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CUBIC VAGUE SOFT SET AND ITS APPLICATION

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ABSTRACT. In this paper, we introduce the notion of cubic vague soft set (CVSS) and cubic vague soft set of $\widetilde{P},(\widetilde{R})$ -order, $\widetilde{P},(\widetilde{R})$-union, and $\widetilde{P},(\widetilde{R})$ -intersection and study their related properties. We also discuss the internal and external cubic vague soft set and some of its properties along with multi-criteria decision making (MCDM) problem.

1. INTRODUCTION

Zadeh [11] proposed the concept of fuzzy set. Soft set (SS) theory was introduced by Molodtsov [7] to deal with uncertainty parameters. Vague set (VS) is an extension of fuzzy sets. Gau et.al [5] defined the notion of vague set, a combination of true membership function (TMF) and false membership function (FMF). Xu et.al [10] proposed vague soft set (VSS) an extension to the SS. They also discussed the basic properties of VSS. Bustince and Burillo [3] developed Vague sets are IFS and its operations Khaleed and Nasruddin [1, 2] defined the notion of interval-valued vague soft sets (IVVSS) and its operations. Also, the concept of generalized interval-valued vague soft set (GIVVSS) was introduced by them. M.K. Sharma [9]developed the study of the mathematical model which takes its base in the theory of IVVS. Jun et.al [6] dealt the concept of cubic set a combination of IVS and FS and its properties. Chinnadurai et.al [4] introduced

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some of the operations of cubic soft matrices. Muhiuddin and Abdullah [8] mentioned the concept of cubic soft BCK/BCI algebras and its properties.

2. PRELIMINARIES

Definition 2.1. *[7] A pair (F, E) is called a soft set(SS) over U where F is a mapping given by $\widetilde{F} : E \to P(\widetilde{U})$. In other words, a SS over $\widetilde{U}$ is a parameterized family of subset of the universal set $\widetilde{U}$.*

Definition 2.2. *[10] A pair (F, X) is called a vague soft set(VSS) over $\widetilde{U}$, where $\widetilde{F}$ is a mapping given by $\widetilde{F} : X \to V(\widetilde{U})$. In other words, a VSS over $\widetilde{U}$ is a parameterized family of VSs of the universal set $\widetilde{U}$. For $e \in X$, $\zeta_{F(e)} : X \to [0,1]^2$ is regarded as the set of e-approximate value of the VSS $(\widetilde{F}, X)$.*

Definition 2.3. *[9] An interval value sets(IVVS) $\widetilde{A^v}$ over a universal set $\widetilde{U}$ is defined as an object of the form $\widetilde{A^v} = \{\langle x_i, [T_{\widetilde{A^v}}(x_i)], [f_{\widetilde{A^v}}(x_i)]\rangle x_i \in X\}$ where $T_{\widetilde{A^v}} : X \to D[0,1]$ and $f_{\widetilde{A^v}} : X \to D[0,1]$ are called "TMF" and "FMF" respectively where $D[0,1]$ is the set of all the intervals within $[0,1]$ or in other words an IVVS can be represented by*

$$\widetilde{A^v} = \langle [(x_i), [\mu_1, \mu_2], [\gamma_1, \gamma_1]] \rangle : x_i \in X$$

where $0 \leq \mu_1 \leq \mu_2 \leq 1$ and $0 \leq \gamma_1 \leq \gamma_2 \leq 1$. For each IVVS $\widetilde{A^v}$, $\pi_{1\widetilde{A^v}}(x_i) = 1 - \mu_{1\widetilde{A^v}}(x_i) - \gamma_{1\widetilde{A^v}}(x_i)$ are called grade of hesitancy of x_i in $\widetilde{A^v}$ respectively.

3. CUBIC VAGUE SOFT SET(CVSS)

In this section we developed CVSS and the operations $\widetilde{P}, (\widetilde{R})$-order, $\widetilde{P}, (\widetilde{R})$-union, $\widetilde{P}, (\widetilde{R})$-intersection on CVSS and investigate some related properties. We also discuss the internal cubic vague soft set (ICVSS)and external cubic vague soft set (ECVSS)and some properties along with example. Let $\widetilde{U}$ and E be the universal set and set of parameters respectively. $CV(\widetilde{U})$ — the set of all CVSSs on $\widetilde{U}$ and $X \subseteq E$.

Definition 3.1. *A pair $(\widetilde{F}, X)$ is called a CVSS over $\widetilde{U}$, where $\widetilde{F}$ is mapping given by $\widetilde{F} : X \to CV(\widetilde{U})$. In other words a CVSSs over $\widetilde{U}$ is a parameterized family of an CVSS of the universal set $\widetilde{U}$. The CVSS $\mathbb{M}$ is given by the equation*

$$\mathbb{M} = \left\{\left\langle M^t_{e_i}(\widetilde{u}), M^{1-f}_{e_i}(\widetilde{u})\right\rangle, \zeta_{e_i}(\widetilde{u})\right\}$$

$\forall\ e_i \in X,\ \widetilde{u} \in \widetilde{U}.$

Example 1.

TABLE 1. Cubic vague soft set

S_i	$\widetilde{F}_{e_1} = \mathbb{M}_1 = [M^t_{e_1}(\widetilde{u}), M^{1-f}_{e_1}(\widetilde{u})]$, $[\zeta^t_{e_1}(\widetilde{u}), \zeta^{1-f}_{e_1}(\widetilde{u})]$	$\widetilde{F}_{e_2} = \mathbb{M}_2 = [M^t_{e_2}(\widetilde{u}), M^{1-f}_{e_2}(\widetilde{u})]$, $[\zeta^t_{e_2}(\widetilde{u}), \zeta^{1-f}_{e_2}(\widetilde{u})]$	$\widetilde{F}_{e_3} = \mathbb{M}_3 = [M^t_{e_3}(\widetilde{u}), M^{1-f}_{e_3}(\widetilde{u})]$, $[\zeta^t_{e_3}(\widetilde{u}), \zeta^{1-f}_{e_3}(\widetilde{u})]$
s_1	$[0.20, 0.30]$ $[0.30, 0.45]$ $[0.15, 0.35]$	$[0.10, 0.20]$ $[0.25, 0.40]$ $[0.15, 0.50]$	$[0.30, 0.40]$ $[0.45, 0.55]$ $[0.35, 0.40]$
s_2	$[0.35, 0.45]$ $[0.45, 0.50]$ $[0.20, 0.40]$	$[0.25, 0.45]$ $[0.30, 0.45]$ $[0.10, 0.40]$	$[0.25, 0.40]$ $[0.35, 0.50]$ $[0.30, 0.55]$
s_3	$[0.10, 0.20]$ $[0.20, 0.35]$ $[0.15, 0.40]$	$[0.15, 0.25]$ $[0.20, 0.35]$ $[0.20, 0.25]$	$[0.20, 0.35]$ $[0.40, 0.50]$ $[0.15, 0.40]$
s_4	$[0.35, 0.50]$ $[0.40, 0.50]$ $[0.40, 0.45]$	$[0.30, 0.40]$ $[0.40, 0.50]$ $[0.30, 0.35]$	$[0.10, 0.30]$ $[0.20, 0.40]$ $[0.15, 0.25]$

Definition 3.2. *A CVSS $(\widetilde{F}, X)$ is said to be an ICVSS, if $\forall\ e_i \in X$, $\widetilde{F}e_i = \mathbb{M}_i$ is so that $M^t_{e_i}(\widetilde{u}) \leq \zeta_{e_i}(\widetilde{u}) \leq {M_{e_i}}^{1-f}(\widetilde{u})\ \forall\ e_i \in X$ and for all $\widetilde{u} \in \widetilde{U}$.*

$M^t_{e_i}(\widetilde{u})$ is denoted $(M^{t^L}_{e_i}(\widetilde{u}), M^{t^U}_{e_i}(\widetilde{u}))$, $\zeta_{e_i}(\widetilde{u})$ is denoted $(\zeta^t_{e_i}(\widetilde{u}), \zeta^{1-f}_{e_i}(\widetilde{u}))$, $M^{1-f}_{e_i}(\widetilde{u})$ is denoted $(M^{1-f^L}_{e_i}(\widetilde{u}), M^{1-f^U}_{e_i}(\widetilde{u}))$.

Definition 3.3. *A CVSS $(\widetilde{F}, X)$ is said to be an ECVSS, if $\forall\ e_i \in X$, $\widetilde{F}(e_i) = \mathbb{M}_i$ is so that $\zeta_{e_i}(\widetilde{u}) \notin (M^t_{e_i}(\widetilde{u}), M^{1-f}_{e_i}(\widetilde{u}))\ \forall\ e_i \in X$ and $\forall\ \widetilde{u} \in \widetilde{U}$.*

Definition 3.4. *Let*

$$(\widetilde{F}, X) = \left\{ \widetilde{F_{e_i}} = \mathbb{M}_i = \left\{ \left\langle \widetilde{u}, [M^t_{e_i}(\widetilde{u}), M^{1-f}_{e_i}(\widetilde{u})], \zeta_{e_i}(\widetilde{u}) \right\rangle |\widetilde{u} \in \widetilde{U} \right\} e_i \in X \right\}$$

and

$$(\widetilde{G}, Y) = \left\{ \widetilde{F_{e_i}} = \mathbb{N}_i = \left\{ \left\langle \widetilde{u}, [N^t_{e_i}(\widetilde{u}), N^{1-f}_{e_i}(\widetilde{u})], \eta_{e_i}(\widetilde{u}) \right\rangle |\widetilde{u} \in \widetilde{U} \right\} e_i \in X \right\}$$

be two CVSS in $\widetilde{U}$, $X, Y \subseteq E$. Then we have the following $(\widetilde{F}, X) = (\widetilde{G}, Y)$ if and only if(iff), the following conditions are satisfied

(i) $X = Y$;

(ii) $\widetilde{F_{e_i}} = \widetilde{G_{e_i}}\ \forall\ e_i \in X$ *iff* $M^t_{e_i}(\widetilde{u}) = N^t_{e_i}(\widetilde{u}), M^{1-f}_{e_i}(\widetilde{u}) = N^{1-f}_{e_i}(\widetilde{u})$ *and* $\zeta_{e_i}(\widetilde{u}) = \eta_{e_i}(\widetilde{u})\ \forall\ e_i \in X$.

Definition 3.5. *If $(\widetilde{F}, X)$ and $(\widetilde{G}, Y)$ are two CVSSs then, we define P-order between then as $(\widetilde{F}, X) \subseteq_{\widetilde{p}} (\widetilde{G}, Y)$ iff the below mentioned conditions are satisfied*

(i) $X \subseteq Y$;

(ii) $\widetilde{F_{e_i}} \leq_{\widetilde{p}} G_{e_i}\ \forall\ e_i \in X$ *iff* $M^t_{e_i}(\widetilde{u}) \leq N^t_{e_i}(\widetilde{u}), M^{1-f}_{e_i}(\widetilde{u}) \leq N^{1-f}_{e_i}(\widetilde{u})$ *and* $\zeta_{e_i}(\widetilde{u}) \leq \eta_{e_i}(\widetilde{u})\ \forall\ \widetilde{u} \in \widetilde{U}$ *corresponding to each* $e_i \in X$.

Definition 3.6. *If $(\widetilde{F}, X)$ and $(\widetilde{G}, Y)$ are two CVSSs then, we define R-order between then as $(\widetilde{F}, X) \subseteq_{\widetilde{R}} (\widetilde{G}, Y)$ iff the below mentioned conditions are satisfied*

(i) $X \subseteq Y$ *and*
(ii) $\widetilde{F_{e_i}} \leq_{\widetilde{R}} \widetilde{G_{e_i}} \; \forall \, e_i \in X$ *iff* $M^t_{e_i}(\widetilde{u}) \leq N^t_{e_i}(\widetilde{u}), M^{1-f}_{e_i}(\widetilde{u}) \leq N^{1-f}_{e_i}(\widetilde{u})$ *and* $\zeta_{e_i}(\widetilde{u}) \geq \eta_{e_i}(\widetilde{u}) \; \forall \, \widetilde{u} \in \widetilde{U}$ *corresponding to each* $e_i \in X$.

Theorem 3.1. *Let*

$$(\widetilde{F}, X) = \widetilde{F}(e_i) = \left\{ \left\langle \widetilde{u}, \left[M^t_{e_i}(\widetilde{u}), M^{1-f}_{e_i}(\widetilde{u})\right], \left[\zeta^t_{e_i}(\widetilde{u}), \zeta^{1-f}_{e_i}(\widetilde{u})\right] \right\rangle | \widetilde{u} \in \widetilde{U}, e_i \in X \right\}$$

be a CVSS in $\widetilde{U}$ which is not an ECVSS. Then there is at least one $e_i \in X$ for which there exists some $x \in X$ such that

$$\zeta^t_{e_i}(\widetilde{u}) \in \left(M^{t^L}_{e_i}(\widetilde{u}), M^{t^U}_{e_i}(\widetilde{u})\right) \quad \textit{and} \quad \zeta^{1-f}_{e_i}(\widetilde{u}) \in \left(M^{1-f^L}_{e_i}(\widetilde{u}), M^{1-f^U}_{e_i}(\widetilde{u})\right).$$

Theorem 3.2. *Let*

$$(\widetilde{F}, X) = \widetilde{F}(e_i) = \left\{ \left\langle \widetilde{u}, \left[M^t_{e_i}(\widetilde{u}), M^{1-f}_{e_i}(\widetilde{u})\right], \left[\zeta^t_{e_i}(\widetilde{u}), \zeta^{1-f}_{e_i}(\widetilde{u})\right] \right\rangle | \widetilde{u} \in \widetilde{U}, e_i \in X \right\}$$

be a CVSS in $\widetilde{U}$. If $(\widetilde{F}, X)$ is both an ICVSS and ECVSS in $\widetilde{U}$, then $\forall \; u \in U$ corresponding to each

$$e_i \in X, \quad \zeta^t_{e_i}(\widetilde{u}) \in \left(M^{t^L}_{e_i}(\widetilde{u}), M^{t^U}_{e_i}(\widetilde{u})\right), \quad \zeta^{1-f}_{e_i}(\widetilde{u}) \in \left(M^{1-f^L}_{e_i}(\widetilde{u}), M^{1-f^U}_{e_i}(\widetilde{u})\right),$$

where

$$M^{t^-}_{e_i}(\widetilde{u}) = \{M^{t^L}_{e_i}(\widetilde{u}) : \widetilde{u} \in \widetilde{U}\}; \quad M^{t^+}_{e_i}(\widetilde{u}) = \{M^{t^U}_{e_i}(\widetilde{u}) : \widetilde{u} \in \widetilde{U}\}$$

and

$$M^{1-f^-}_{e_i}(\widetilde{u}) = \{M^{1-f^L}_{e_i}(\widetilde{u}) : \widetilde{u} \in \widetilde{U}\}; \quad M^{1-f^+}_{e_i}(\widetilde{u}) = \{M^{1-f^U}_{e_i}(\widetilde{u}) : \widetilde{u} \in \widetilde{U}\}.$$

3.1. **Value Extraction From CVSS.**

Definition 3.7. *For each CVSS its value function is defined as*

$$V_f = \left(\frac{L_v(u) + U_v(u)}{2} + \zeta_v(u) \right),$$

which can be considered as a meaningful value represents the level of vagueness of the corresponding elements. The element with least vague score value is consider as the best choice.

4. Application of cubic vague soft set

Consider the following MCDM problems.

An individual is interested to buy a bike and approaches three different bike showrooms. $U = \{b_1, b_2, b_3\}$ by considering six criteria $E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$, where $e_1 = ultra\ model$, $e_2 = new\ model$, $e_3 = old\ model$, $e_4 = full\ payment$, $e_5 = monthly\ payment$ and $e_6 = half\ yearly\ payment$. Let $X, Y \subseteq E$. Let $\mathcal{X}$ represent the bike model $\{ultra\ model,\ new\ model\ and\ old\ model\}$ and Y represents the payment mode $\{full\ payment,\ monthly\ payment\ ,\ half\ year\ payment\}$. Each alternative is represented as CVSS. Let (F, X) describe the attributes bike model and (G, Y) describe the attributes payment mode.

4.1. **Algorithm to solve MCDM problem.**

Step(1) Construct the CVSS.
Step(2) Determine the TMF and FMF of (F, X) and (G, Y).
Step(3) Compute the R-union TMF.
Step(4) Compute the R-intersection FMF.
Step(5) Find the value function $V_f = \left(\frac{L_v(u)+U_v(u)}{2} + \zeta_v(u)\right)$.
Step(6) Determine the row-column sums TMF and FMF.
Step(7) Compute the final score function.

Step(1) Construct the cubic vague soft set(CVSS).
(F, X) describe the attributes of bike models, Ultra model=
$\langle b_1, \{[0.6, 0.7], [0.7, 0.8]\}, [0.5, 0.7]\rangle, \langle b_2, \{[0.2, 0.5], [0.3, 0.6]\}, [0.6, 0.7]\rangle,$
$\langle b_3, \{[0.4, 0.7], [0.5, 0.8]\}, [0.5, 0.6]\rangle$,
New model=
$\langle b_1, \{[0.2, 0.5], [0.4, 0.6]\}, [0.7, 0.8]\rangle,$
$\langle b_2, \{[0.3, 0.6], [0.5, 0.7]\}, [0.4, 0.6]\rangle, \langle b_3, \{[0.1, 0.5], [0.3, 0.8]\}, [0.4, 0.7]\rangle$,
Old model= $\langle b_1, \{[0.3, 0.4], [0.5, 0.6]\}, [0.2, 0.5]\rangle,$
$\langle b_2, \{[0.6, 0.8], [0.7, 0.9]\}, [0.4, 0.7]\rangle, \langle b_3, \{[0.5, 0.6], [0.7, 0.8]\}, [0.3, 0.5]\rangle$.
(G, Y) describe the attributes of payment mode, Full payment=
$\langle b_1, \{[0.5, 0.8], [0.6, 0.9]\}, [0.7, 1]\rangle, \langle b_2, \{[0.6, 0.9], [0.8, 1]\}, [0.3, 0.5]\rangle,$
$\langle b_3, \{[0.4, 0.6], [0.5, 0.7]\}, [0.3, 0.4]\rangle$,
Monthly payment= $\langle b_1, \{[0.2, 0.5], [0.3, 0.6]\}, [0.4, 0.7]\rangle,$
$\langle b_2, \{[0.3, 0.5], [0.4, 0.8]\}, [0.5, 0.6]\rangle, \langle b_3, \{[0.4, 0.6], [0.5, 0.9]\}, [0.1, 0.5]\rangle$,
Half year payment= $\langle b_1, \{[0.7, 0.8], [0.8, 1]\}, [0.4, 0.6]\rangle,$

$\langle b_2, \{[0.4, 0.7], [0.5, 0.8]\}, [0.3, 0.6]\rangle, \langle b_3, \{[0.2, 0.4], [0.3, 0.7]\}, [0.6, 0.8]\rangle$.

The following steps of algorithm and caring out the calculation the final result of step-7 final score is obtain as given below.

Step(7) Compute the final value function.

TABLE 2. Final score

b	$TMF(T)$	$FMF(F)$	$Finalscore(T-F)$
b_1	-0.2	0.05	-0.15
b_2	0.7	0	0.7
b_3	-0.5	-0.05	-0.45

From Table 2, the element with least final score value is consider as the best choice. Hence the individual select b_3 showroom to purchase a bike.

5. CONCLUSION

This manuscript deals with the concept of CVSS introduced the concept of CVSS as an extension to the IVVSS. Also we have proved the properties of $\widetilde{P}, (\widetilde{R})$ order $\widetilde{P}, (\widetilde{R})$ union and intersection, $\widetilde{P}, (\widetilde{R})$ equality and subset. An application is illustrated to prove the reliability of CVSS in MCDM problem.

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