Advances in Mathematics: Scientific Journal **9** (2020), no.4, 1623–1636
ISSN: 1857-8365 (printed); 1857-8438 (electronic)
https://doi.org/10.37418/amsj.9.4.18 Spec. Issue on NCFCTA-2020

NUMERICAL SOLUTIONS OF DOUBLE DIFFUSIVE CONVECTIVE FLOW PAST A CHEMICAL REACTIVE VERTICALLY INCLINED INFINITE PLATE WITH HEAT SOURCE/SINK

R. M. KANNAN AND B. PULLEPU[1]

ABSTRACT. A numerical solution is presented for the effects of chemical reaction, the heat source/sink, Schmidt number and Prandtl number on double diffusion natural convective flow along a vertically inclined infinite plate. The governing non-dimensional equations are solved using, iterative, tri-diagonal, implicit finite-difference scheme. Representative outcomes of the fluid velocity, temperature and concentration profiles are displayed in graph.

1. INTRODUCTION

Coupled temperature also mass transfer performance a significant part in industrial productions aimed at the model of fins, steel undulating, atomic power plants, gas turbines and in numerable impulsion strategies for airplanes, weaponries, spaceships strategy, solar energy gatherers and strategy of chemical treating apparatus, satellites and planetary vehicles. There exist binary forms of chemical responses such as identical or unrelated. These reactions are based on the environment such as in interface or in a distinct phase quantity. In combined structures the reaction is heterogeneous, consider the region at a boundary level, while, the response is similar when this takes domicile in a solution. Chamber and Young [1] studied first order organic response adjacent

[1]*corresponding author*

2010 *Mathematics Subject Classification.* 76R10, 80M20, 76M20.
Key words and phrases. Chemical reaction, Finite difference method, natural convection, Heat source/sink, upright inclined infinite plate, UWC/UWT.

to a horizontal platter. Soundalgekar et. al [2] obtained an particular result of the movement for an non-compressible viscidliquid over an impetuously accelerated infinite upright plate in the occurrence of external corpus below the subsequent circumstances: i) flexible plate thermal and ii) Constant heat flux. Gupta et.al [3] has analysed natural transmission over single linearly speeded upright plate in the occurrence of viscid dissipative thermal consuming Laplace-transform technique. Williams and Kafoussius calculated the result of thermal-dependent gluiness on the natural transmission laminar periphery layer moving along a horizontal uniform platter. Hassanien et. al explored in constant viscosity and temperature transfer properties on joined thermal besides mass transmission now mixed convection over a UMF/UHF segment in permeable medium. EL.Kabeir and Abdou [4] focused the properties of chemical reaction, mass and heat transmission on MHD movement over a horizontal uniform cone side in micro glacial liquids with thermal source/sink outcomes. Chen et. al [5] explored the free transmission on horizontal, vertical and inclined plates with different combinations of non-uniform surface thermal or variable thermal flux. The combined properties of mass and thermal transmission conceded through a mathematical study of unsteady natural convection movement past an impetuously moving horizontal platter with isothermal temperature and mass flux is discussed by Muthucumarasamy and Ganesan [6]. Raptis et. al [7] attained the terms for the change in displacement and temperature for stable two dimensional thermal and mass transfer movement of an non-compressible glutinous liquid through a permeable substance confined via a upright infinite area with constant suction. Loganathan and Sivapoornapriya [8] analysed viscid dissipation properties unstable free convective movement in finite upright plate with isothermal mass and Heat flux. Hence, it is projected to analyze an unstable2D, laminar movement with thermal and mass transmission outcomes over an infinite upright inclined plate in the occurrence of permeable spectrum along with heat generation/absorption properties subject to UWT/UWC. The dimensionless leading calculations are solved using the implicit finite difference scheme of Crank-Nicolson type. In order to verify the correctness of our numeral outcomes, the current study is matched with the existing theoretic result of Soundalgekar and Patil [9] and they are originate to view in excellent conformation.

2. Mathematical Investigation

Cogitate an unstable, 2D, laminar free convective movement over an infinite vertical inclined plate in the occurrence of permeable medium under the effect of heat generation/absorption, focus to UWT/UWC. Consider the x-axis is engaged the plate in the upright ascendant and the y-axis is consider normal to the platter. At period $t^{'} \leq 0$, the plate and liquid are at the similar thermal $T^{\prime}_{\infty}$ and concentration $C^{\prime}_{\infty}$. At time $t^{'} > 0$ the temperature of the platter is raised to $T^{\prime}_{\infty}$ and concentration C_{∞}. By under Boussinesq's estimate the leading equations,

$$\frac{\partial u}{\partial t'} = g\beta\cos\phi(T^{'} - T^{'}_{\infty}) + \nu\frac{\partial^2 u}{\partial y^2} + g\beta_C(C^{'} - C^{'}_{\infty})\cos\phi \tag{2.1}$$

impetus, dynamism and concentration calculations are specified by

$$\frac{\partial T^{'}}{\partial t'} = \alpha\frac{\partial^2 T^{'}}{\partial y^2} + \frac{Q_0}{\rho C_P}(T^{'} - T^{'}_{\infty})\,. \tag{2.2}$$

$$\frac{\partial C^{'}}{\partial t'} = D\frac{\partial^2 C^{'}}{\partial y^2} + K_C(C^{'} - C^{'}_{\infty})\,. \tag{2.3}$$

Using the following B.C's:

$$\left.\begin{array}{l} t^{'} \leq 0 : u = 0, T^{'} = T^{'}_{\infty}, C^{'} = C^{'}_{\infty}, \forall\,\text{t and y} \\ t^{'} > 0 : u = u_0, T^{'} = T_w, C^{'} = C_w \text{ at}\, y = 0 \\ u \to 0, T^{'} \to T^{'}_{\infty}, C^{'} \to C^{'}_{\infty}\,\text{as}\, y \to \infty \end{array}\right\} \tag{2.4}$$

On presenting the subsequent dimensionless values:

$$\left.\begin{array}{l} U = \dfrac{u}{u_0}, Y = \dfrac{yu_0^2}{\nu}, t = \dfrac{tu_0^2}{\nu}, \Delta = \dfrac{Q_0\nu}{\rho C_P u_0^2}, \\ T = \dfrac{T^{'} - T^{'}_{\infty}}{T^{'}_w - T^{'}_{\infty}}, Gr = \dfrac{g\beta\nu(T^{'}_w - T^{'}_{\infty})}{u_0^3}, Pr = \dfrac{\nu}{\alpha}, \\ C = \dfrac{C^{'} - C^{'}_{\infty}}{C^{'}_w - C^{'}_{\infty}}, Gr_c = \dfrac{g\beta_c\nu(C^{'}_w - C^{'}_{\infty})}{u_0^3}, Sc = \dfrac{\nu}{D} \end{array}\right\}$$

In equation (2.1 - 2.4) principals to

$$\frac{\partial U}{\partial t} = G_r T\cos\phi + \frac{\partial^2 U}{\partial Y^2} + G_c C\cos\phi \tag{2.5}$$

$$\frac{\partial T}{\partial t} = \frac{1}{Pr} + \frac{\partial^2 T}{\partial Y^2} + \Delta T \tag{2.6}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc} + \frac{\partial^2 C}{\partial Y^2} - \lambda C\,. \tag{2.7}$$

The preliminary and boundary condition in non-dimensional form

$$\left.\begin{array}{l} t \leq 0 : U = 0, T = 0, C = 0 \text{ for all } Y \\ t > 0 : U = 1, T = 1, \text{and } C = 1 \text{ at } Y = 0 \\ U = 0, T = 0 \text{ at } X = 0 \\ U \to 0, T \to 0, C \to 0 \text{ as } Y \to \infty \end{array}\right\}. \tag{2.8}$$

3. Numerical Procedure

The above combined non-linear equations (2.5 - 2.7) be resolved by implicit finite difference method. We express the equations (2.5 - 2.7) by implementing finite difference method of Crank-Nicholson category.

The equivalent finite-difference equation exist as follows:

$$\begin{aligned} \frac{1}{\Delta t}[U_j^{k+1} - U_j^k] &= G_r\left(\frac{T_j^{k+1} + T_j^k}{2}\right) + G_c\left(\frac{C_j^{k+1} + C_j^k}{2}\right) \\ &+ \frac{1}{2(\Delta Y)^2}[U_{j-1}^{k+1} - 2U_j^{k+1} + U_{j+1}^{k+1} + U_{j-1}^k - 2U_j^k + U_{j+1}^k] \end{aligned} \tag{3.1}$$

$$\begin{aligned} \left[\frac{T_j^{k+1} - T_j^k}{\Delta T}\right] &= \frac{1}{Pr}\frac{1}{2(\Delta Y)^2}\left[T_{j-1}^{k+1} - 2T_j^{k+1} + T_{j+1}^{k+1} + T_{j-1}^k - 2T_j^k + T_{j+1}^k\right] \\ &+ \Delta\left(\frac{T_j^{k+1} + T_j^k}{2}\right) \end{aligned} \tag{3.2}$$

$$\begin{aligned} \left[\frac{C_j^{k+1} - C_j^k}{\Delta T}\right] &= \frac{1}{2Sc(\Delta Y)^2}\left[C_{j-1}^{k+1} - 2C_j^{k+1} + C_{j+1}^{k+1} + C_{j-1}^k - 2C_j^k + C_{j+1}^k\right] \\ &- \lambda\left(\frac{C_j^{k+1} + C_j^k}{2}\right) \end{aligned}$$

The integration area is divided as a square with edges $t_{\max}(= 5)$ then $Y_{\max}(= 14)$, wherever $Y_{\max}$ resembles to $y = \infty$ which deceptions exact well outer the impetus and energy periphery layers. The supreme of Y was taken as 14 afterward some initial inquiries so that the past binary of the periphery circumstances (2.8) are fulfilled. Now, the j-along the Y direction, and the subscript n beside the t track. At any one-time stage, the quantities U_j^k and V_j^k performing now the difference equation are presented as coefficients. The quantities U, V, and T are acknowledged at entirely network points at $t = 0$ in the preliminary circumstances.

The calculations of U, V,T and C at the period level $(k + 1)$ via the quantities at the earlier period level (k) are agreed out as monitors:

The finite transformation Eq. (3.1) at each inner lattice point on a specific i-stage establishes a tridiagonal scheme of calculations. In schemes of calculations are workout applying the Thomas algorithm as labeled in the effort of Carnahan et al [10].Thus, the quantities of Care initiate at each at each lattice point for a specific j at the $(k+1)^{th}$ period level. Similarly, the quantities of T at the $(k+1)^{th}$ time level in Eq. (3.2), Via the quantities of C and T at $(k+1)^{th}$ stage in equation (3.1), the quantities of U at (k+1)th stage are establish in a comparable way. Thus, the quantities of C, T and U are identified on a specific j-level. Lastly, the quantities of V are designed clearly using Eq. (3.1) at each lattice point on an individual i-stage at the $(k + 1)^{th}$ period stage. This procedure is repetitive for different j-levels. Thus, the quantities of C, T, U, and V are identified at all lattice points in the square area at the $(k + 1)^{th}$ stage.

In an analogous technique calculations are conceded out by acting along the j-direction. Later calculating the quantities conforming to every j at a period level, the quantities at the subsequent period level are calculated in a like way. Calculations are frequent awaiting the stable-condition is stretched. The stable-state answer is expected to have attained, when the complete variance among the quantities of U, as well as heat T and concentration C at two sequential time stages is smaller than 10-5 at all lattice points. The finite difference system is unreservedly steady. In confined truncation inaccuracy is $O(\Delta t^2 + \Delta Y^2)$ and it leads to null as Δt and ΔY tends to null. Therefore the system is well-matched. Stability and compatibility confirm convergence.

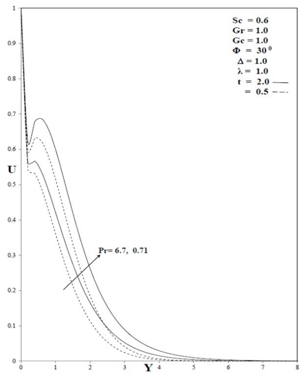

FIGURE 1. Velocity profile for various values of Pr.

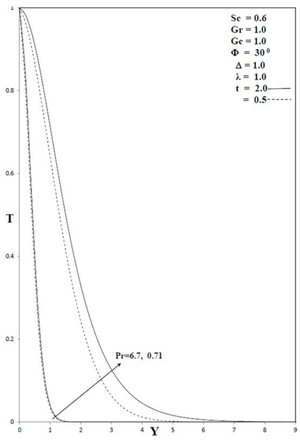

FIGURE 2. Temperature profile for various values of Pr.

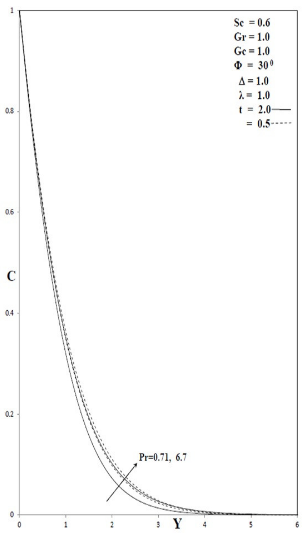

FIGURE 3. Concentration profile for various values of Pr.

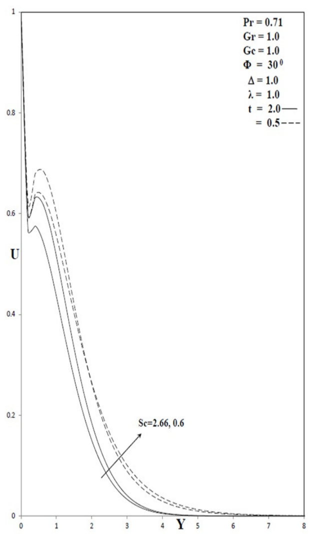

FIGURE 4. Velocity profile for various values of Sc.

4. CONCLUDING REMARKS

To get the substantial nature of the problematic the numeral principles of various constraints like Prandtl numeral, Schmidt number, thermal source/sink constraint and chemical reaction constraint are accessible clear by graph. For

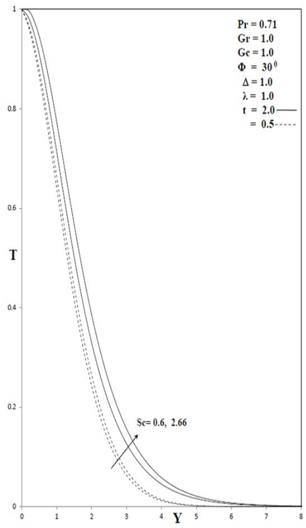

FIGURE 5. Temperature profile for various values of Sc.

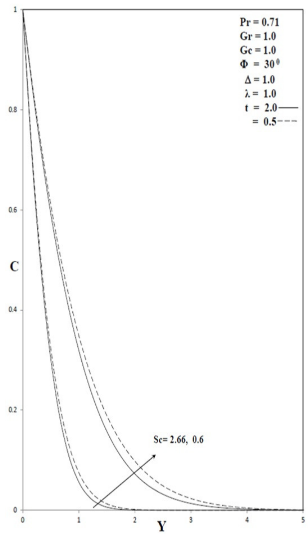

FIGURE 6. Concentration profile for various values of Sc.

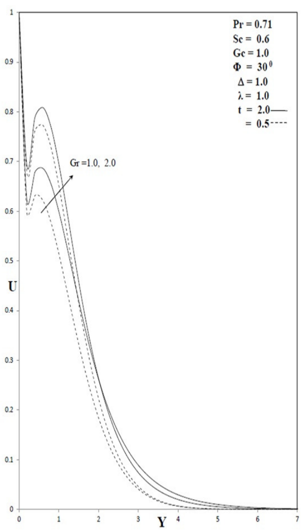

FIGURE 7. Velocity profile for various values of Gr.

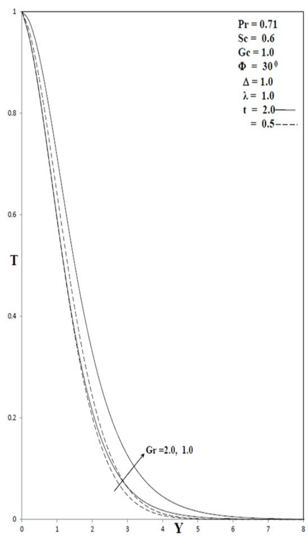

FIGURE 8. Temperature profile for various values of Gr.

the scheming part varying the quantities of different constraints express above expression the numeral quantities of velocity and thermal are obtained till they congregate to natural stream boundary circumstances.

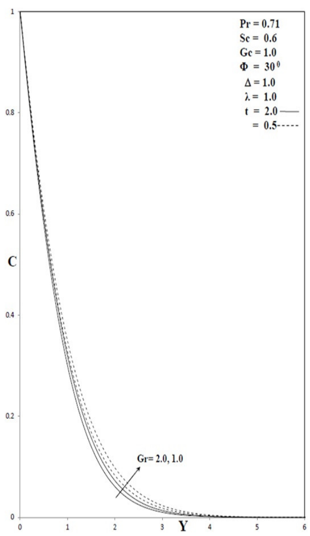

FIGURE 9. Concentration profile for various values of Gr.

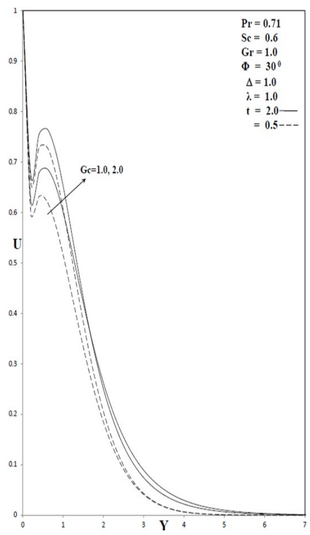

FIGURE 10. Velocity profile for various values of Gc.

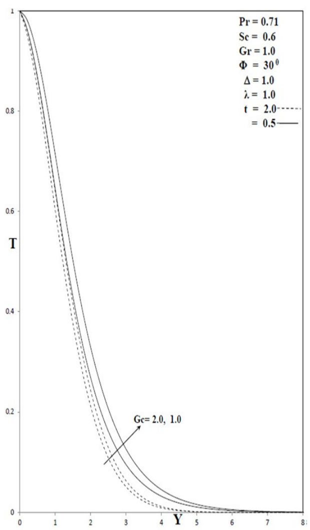

FIGURE 11. Temperature profile for various values of Gc.

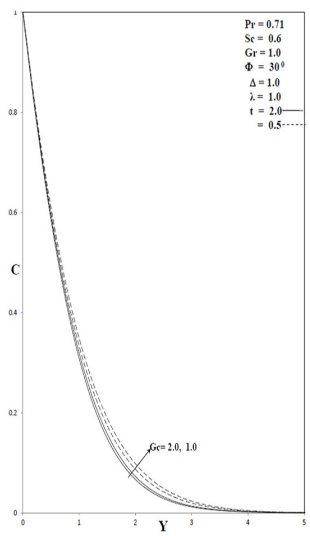

FIGURE 12. Concentration profile for various values of Gc.

In displays 1a -1c depict the unsteady velocity, thermal and concentration outlines in place of different quantities of Prandtl number (Pr). As shown in the (fig1 (a)) decreasing values of Pr velocity increases. Figure 1(b) represents the

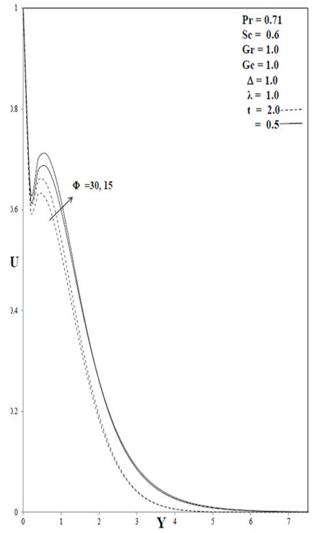

FIGURE 13. Velocity profile for various values of ϕ.

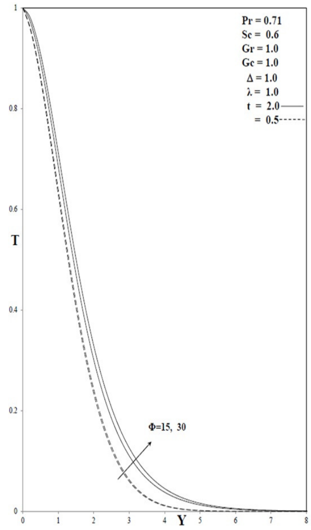

FIGURE 14. Temperature profile for various values of ϕ.

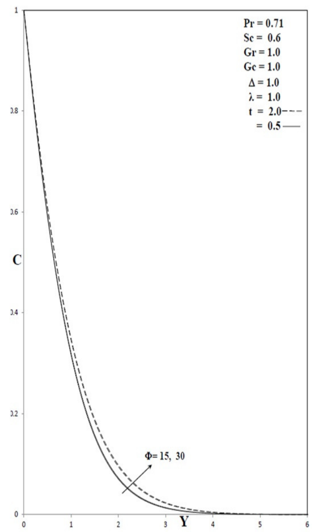

FIGURE 15. Concentration profile for various values of ϕ.

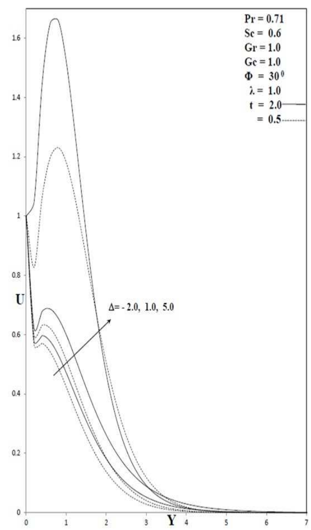

FIGURE 16. Velocity profile for various values of Δ.

outcome of Pr on the thermal field. It shows that the thermal growths for minor quantities of Pr, the thermal boundary layer reductions for bigger quantities of

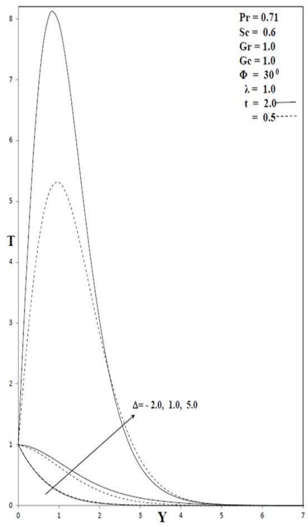

FIGURE 17. Temperature profile for various values of Δ.

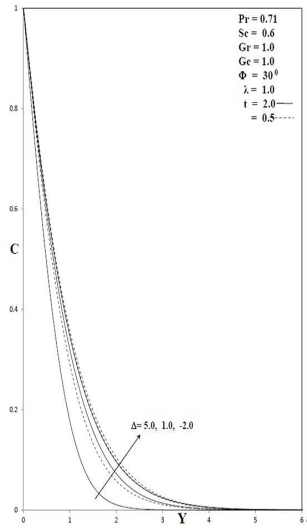

FIGURE 18. Concentration profile for various values of Δ.

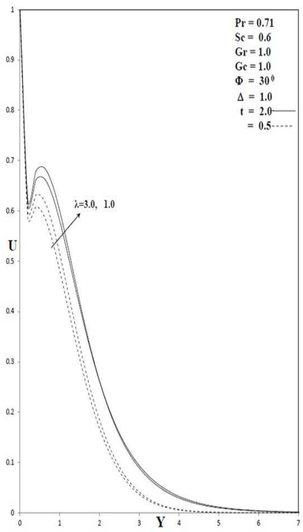

FIGURE 19. Velocity profile for various values of λ.

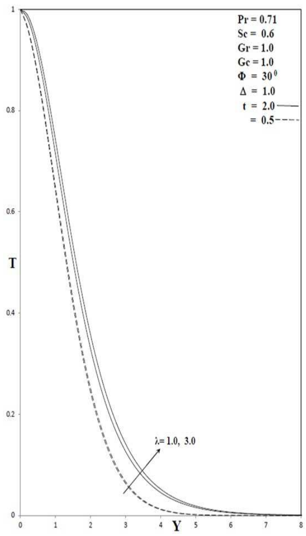

FIGURE 20. Temperature profile for various values of λ.

Pr. In fig 1 (c) it is view that the effect exists reversed in the case of concentration profile.

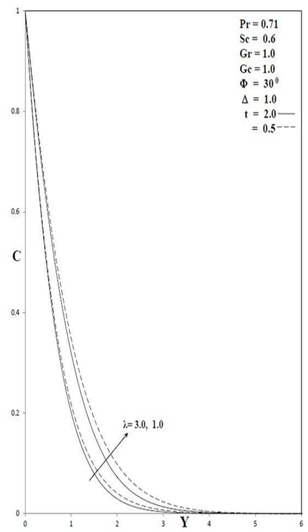

FIGURE 21. Concen-tration profile for various values of λ.

Figures 2(a) - 2(c) analyzed the properties of change in displacement, thermal and concentration outlines for different quantities of Schmidt numeral Sc. As the Sc growths, concentration drops. This sources the concentration resistance properties to drop. The velocity declines with a growth in Sc (fig 2 (a)). Whereas the thermal rises for bigger quantities of Sc as shown from figure 2(b). Graph 2 (c) illustrations that the concentration reductions for higher quantities of Sc with growing Schmidt numeral the velocity is reduced through the boundary layer; that is, the movement is delayed.

Figures 3 (a)-3 (c) shows in effects of the Grashof numeral Gr on change in displacement, thermal and concentration allocations. In fig 3a shows the velocity is enlarged during the boundary layer with a growth in Gr. Figures 3 (b) and 3 (c) shows the effect is decreased in mutually the cases (concentration and temperature).

Consider velocity, temperature and concentration outlines for various quantities of Gc are displayed in fig: 4(a)-4(c). The upper quantities of Gc, the velocity increases 4(a), then the tendency is reversed in the circumstance of both thermal and concentration are exposed in figure 4b and 4c.

Discussed velocity, temperature and attentiveness summaries for different quantities of angles are exposed now figures 5 (a)-5 (c). It is studied that velocity is increased, when angle ϕ decreased 5(a), but the effect is reversed in the cases of both thermal and concentration field are presented graphically shows fig. 5(b) and 5 c individually.

The figures 6 (a)-6 (c) illustrates effect of change in displacement, temperature and concentration outlines against various quantities of heat source/sink constraint Δ. The increasing quantities of Δ denote the occurrence of heat source and the diminishing quantities relate to thermal sink. It is viewed fig 6(a) and 6(b) that the outcome of Δ on the velocity and temperature circulation. It is viewed from above graphs heat is produced the buoyancy force upturns which brings the flow rate to increase and giving rise to the velocity outlines and temperature field. From 6 (c) analyzed the concentration diminutions and the period taken to touch the stable state is raised when Δ rise.

Fig 7 (a)-7 (c) analyzed the properties of change in displacement, thermal and concentration outlines for various quantities of chemical reaction constraint λ. It is appreciated that the velocity field of air (Pr = 0.71) growths in the case of reproductive reaction, but it reductions in the case of destructive reaction. Change of displacement and boundary layer width with a growth in λ (figure 7(a)), whereas thermal rises for bigger quantities of λ (figure 7(b)) the steady state-state concentration outlines for various quantities of λ exist in fig 7 (c). The above graphs witnessed that the concentration profiles rises as the chemical reaction constraint λ growths in the case of generative reaction, but it reductions as λ increases in the case of destructive response.

5. CONCLUSION

This article analyzed the terminologies for the velocity and temperature by Crank-Nicolson method. The different outcomes similar heat source/sink parameter, Prandtl number, chemical reaction constraint, Mass Grashof number, Grashof number and Schmidt number are workout and investigated explicitly. Deductions of the study are as monitors:

(1) The liquid velocity rises for upper quantities of Δ, G_r, G_c and lesser quantities of Pr, λ, Sc, ϕ.

(2) The thermal field growths, for bigger quantities of $\Delta, Sc, \phi, \lambda$ and minor quantities of Pr, G_r, G_c.

(3) The concentration of species reductions for lesser quantities of Δ, Sc, G_r, G_c, λ and bigger quantities of ϕ, Pr.

(4) The period attain to grasp stable state rises with growing Δ, Pr, λ and Sc.

References

[1] P. L. Chamber, J. D. Young: *On the diffusion of a chemically reactive species in a laminar boundary layer flow*, The Physics of Fluids, **1** (1956), 48 – 54.

[2] V. M. Soundalgekar, N. S. Birajdar, Darwhekar : *Mass transfer effects on the flow past an impulsively started infinite vertical plate with variable temperature of constant heat flux*, Astrophysics and Space Sciences, **100** (1984), 159 – 164.

[3] A. S. Gupta, I. Pop, V. M. Soundalgekar: *Free convection effects on the flow past an accelerated vertical plate in an incompressible dissipative fluid*, Rev. Roum. Sci. Techn. Mec. Apl., **24**(4) (1979), 561–568.

[4] S. M. M. EL-Kabeir, M. M. M. Abdou: *Chemical reaction, heat and mass transfer on MHD flow over a vertical isothermal cone surface in micro polar fluids with heat generation/absorption*, Applied Mathematical Sciences, **34** (2007), 1663 – 1674.

[5] T. S. Chan, H .C. Tien, B. F. Armaly: *Natural convection on horizontal, inclined and vertical plates with variable surface temperature heat flux*, International Journal of Heat Mass Transfer, **29** (1986), 1465 – 1478.

[6] R. Muthucumaraswamy, P. Ganesan: *First order chemical reaction on flow past an impulsively started vertical plate with uniform heat and mass flux*, Acta Mechanica, **147** (2001), 45 – 57.

[7] A. Raptis, G. Tzivenidis, N. Kafousia: *Free convection and mass transfer flow through a porous medium bounded by an infinite vertical limiting surface with constant suction*, Letters in Heat and Mass Transfer, **8** (1982), 417 – 424.

[8] P. Loganthan, C. Sivapoornapriya: *Viscous dissipation effects on unsteady natural convective flow past an infinite vertical plate with uniform heat and mass flux*, Wseas Transactions on Heat and Mass Transfer, **9** (2014), 63 – 73.

[9] V. M. Soundalgekar, M. R. Patil: *Stokes problem for a vertical infinite plate with variable temperature*, Astrophysics and Space Science, **59**(1978), 503 – 506.

[10] B. Carnahan, H. A. Luther, J.O. Wikes: Applied Numerical Methods, John Wiley and Sons, 1969.

Department of Mathematics
SRM Institute of Science and Technology
Kattankulathur-603203, Tamil Nadu, India
E-mail address: kannanrm19@yahoo.in

Department of Mathematics
SRM Institute of Science and Technology
Kattankulathur-603203, Tamil Nadu, India
E-mail address: bapujip@yahoo.com