z)-(0-y)), \zeta_B(z)\} \zeta_{A\ \times_1\ B}(y-x) \geq \min\{\zeta_A((x-z)-(0-y)) \cdot \zeta_B((x-z)-(0-y)), \zeta_A(z) \cdot \zeta_B(z)\} \zeta_{A\ \times_1\ B}(y-x) \geq \min\{\zeta_{A\ \times_1\ B}((x-z)\ -(0-y)), \zeta_{A\ \times_1\ B}(z)\}$.

iii) $\Upsilon_A(0) \cdot \Upsilon_B(0) \leq \Upsilon_A(x) \cdot B(x) \Upsilon_{A \times_1 B}(0) \leq \Upsilon_{A \times_1 B}(x)$.

iv) $\Upsilon_A(y-x) \cdot \Upsilon_B(y-x) \leq \max\{\Upsilon_A((x-z)-(0-y)), \Upsilon_A(z)\} \cdot \max\{\Upsilon_B((x-z)-(0-y)), \Upsilon_B(z)\} \Upsilon_{A \times_1 B}(y-x) \leq \max\{\Upsilon_A((x-z)-(0-y)) \cdot \Upsilon_B((x-z)-(0-y)), \Upsilon_A(z) \cdot \Upsilon B(z)\} \Upsilon_{A \times_1 B}(y-x) \leq \max\{\Upsilon_{A \times_1 B}((x-z)-(0-y)), \Upsilon_{A \times_1 B}(z)\}$

By using Necessity Operator and Possibility Operator:

i) $\Box(\zeta_{A \times_1 B}(0)) \geq \Box(\zeta_{A \times_1 B}(x))$

$1 - \zeta_{\Box (A \times_1 B)}(0) \leq 1 - \zeta_{\Box (A \times_1 B)}(x)$

$\Upsilon_{\Box (A \times_1 B)}(0) \leq \Upsilon_{\Box (A \times_1 B)}(x) \forall x \in X$.

ii) $\diamond(\Upsilon_{A \times_1 B}(0)) \leq \diamond(\Upsilon_{A \times_1 B}(x))$

$1 - \Upsilon_{\diamond (A \times_1 B)}(0) \geq 1 - \Upsilon_{\diamond (A \times_1 B)}(x)$

$\zeta_{\diamond (A \times_1 B)}(0) \geq \zeta_{\diamond (A \times_1 B)}(x) \forall x \in X$.

iii) $\Box(\zeta_{A \times_1 B}(y-x)) \geq \Box\{min\{\zeta_{A \times_1 B}((x-z)-(0-y)), \zeta_{A \times_1 B}(z)\}\}$

$1 - \zeta_{\Box(A \times_1 B)}(y-x) \leq 1 - \min\{1 - \zeta_{\Box(A \times_1 B)}((x-z)-(0-y)), 1 - \zeta_{\Box(A \times_1 B)}(z)\}$

$\Upsilon_{\Box(A \times_1 B)}(y-x) \leq \max\{\Upsilon_{\Box(A \times_1 B)}((x-z)-(0-y)), \Upsilon_{\Box (A \times_1 B)}(x)\} \forall x \in X$.

iv) $\diamond(\Upsilon_{A \times_1 B}(y-x)) \leq \diamond\{max\{\Upsilon_{A \times_1 B}((x-z)-(0-y)), \Upsilon_{A \times_1 B}(z)\}\}$

$1 - \Upsilon_{\diamond(A \times_1 B)}(y-x) \geq 1 - \max\{1 - \Upsilon_{\diamond(A \times_1 B)}((x-z)-(0-y)), 1 - \Upsilon_{\diamond(A \times_1 B)}(z)\}$

$\zeta_{\diamond (A \times_1 B)}(y-x) \geq \min\{\zeta_{\diamond (A \times_1 B)}((x-z)-(0-y)), \zeta_{\diamond (A \times_1 B)}(x)\} \forall x \in X$. □

Theorem 5.2. *If A and B are Intuitionistic fuzzy A-ideal of subtraction of MS-Algebra, then $\Box(A \times_2 B)$ is also intuitionistic fuzzy A-ideal of subtraction MS-Algebra.*

Proof. It is trivial. □

Theorem 5.3. *If A and B are Intuitionistic fuzzy A-ideal of subtraction of MS-Algebra, then $\Box(A \times_3 B)$ is also intuitionistic fuzzy A-ideal of subtraction MS-Algebra.*

Proof. If A and B are Intuitionistic fuzzy A-ideal,

i) $\zeta_A(0) \cdot \zeta_B(0) \geq \zeta_A(x) \cdot \zeta_B(x)$ and $\Upsilon_A(0) + \Upsilon_B(0) - \Upsilon_A(0) \cdot \Upsilon_B(0) \leq \Upsilon_A(x) + \Upsilon_B(x) - \Upsilon_A(x) \cdot \Upsilon_B(x), \forall x \in X$.

ii) $\zeta_A(y-x) \cdot \zeta_B(y-x) \geq \min\{\zeta_A((x-z)-(0-y)), \zeta_A(z)\} \cdot \min\{\zeta_B((x-z)-(0-y)), \zeta_B(z)\}$.

iii) $\Upsilon_A(y-x)+\Upsilon_B(y-x)-\Upsilon_A(y-x)\cdot\Upsilon_B(y-x)\leq\max\{\Upsilon_A((x-z)-(0-y)),\Upsilon_A(z)\}+\max\{\Upsilon_B((x-z)-(0-y)),\Upsilon_B(z)\}-\max\{\Upsilon_A((x-z)-(0-y)),\Upsilon_A(z)\}\cdot\max\{\Upsilon_B((x-z)-(0-y)),\Upsilon_B(z)\}$.

i)$\zeta A(0)\cdot\zeta_B(0)\geq A(x)\cdot\zeta_B(x)\zeta_{A\ \times_3\ B}(0)\geq\zeta_{A\ \times_3\ B}(x)$.

ii) $\zeta_A(y-x)\cdot\zeta_B(y-x)\geq\min\{\zeta_A((x-z)-(0-y)),\zeta_A(z)\}\cdot\min\{\zeta_B((x-z)-(0-y)),\zeta_B(z)\}$

$\zeta_{A\ \times_3\ B}(y-x)\geq\min\{\zeta_A((x-z)-(0-y))\cdot\zeta_B((x-z)-(0-y)),\zeta_A(z)\cdot\zeta_B(z)\}$

$\zeta_{A\ \times_3\ B}(y-x)\geq\min\{\zeta_{A\ \times_3\ B}((x-z)\ -(0-y)),\ \zeta_{A\ \times_3\ B}(z)\}$.

iii) $\Upsilon_A(0)+\Upsilon_B(0)-\Upsilon_A(0)\cdot\Upsilon_B(0)\leq\Upsilon_A(x)+\Upsilon_B(x)-\Upsilon_A(x)\cdot\Upsilon_B(x)$

$\Upsilon_{A\ \times_3\ B}(0)\leq\Upsilon_{A\ \times_3\ B}(x)$.

iv)$\Upsilon_A(y-x)+\Upsilon_B(y-x)-\Upsilon_A(y-x)\cdot\Upsilon_B(y-x)\leq\max\{\Upsilon_A((x-z)-(0-y)),\Upsilon_A(z)\}+\max\{\Upsilon_B((x-z)-(0-y)),\Upsilon_B(z)\}-\max\{\Upsilon_A((x-z)-(0-y)),\Upsilon_A(z)\}\cdot\max\{\Upsilon_B((x-z)-(0-y)),\Upsilon_B(z)\}$

$\Upsilon_{A\ \times_3\ B}(y-x)\leq\max\{\Upsilon_A((x-z)-(0-y))+\Upsilon_B((x-z)-(0-y))-\ \Upsilon_A((x-z)-(0-y))\cdot\Upsilon_B((x-z)-(0-y)),\Upsilon_A(z)+\Upsilon B(z)-\Upsilon_A(z)\cdot\Upsilon B(z)\}$

$\Upsilon_{A\ \times_3\ B}(y-x)\leq\max\{\Upsilon_{A\ \times_3\ B}((x-z)\ -(0-y)),\ \Upsilon_{A\ \times_3\ B}(z)\}$.

By using Necessity Operator and Possibility Operator:

i) $\Box(\zeta_{A\ \times_3\ B}(0))\geq\Box(\zeta_{A\ \times_3\ B}(x))$

$1-\zeta_{\Box\ (A\ \times_3\ B)}(0)\leq 1-\zeta_{\Box\ (A\ \times_3\ B)}(x)$

$\Upsilon_{\Box\ (A\ \times_3\ B)}(0)\leq\Upsilon_{\Box\ (A\ \times_3\ B)}(x)\forall x\in X$.

ii) $\diamond(\Upsilon_{A\ \times_3\ B}(0))\leq\diamond(\Upsilon_{A\ \times_3\ B}(x))$

$1-\Upsilon_{\diamond\ (A\ \times_3\ B)}(0)\geq 1-\Upsilon_{\diamond\ (A\ \times_3\ B)}(x)$

$\zeta_{\diamond\ (A\ \times_3\ B)}(0)\geq\zeta_{\diamond\ (A\ \times_3\ B)}(x)\forall x\in X$.

iii) $\Box(\zeta_{A\ \times_3\ B}(y-x))\geq\Box\{min\{\zeta_{A\ \times_3\ B}((x-z)\ -(0-y)),\ \zeta_{A\ \times_3\ B}(z)\}\}$

$1-\zeta_{\Box(A\ \times_3\ B)}(y-x)\leq 1-\min\{1-\ \zeta_{\Box(A\ \times_3\ B)}((x-z)\ -(0-y)),1-\zeta_{\Box(A\ \times_3\ B)}(z)\}$

$\Upsilon_{\Box\ (A\ \times_3\ B)}(y-x)\leq\max\{\Upsilon_{\Box(A\ \times_3\ B)}((x-z)\ -(0-y)),\Upsilon_{\Box\ (A\ \times_3\ B)}(x)\}\forall x\in X$.

iv) $\diamond(\Upsilon_{A\ \times_3\ B}(y-x))\leq\diamond\{max\{\Upsilon_{A\ \times_3\ B}((x-z)\ -(0-y)),\ \Upsilon_{A\ \times_3\ B}(z)\}\}$

$1-\Upsilon_{\diamond(A\ \times_3\ B)}(y-x)\geq 1-\max\{1-\ \Upsilon_{\diamond(A\ \times_3\ B)}((x-z)\ -(0-y)),1-\Upsilon_{\diamond(A\ \times_3\ B)}(z)\}$

$\zeta_{\diamond(A\ \times_3\ B)}(y-x)\geq\min\{\zeta_{\diamond\ (A\ \times_3\ B)}((x-z)\ -(0-y)),\ \zeta_{\diamond\ (A\ \times_3\ B)}(x)\}\forall x\in X$. $\Box$

Theorem 5.4. *If A and B are Intuitionistic fuzzy A-ideal of subtraction of MS-Algebra, then $\Box(A \times_4 B)$ is also intuitionistic fuzzy A-ideal of subtraction MS-Algebra.*

Proof. It is straightforward. $\Box$

Theorem 5.5. *If A and B are Intuitionistic fuzzy A-ideal of subtraction of MS-Algebra, then $\Box(A \times_5 B)$ is also intuitionistic fuzzy A-ideal of subtraction MS-Algebra.*

Proof. If A and B are Intuitionistic fuzzy A-ideal,

i) $\max(\zeta_A(0), \zeta_B(0)) \geq \max(\zeta_A(x), \zeta_B(x))$ and $\min(\Upsilon_A(0), \Upsilon_B(0)) \leq \min(\Upsilon_A(x), \Upsilon_B(x)) \forall x \in X$.

ii) $\max(\zeta_A(y-x), \zeta_B(y-x)) \geq \min\{\max\{\zeta_A((x-z)-(0-y)), \zeta_A(z)\}, \max\{\zeta_B((x-z)-(0-y)), \zeta_B(z)\}\}$.

iii) $\min(\Upsilon_A(y-x), \Upsilon_B(y-x)) \leq \max\{\min\{\Upsilon_A((x-z)-(0-y)), \Upsilon_A(z)\}, \min\{\Upsilon_B((x-z)-(0-y)), \Upsilon_B(z)\}\}$.

i) $\max(\zeta A(0) \cdot \zeta_B(0)) \geq \max(\zeta_A(x) \cdot \zeta_B(x)) \zeta_{A \times_5 B}(0) \geq \zeta_{A \times_5 B}(x)$.

ii) $\max(\zeta_A(y-x), \zeta_B(y-x)) \geq \min\{\max\{\zeta_A((x-z)-(0-y)), \zeta_A(z)\}, \max\{\zeta_B((x-z)-(0-y)), \zeta_B(z)\}\}$

$\zeta_{A \times_5 B}(y-x) \geq \min\{\max\{\zeta_A((x-z)-(0-y)) \cdot \zeta_B((x-z)-(0-y)), \zeta_A(z) \cdot \zeta_B(z)\}\}$

$\zeta_{A \times_5 B}(y-x) \geq \min\{\zeta_{A \times_5 B}((x-z) \ -(0-y)), \ \zeta_{A \times_5 B}(z)\}$.

iii) $\min(\Upsilon_A(0), \Upsilon_B(0)) \leq \min(\Upsilon_A(x), \Upsilon_B(x))$

$\Upsilon_{A \times_5 B}(0) \leq \Upsilon_{A \times_5 B}(x)$.

iv) $\min(\Upsilon_A(y-x), \Upsilon_B(y-x)) \leq \max\{\min\{\Upsilon_A((x-z)-(0-y)), \Upsilon_A(z)\}, \min\{\Upsilon_B((x-z)-(0-y)), \Upsilon_B(z)\}\}$.

$\Upsilon_{A \times_5 B}(y-x) \leq \max\{\min\{\Upsilon_A((x-z)-(0-y)) \cdot \Upsilon_B((x-z)-(0-y)), \ \Upsilon_A(z) \cdot \Upsilon B(z)\}\}$

$\Upsilon_{A \times_5 B}(y-x) \leq \max\{\Upsilon_{A \times_5 B}((x-z) \ -(0-y)), \ \Upsilon_{A \times_5 B}(z)\}$.

By using Necessity Operator and Possibility Operator.

i) $\Box(\zeta_{A \times_5 B}(0)) \geq \Box(\zeta_{A \times_5 B}(x))$

$1 - \zeta_{\Box \ (A \times_5 B)}(0) \leq 1 - \zeta_{\Box \ (A \times_5 B)}(x)$

$\Upsilon_{\Box \ (A \times_5 B)}(0) \leq \Upsilon_{\Box \ (A \times_5 B)}(x) \forall x \in X$.

ii) $\diamond(\Upsilon_{A \times_5 B}(0)) \leq \diamond(\Upsilon_{A \times_5 B}(x))$

$1 - \Upsilon_{\diamond \ (A \times_5 B)}(0) \geq 1 - \Upsilon_{\diamond \ (A \times_5 B)}(x)$

$\zeta_{\diamond \ (A \times_5 B)}(0) \geq \zeta_{\diamond \ (A \times_5 B)}(x) \forall x \in X$.

iii) $\Box(\zeta_{A \times_5 B}(y-x)) \geq \Box\{\min\{\zeta_{A \times_5 B}((x-z) \ -(0-y)), \ \zeta_{A \times_5 B}(z)\}\}$

$1 - \zeta_{\Box(A \times_5 B)}(y-x) \leq 1 - \min\{1 - \ \zeta_{\Box(A \times_5 B)}((x-z) \ -(0-y)), 1 -$

$\zeta_{\square(A\ \times_5\ B)}(z)\}$

$\Upsilon_{\square\ (A\ \times_5\ B)}(y-x) \leq \max\{\Upsilon_{\square\ (A\ \times_5\ B)}((x-z)\ -(0-y)),\ \Upsilon_{\square\ (A\ \times_5\ B)}(x)\} \forall x \in X.$

iv) $\diamond(\Upsilon_{A\ \times_5\ B}(y-x)) \leq \diamond\{\max\{\Upsilon_{A\ \times_5\ B}((x-z)\ -(0-y)),\ \Upsilon_{A\ \times_5\ B}(z)\}\}$

$1-\Upsilon_{\diamond(A\ \times_5\ B)}(y-x) \geq 1-\max\{1-\ \Upsilon_{\diamond(A\ \times_5\ B)}((x-z)\ -(0-y)), 1-\Upsilon_{\diamond(A\ \times_5\ B)}(z)\}$

$\zeta_{\diamond\ (A\ \times_5\ B)}(y-x) \geq \min\{\zeta_{\diamond\ (A\ \times_5\ B)}((x-z)\ -(0-y)),\ \zeta_{\diamond(A\ \times_5\ B)}(x)\} \forall x \in X.$ $\square$

6. CONCLUSION

In this paper we have introduced the Cartesian product over Intuitionistic fuzzy A-ideals of subtraction MS-Algebra and we have proved some of their properties.

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