Advances in Mathematics: Scientific Journal **9** (2020), no.4, 1561–1567
ISSN: 1857-8365 (printed); 1857-8438 (electronic)
https://doi.org/10.37418/amsj.9.4.10 Spec. Issue on NCFCTA-2020

COMPLEX CUBIC SET AND THEIR PROPERTIES

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ABSTRACT. In this manuscript, we introduce the notion of complex cubic sets (CCS). CCS is a combination of the complex-interval valued fuzzy set (CIVFS) and complex fuzzy set (CFS). In CCS, there is an advantage to provide the membership grade in complex numbers. Also, we have defined internal complex cubic set (ICCS) and external complex cubic set (ECCS). We have introduced some new operation on P(R)-order, P(R)-union, P(R)-intersection and discussed some of its properties. A multi criteria decision making (MCDM) problem is illustrated to deal with today's complexity structure.

1. INTRODUCTION

Zadeh [6,7] introduced fuzzy set (FS) and interval-valued fuzzy set (IVFS). Jun et.al [5] defined the concept of cubic set which is a combination of IVFS and FS. They also investigated the properties of internal and external cubic sets and Chinnadurai [1] introduced cubic soft matrices. Ramot[3] proposed the concept of CFS. In CFS the amplitude and the phase term represent the membership grade for an element. Greenfield [2] introduced the concept of CIVFS. In CIVFS the amplitude and the phase term membership grade are in interval for each element.

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2010 *Mathematics Subject Classification.* 47S40.

Key words and phrases. complex fuzzy set, interval valued complex fuzzy set, complex cubic set.

Here, we introduce the concept and properties of ICCS, ECCS, complement and P(R)-order, union and intersection. Finally, a MCDM problem is provided to prove the effectiveness of CCS.

2. PRELIMINARIES

We discuss some of the basic concepts which are necessary for the study. Here every element z contained in a non-empty set $\vec{X}$, i.e., $(z \in \vec{X})$.

Definition 2.1. *[7] A fuzzy set $\mathcal{F}$ in $\vec{X}$ is defined as $\mathcal{F} = \left\{z, \mu_{\mathcal{F}}(z) : z \in \vec{X}\right\}$ where $\mu_{\mathcal{F}}(z)$ is called the membership value of z in $\mathcal{F}$ and $0 \leq \mu_{\mathcal{F}}(z) \leq 1$.*

Definition 2.2. *[6] Let a function $\mathcal{F} : \vec{X} \to [\mathcal{I}]$ is called an interval-valued fuzzy set(IVF) in $\vec{X}$. Let $[\mathcal{I}]^{\vec{X}}$ stand for the set of all IVF sets in $\vec{X}$. For every $\mathcal{F} \in [\mathcal{I}]^{\vec{X}}$ and $z \in \vec{X}$, $\mathcal{F}(z) = [\mathcal{F}^-(z), \mathcal{F}^+(z)]$ is called the degree of membership of an element z to $\mathcal{F}$, where $\mathcal{F}^- : \vec{X} \to \mathcal{I}$ and $\mathcal{F}^+ : \vec{X} \to \mathcal{I}$ are fuzzy sets in $\vec{X}$ which are called a lower and upper fuzzy set in $\vec{X}$, resprctively. We denote $\mathcal{F} = [\mathcal{F}^-, \mathcal{F}^+]$.*

Definition 2.3. *[5] A cubic set is the combination of IVF and fuzzy set in $\vec{X}$. We mean a structure $\mathcal{F} = \left\{\langle z, F(z), \lambda(z)\rangle \,|z \in \vec{X}\right\}$ in which $F(z)$ is an IVF in $\vec{X}$ and $\lambda(z)$ is a fuzzy set in $\vec{X}$. A cubic set $\mathcal{F} = \left\{\langle z, F(z), \lambda(z)\rangle \,|z \in \vec{X}\right\}$ is simply denoted by $\mathcal{F} = \langle F, \lambda\rangle$.*

Definition 2.4. *[3, 4] A complex fuzzy set(CFS) $\mathcal{F}$ defined on a universe of discourse $\vec{X}$ is characterized by a membership function $\mu_{\mathcal{F}}(z)$ that assigns a complex-valued grade of membership in $\mathcal{F}$ to any element $z \in \vec{X}$. By definition, all values of $\mu_{\mathcal{F}}(z)$ lie within the unit circle in the complex plane are expressed by $\mu_{\mathcal{F}}(z) = \beta_{\mathcal{F}}(z)e^{i\theta_{\mathcal{F}}(z)}$, where $i = \sqrt{-1}$, $\beta_{\mathcal{F}}(z)$ and $\theta_{\mathcal{F}}(z)$ are both real-valued, $\beta_{\mathcal{F}}(z) \in [0, 1]$ and $\theta_{\mathcal{F}}(z) \in [0, 2\pi]$. A complex fuzzy set $\mathcal{F}$ is the form*

$$\mathcal{F} = \left\{(z, \mu_{\mathcal{F}}(z)) : z \in \vec{X}\right\} = \left\{\left(z, \beta_{\mathcal{F}}(z)e^{i\theta_{\mathcal{F}}(z)}\right) : z \in \vec{X}\right\}.$$

Definition 2.5. *[2] An complex interval valued fuzzy set(CIVFS) $\mathcal{F}$ on $\vec{X}$ is a mapping given by*

$$\mathcal{F} : \vec{X} \to \{\ddot{a}|\ddot{a} \in \mathcal{C} : |\ddot{a}| \leq 1\} \times \{\ddot{a}|\ddot{a} \in \mathcal{C} : |\ddot{a}| \leq 1\}.$$

For all $z \in \vec{X}, \beta_{\mathcal{F}}(z) = \left[\beta^-_{\mathcal{F}}(z), \beta^+_{\mathcal{F}}(z)\right]$ is called the complex degree of membership of an element $\ddot{a}$. Here, $\beta^-_{\mathcal{F}}(z)$ and $\beta^+_{\mathcal{F}}(z)$ are referred to as the lower *and upper bounds of the membership function of* z*, with* $\beta_{\mathcal{F}}^{-}(z), \beta_{\mathcal{F}}^{+}(z) : \vec{X} \to \{\ddot{a} | \ddot{a} \in \mathcal{C} : |\ddot{a}| \leq 1\}$ *where* $\beta_{\mathcal{F}}^{-}(z) = r^{-}(z)e^{i\theta-}$ *and* $\beta_{\mathcal{F}}^{+}(z) = r^{+}(z)e^{i\theta+}$ *such that* $1 \geq r^{+} \geq r^{-} \geq 0$ *and* $2\pi \geq \theta^{+} \geq \theta^{-} \geq 0$.

3. COMPLEX CUBIC SET(CCS)

In this section we define complex cubic set and investigate some properties of internal,external CCS. Let S be a non empty set for all elements $z \in S$.

Definition 3.1. *A complex cubic set in* S *we mean a structure* $\mathcal{C}_k = \left\{\left\langle z, \left[\eta_k^{-}(z), \eta_k^{+}(z)\right] \mu_k(z)\right\rangle | z \in S\right\}$. *The CCS is a combination of IVCFS and CFS. Then CCS of* $\mathcal{C}_k$ *is defined on* S *can be represented as,* $\mathcal{C}_k = \left\{\left\langle z, \left[q_k^{-}(z)e^{i\theta_k^{-}(z)}, q_k^{+}(z)e^{i\theta_k^{+}(z)}\right] \nu_k(z)e^{i\theta_k(z)}\right\rangle | z \in S\right\}$. *The amplitude terms* $q_k^{-}, q_k^{+}, \nu_k \in [0,1]$ *and satisfy the inequality* $1 \geq q_k^{+} \geq q_k^{-} \geq 0$. *On the other hand, the phase terms* $\theta_k^{-}, \theta_k^{+}, \theta_k \in [0, 2\pi]$ *and satisfy the inequality* $2\pi \geq \theta_k^{+} \geq \theta_k^{-} \geq 0$.

Definition 3.2. *A complex cubic set,*

$$\mathcal{C}_k = \left\{\left\langle z, \left[q_k^{-}(z)e^{i\theta_k^{-}(z)}, q_k^{+}(z)e^{i\theta_k^{+}(z)}\right] \nu_k(z)e^{i\theta_k(z)}\right\rangle | z \in S\right\}$$

is said to be an internal complex cubic set(ICCS) if the amplitude term $q_k^{+}(z) \geq \nu_k(z) \geq q_k^{-}(z)$ *and the phase term* $\theta_k^{+}(z) \geq \theta_k(z) \geq \theta_k^{-}(z)$ *for all* $z \in S$.

Definition 3.3. *A complex cubic set,*

$$\mathcal{C}_k = \left\{\left\langle z, \left[q_k^{-}(z)e^{i\theta_k^{-}(z)}, q_k^{+}(z)e^{i\theta_k^{+}(z)}\right] \nu_k(z)e^{i\theta_k(z)}\right\rangle | z \in S\right\}$$

is said to be an external complex cubic set(ECCS) if the amplitude term $\nu_k(z) \notin \left(q_k^{-}(z), q_k^{+}(z)\right)$ *and the phase term* $\theta_k(z) \notin \left(\theta_k^{-}(z), \theta_k^{+}(z)\right)$ *for all* $z \in S$.

Example 1. *Let* $\mathcal{C}_1 = \left\langle \left[0.2e^{i(0.82\pi)}, 0.51e^{i(1.2\pi)}\right], \mu_1(z)\right\rangle$ *be a complex cubic set in* S. *Then* $\mathcal{C}_1$ *is said to be ICCS if* $\mu_1(z) = 0.3e^{i(1\pi)}$ *and* $\mathcal{C}_1$ *is said to be ECCS if* $\mu_1(z) = 0.6e^{i(1.4\pi)}$ *for all* $z \in S$.

Theorem 3.1. *Let,* $\mathcal{C}_k = \left\{\left\langle z, \left[q_k^{-}(z)e^{i\theta_k^{-}(z)}, q_k^{+}(z)e^{i\theta_k^{+}(z)}\right] \nu_k(z)e^{i\theta_k(z)}\right\rangle | z \in S\right\}$ *be a CCS in* S. *If* $\mathcal{C}_k$ *is both an ICCS and ECCS, then*

$$\nu_k(z)e^{i\theta_k(z)} \in \left(q_k^{+}(z)e^{i\theta_k^{+}(z)} \cup q_k^{-}(z)e^{i\theta_k^{-}(z)}\right).$$

Proof. Assume that $\mathcal{C}_k$ is an internal complex cubic set then we have $q_k^-(z)e^{i\theta_k^-(z)} \leq \nu_k(z)e^{i\theta_k(z)} \leq q_k^+(z)e^{i\theta_k^+(z)}$ and $\mathcal{C}_k$ is an external complex cubic set then we have $\nu_k(z)e^{i\theta_k(z)} \notin \left(q_k^-(z)e^{i\theta_k^-(z)}, q_k^+(z)e^{i\theta_k^+(z)}\right)$, for all $z \in S$. Thus $\nu_k(z)e^{i\theta_k(z)} = q_k^-(z)e^{i\theta_k^-(z)}$ (or) $\nu_k(z)e^{i\theta_k(z)} = q_k^+(z)e^{i\theta_k^+(z)}$, and so

$$\nu_k(z)e^{i\theta_k(z)} \in \left(q_z^+(x)e^{i\theta_k^+(z)} \cup q_k^-(z)e^{i\theta_k^-(z)}\right).$$

$\square$

Definition 3.4. *If* $\mathcal{C}_1 = \left\langle \left[q_1^-(z)e^{i\theta_1^-(z)}, q_1^+(z)e^{i\theta_1^+(z)}\right] \nu_1(z)e^{i\theta_1(z)} \right\rangle$ *and* $\mathcal{C}_2 = \left\langle \left[q_2^-(z)e^{i\theta_2^-(z)}, q_2^+(z)e^{i\theta_2^+(z)}\right] \nu_2(z)e^{i\theta_2(z)} \right\rangle$ *be complex cubic set in* S*. Then we define*

(a) P-order: $\mathcal{C}_1 \subseteq \mathcal{C}_2 \Rightarrow \left[q_1^-(z)e^{i\theta_1^-(z)}, q_1^+(z)e^{i\theta_1^+(z)}\right] \subseteq \left[q_2^-(z)e^{i\theta_2^-(z)}, q_2^+(z)e^{i\theta_2^+(z)}\right]$ *and* $\nu_1(z)e^{i\theta_1(z)} \leq \nu_2(z)e^{i\theta_2(z)}$ *,for all* $z \in S$.

(b) R-order: $\mathcal{C}_1 \subseteq \mathcal{C}_2 \Rightarrow \left[q_1^-(z)e^{i\theta_1^-(z)}, q_1^+(z)e^{i\theta_1^+(z)}\right] \subseteq \left[q_2^-(z)e^{i\theta_2^-(z)}, q_2^+(z)e^{i\theta_2^+(z)}\right]$ *and* $\nu_1(z)e^{i\theta_1(z)} \geq \nu_2(z)e^{i\theta_2(z)}$*, for all* $z \in S$.

Definition 3.5. *For any* $\mathcal{C}_k = \left\{\left\langle z, \left[q_k^-(z)e^{i\theta_k^-(z)}, q_k^+(z)e^{i\theta_k^+(z)}\right] \nu_k(z)e^{i\theta_k(z)} \right\rangle | z \in S\right\}$ *is a CCS in* S*, where* $i \in \mathcal{N}(\mathcal{N} = 1, 2, ...n)$ *we define,*

$$(1)\underline{(P-union)}\ \overset{P}{\bigcup_{i\in\mathcal{N}}} \mathcal{C}_k = \left\langle \bigcup_{i\in\mathcal{N}} \left[q_k^-(z)e^{i\theta_k^-(z)}, q_k^+(z)e^{i\theta_k^+(z)}\right], \bigvee_{i\in\mathcal{N}} \nu_k(z)e^{i\theta_k(z)} \right\rangle$$

$$(2)\underline{(R-union)}\ \overset{R}{\bigcup_{i\in\mathcal{N}}} \mathcal{C}_k = \left\langle \bigcup_{i\in\mathcal{N}} \left[q_k^-(z)e^{i\theta_k^-(z)}, q_k^+(z)e^{i\theta_k^+(z)}\right], \bigwedge_{i\in\mathcal{N}} \nu_k(z)e^{i\theta_k(z)} \right\rangle$$

$$(3)\underline{(P-intersection)}\ \overset{P}{\bigcap_{i\in\mathcal{N}}} \mathcal{C}_k = \left\langle \bigcap_{i\in\mathcal{N}} \left[q_k^-(z)e^{i\theta_k^-(z)}, q_k^+(z)e^{i\theta_k^+(z)}\right], \bigwedge_{i\in\mathcal{N}} \nu_k(z)e^{i\theta_k(z)} \right\rangle$$

$$(4)\underline{(R-intersection)}\ \overset{R}{\bigcap_{i\in\mathcal{N}}} \mathcal{C}_k = \left\langle \bigcap_{i\in\mathcal{N}} \left[q_k^-(z)e^{i\theta_k^-(z)}, q_k^+(z)e^{i\theta_k^+(z)}\right], \bigvee_{i\in\mathcal{N}} \nu_k(z)e^{i\theta_k(z)} \right\rangle$$

Definition 3.6. *Given a complex cubic set* $\mathcal{C}_k$ *in* S*, the complement of* $\mathcal{C}_k$ *is denoted by* $(\mathcal{C}_k)^c$ *and is defined as follows,* $q_k^-(z)e^{i\theta_k^-(z)} = 1 - q_k^+(z)e^{i[2\pi-\theta_k^+(z)]}, q_k^+(z)e^{i\theta_k^+(z)} = 1 - q_k^-(z)e^{i[2\pi-\theta_k^-(z)]}$ *and* $\nu_k(z)e^{i\theta_k(z)} = 1 - \nu_k(z)e^{i[2\pi-\theta_k(z)]}$*. We have,*

$$(\mathcal{C}_k)^c = \left\langle \left[1 - q_k^+(z)e^{i[2\pi-\theta_k^+(z)]}, 1 - q_k^-(z)e^{i[2\pi-\theta_k^-(z)]}\right], 1 - \nu_k(z)e^{i[2\pi-\theta_k(z)]} \right\rangle,$$

for all $z \in S$.

4. APPLICATION OF COMPLEX CUBIC SOFT SETS IN DETERMINING THE LEVEL OF CONTAMINATION IN PROCESSED MILK

In this section we present a problem in approximating the level of contamination in processed milk using CCS. An algorithm is developed for the same. The working of the algorithm is illustrated with an example.

Definition 4.1. *For each element $x \in X$, the value function is defined as,*

$$\mathcal{VC}_k = \left| \left(\frac{r^- + r^+}{2} - r^f \right) .e^{i\left(\frac{\theta^- + \theta^+}{2} - \theta\right)} \right|$$

for $(k = 1, 2, ..., n)$.

Statement of the problem:
Let U = $\{m_1, m_2, .., m_n\}$ be the list of processed milk samples taken into consideration. Let E = $\{e_1, e_2, .., e_m\}$ be the parameters based on which the selection of contamination is to be finalized. Let F = $\{f_1, f_2, .., f_k\}$ be the set of experts from food and safety organization. Each expert present the membership values in CCS form. The CCS's are represented as $(\mathcal{C}_1, \mathcal{C}_2, .., \mathcal{C}_k)$. The problem is to convert the CCS's into significant value which provides the level of contamination in processed milk from the given list.

Algorithm:
Step:1 Identify the processed milk samples and the parameters.
Step:2 Form the CCS $(\mathcal{C}_1, \mathcal{C}_2, .., \mathcal{C}_k)$ for each expert.
Step:3 Using Definition 4.1 calculate the value function $\mathcal{VC}_1, \mathcal{VC}_2, .., \mathcal{VC}_k$.
Step:4 Evaluate the total value by summing the values.
Step:5 Order the values in descending order, the sample with maximum value confirms that there is a significant contamination in processed milk.

Case Study:
A group of experts from food and safety organization are in the process of testing the milk sample individually and independently based on the set of parameters.

1. Let U = $\{m_1, m_2, m_3, m_4\}$ be the list of processed milk samples and E= $\{e_1, e_2, e_3\}$ be the list of parameters related to contamination. Here e_1 = aflatoxin-M1, e_2 = antibiotics, e_3 = pesticides.

2. Let (f_1) be an expert. Let the expert inspect the milk samples based on the parameter set and provide their observation details in CCSS $(\mathcal{C}_1)$ by applying Definition 3.1.

	e_1	e_2
m_1	$\left\langle [0.5e^{i(0.6\pi)}, 0.8e^{i(0.9\pi)}], 0.1e^{i(0.2\pi)} \right\rangle$	$\left\langle [0.6e^{i(0.7\pi)}, 0.7e^{i(0.9\pi)}], 0.5e^{i(0.6\pi)} \right\rangle$
m_2	$\left\langle [0.3e^{i(0.4\pi)}, 0.9e^{i(1.2\pi)}], 0.4e^{i(0.6\pi)} \right\rangle$	$\left\langle [0.7e^{i(0.8\pi)}, 0.8e^{i(1.2\pi)}], 0.3e^{i(0.7\pi)} \right\rangle$
m_3	$\left\langle [0.41e^{i(0.6\pi)}, 0.7e^{i(0.8\pi)}], 0.3e^{i(1.2\pi)} \right\rangle$	$\left\langle [0.3e^{i(0.4\pi)}, 0.8e^{i(0.9\pi)}], 0.41e^{i(0.7\pi)} \right\rangle$
m_4	$\left\langle [0.04e^{i(0.15\pi)}, 0.3e^{i(0.41\pi)}], 0.5e^{i(0.8\pi)} \right\rangle$	$\left\langle [0.35e^{i(0.8\pi)}, 0.4e^{i(1.6\pi)}], 0.6e^{i(0.9\pi)} \right\rangle$

	e_3
m_1	$\left\langle [0.2e^{i(0.4\pi)}, 0.5e^{i(0.7\pi)}], 0.1e^{i(0.7\pi)} \right\rangle$
m_2	$\left\langle [0.3e^{i(0.5\pi)}, 0.6e^{i(0.7\pi)}], 0.1e^{i(0.5\pi)} \right\rangle$
m_3	$\left\langle [0.13e^{i(0.6\pi)}, 0.4e^{i(1.2\pi)}], 0.07e^{i(0.3\pi)} \right\rangle$
m_4	$\left\langle [0.4e^{i(1.6\pi)}, 0.71e^{i(2\pi)}], 0.18e^{i(0.3)} \right\rangle$

3. Value function is calculated by using Definition 4.1

$$\mathcal{VC}_1 = \begin{pmatrix} 0.04 & 0.28 & 0.19 \\ 0.15 & 0.25 & 0.14 \\ 0.44 & 0.43 & 0.17 \\ 0.03 & 0.17 & 0.23 \end{pmatrix}$$

4. The total score for each milk sample is calculated and presented as,

$$TS_j = \begin{pmatrix} 1.44 \\ 1.51 \\ 1.61 \\ 1.34 \end{pmatrix}$$

Arrange the milk samples based on the final score and assign rank for each sample.

Tabular representation of sample's total score values

TS_j	m_i	Score	Rank
TS_1	m_3	1.61	1
TS_2	m_2	1.51	2
TS_3	m_1	1.44	3
TS_4	m_4	1.34	4

We predict from the above tabular representation that the tested processed milk sample m_3 is the most contaminated and unsafe for daily usage.

CONCLUSION

In this manuscript, we have defined the notion of CCS to deal with uncertainty. Also, a MCDM problem is proposed to demonstrate the reliability and validity of the tool.

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