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SUBSYSTEMS OF INTERVAL NEUTROSOPHIC AUTOMATA

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ABSTRACT. In this paper, we introduce subsystem of interval neutrosophic automaton with example. We introduce different types of subsystems of interval neutrosophic automaton and discuss the properties of subsystems of interval neutrosophic automaton and establish the connection between different types of subsystems.

1. INTRODUCTION

The theory of neutrosophy and neutrosophic set was introduced by Florentin Smarandache in 1999 [2]. A neutrosophic set N is classified by a Truth membership function $T_N,$ Indeterminacy membership function $I_N,$ and Falsity membership function $F_N,$ where $T_N, I_N,$ andF_N are real standard and non-standard subsets of $]0^-, 1^+[$. Wang $etal.$ [3] introduced the notion of interval-valued neutrosophic sets. The concept of interval neutrosophic finite state machine was introduced by Tahir Mahmood [1]. In this paper, we introduced the concept of subsystem of interval neutrosophic automata.

Also, we introduced some other subsystems of interval neutrosophic automata and discussed their properties. We establish a necessary and sufficient condition for interval neutrosophic subset N_Q of Q to be a subsystem of interval neutrosophic automaton.

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2. Preliminaries

Definition 2.1. *[2] Let U be the universe of discourse. A neutrosophic set (NS) N in U is $N = \{\langle x, (T_N(x), I_N(x), F_N(x))\rangle, x \in U,\ T_A, I_A, F_A \in]0^-, 1^+[\ \}$ and with the condition $0^- \leq sup\, T_N(x) + sup\, I_N(x) + sup\, F_N(x) \leq 3^+$. We need to take the interval $[0,1]$ for technical applications instead of $]0^-, 1^+[$.*

Definition 2.2. *[3] Let U be a universal set. An interval neutrosophic set (INS for short) is of the form*

$$N = \{\langle \alpha_N(x), \beta_N(x), \gamma_N(x)\rangle \,|x \in U\} = \{\langle x, [\inf\ \alpha_N(x), \sup\ \alpha_N(x)],$$
$$[\inf\ \beta_N(x), \sup\ \beta_N(x)], [\inf\ \gamma_N(x), \sup \gamma_N(x)]\rangle \,|x \in U\},$$

where $\alpha_N(x),\ \beta_N(x)$, and $\gamma_N(x) \subseteq [0,1]$ and the condition that

$$0 \leq \sup\ \alpha_N(x) +\ \sup\ \beta_N(x) +\ \sup\ \gamma_N(x) \leq\ 3.$$

3. Interval Neutrosophic Automata

Definition 3.1. *[1] $M = (Q,\ \Sigma,\ N)$ is called interval neutrosophic automaton $(INAforshort)$, where Q and Σ are non-empty finite sets called the set of states and input symbols respectively, and $N = \{\langle \alpha_N(x), \beta_N(x), \gamma_N(x)\rangle\}$ is an INS in $Q \times \Sigma \times Q$. The set of all words of finite length of Σ is denoted by Σ^*. The empty word is denoted by ϵ, and the length of each $x \in \Sigma^*$ is denoted by $|x|$.*

Definition 3.2. *[1] $M = (Q,\ \Sigma,\ N)$ be an INA. Define an INS $N^* = \{\langle \alpha_{N^*}(x), \beta_{N^*}(x), \gamma_{N^*}(x)\rangle\}$ in $Q \times \Sigma^* \times Q$ by*

$$\alpha_{N^*}(q_i,\ \epsilon,\ q_j) = \begin{cases} [1,1] & \text{if } q_i = q_j \\ [0,0] & \text{if } q_i \neq q_j \end{cases}$$

$$\beta_{N^*}(q_i,\ \epsilon,\ q_j) = \begin{cases} [0,0] & \text{if } q_i = q_j \\ [1,1] & \text{if } q_i \neq q_j \end{cases}$$

$$\gamma_{N^*}(q_i,\ \epsilon,\ q_j) = \begin{cases} [0,0] & \text{if } q_i = q_j \\ [1,1] & \text{if } q_i \neq q_j \end{cases}$$

$$\alpha_{N^*}(q_i, w, q_j) = \alpha_{N^*}(q_i, xy, q_j) = \vee_{q_r \in Q}[\alpha_{N^*}(q_i, x, q_r) \cup \alpha_{N^*}(q_r, y, q_j)],$$
$$\beta_{N^*}(q_i, w, q_j) = \beta_{N^*}(q_i, xy, q_j) = \vee_{q_r \in Q}[\beta_{N^*}(q_i, x, q_r) \cup \beta_{N^*}(q_r, y, q_j)],$$
$$\gamma_{N^*}(q_i, w, q_j) = \gamma_{N^*}(q_i, xy, q_j) = \vee_{q_r \in Q}[\gamma_{N^*}(q_i, x, q_r) \cup \gamma_{N^*}(q_r, y, q_j)],$$

$\forall q_i, q_j \in Q, w = xy, x \in \Sigma^*$ *and* $y \in \Sigma$.

Definition 3.3. *Let* $M = (Q, \Sigma, N)$ *be an interval neutrosophic automaton. Let* N_Q *be a interval neutrosophic subset of* Q*, and for each* $q_i \in Q$*,*

$$N_Q = \{\langle \alpha_{N_Q}(q_i),\ \beta_{N_Q}(q_i),\ \gamma_{N_Q}(q_i)\rangle\} = \{\langle q_i, [\inf \alpha_{N_Q}(q_i), \sup \alpha_{N_Q}(q_i)], [\inf \beta_{N_Q}(q_i), \sup \beta_{N_Q}(q_i)], [\inf \gamma_{N_Q}(q_i), \sup \gamma_{N_Q}(q_i)]\rangle\}.$$

Then (Q, N_Q, Σ, N) *is called a subsystem of* M *and it is denoted by* N_Q *if* $\forall q_i, q_j \in Q$ *and* $x \in \Sigma$ *such that* $\alpha_{N_Q}(q_j) \geq \vee_{q_i \in Q}\{\alpha_{N_Q}(q_i) \wedge \alpha_N(q_i, x, q_j)\}$, $\beta_{N_Q}(q_j) \leq \wedge_{q_i \in Q}\{\beta_{N_Q}(q_i) \vee \beta_N(q_i, x, q_j)\}$ *and* $\gamma_{N_Q}(q_j) \leq \wedge_{q_i \in Q}\{\gamma_{N_Q}(q_i) \vee \gamma_N(q_i, x, q_j)\}$.

Definition 3.4. *Let* $M = (Q, \Sigma, N)$ *be an interval neutrosophic automaton and* $N_Q = \{\langle \alpha_{N_Q}, \beta_{N_Q}, \gamma_{N_Q}\rangle\}$ *be an interval neutrosophic subset of* Q. *Let* $q_j \in Q$ *and for all* $x \in \Sigma^*$*, define an interval neutrosophic subset* $N_Q x$ *of* Q *by,* $(\alpha_{N_Q} x)(q_j) = \vee_{q_i \in Q}\{\alpha_{N_Q}(q_i) \wedge \alpha_N(q_i, x, q_j)\}$ $(\beta_{N_Q} x)(q_j) = \wedge_{q_i \in Q}\{\beta_{N_Q}(q_i) \vee \beta_N(q_i, x, q_j)\}$ *and* $(\gamma_{N_Q} x)(q_j) = \wedge_{q_i \in Q}\{\gamma_{N_Q}(q_i) \vee \gamma_N(q_i, x, q_j)\}$.

4. Properties of Subsystems of Interval Neutrosophic Automata

Theorem 4.1. *Let* $M = (Q, \Sigma, N)$ *be an interval neutrosophic automaton. Let* $N_Q = \{\langle \alpha_{N_Q}, \beta_{N_Q}, \gamma_{N_Q}\rangle\}$ *be an interval neutrosophic subset of* Q. *Then* N_Q *is a subsystem of* M *if and only if* $\forall q_i, q_j \in Q, \forall x \in \Sigma^*$,

$\alpha_{N_Q}(q_j) \geq \vee_{q_i \in Q}\{\alpha_{N_Q}(q_i) \wedge \alpha_N(q_i, x, q_j)\}$,
$\beta_{N_Q}(q_j) \leq \wedge_{q_i \in Q}\{\beta_{N_Q}(q_i) \vee \beta_N(q_i, x, q_j)\}$ *and*
$\gamma_{N_Q}(q_j) \leq \wedge_{q_i \in Q}\{\gamma_{N_Q}(q_i) \vee \gamma_N(q_i, x, q_j)\}$.

Proof. Suppose N_Q is a subsystem of M. Let $q_i, q_j \in Q$ and $x \in \Sigma^*$. We prove the result by induction on $|x| = n$. If $n = 0$, then $x = \epsilon$. Now if $q_i = q_j$, then $\alpha_{N_Q}(q_j) \wedge \alpha_{N*}(q_i, \epsilon, q_j) = \alpha_{N_Q}(q_j)$, $\beta_{N_Q}(q_j) \vee \beta_{N*}(q_i, \epsilon, q_j) = \beta_{N_Q}(q_j)$, and $\gamma_{N_Q}(q_j) \vee \gamma_{N*}(q_i, \epsilon, q_j) = \gamma_{N_Q}(q_j)$. Now if $q_i \neq q_j$, then $\alpha_{N_Q}(q_i) \wedge \alpha_{N^*}(q_i, \epsilon, q_j) \leq \alpha_{N_Q}(q_j)$, $\beta_{N_Q}(q_i) \vee \beta_{N^*}(q_i, \epsilon, q_j) \geq \beta_{N_Q}(q_j)$, and $\gamma_{N_Q}(q_i) \vee \gamma_{N^*}(q_i, \epsilon, q_j) \geq \gamma_{N_Q}(q_j)$.

Therefore, the result is true for $n = 0$.

Suppose the result is true for all $y \in \Sigma^*$ such that $|y| = n-1, n > 0$. Let $x = ya, |y| = n-1, y \in \Sigma^*, a \in \Sigma$. Then

$$\begin{aligned}
&\vee_{q_i \in Q}\{\alpha_{N_Q}(q_i) \wedge \alpha_{N^*}(q_i, x, q_j)\} = \vee_{q_i \in Q}\{\alpha_{N_Q}(q_i) \wedge \alpha_{N^*}(q_i, ya, q_j)\} \\
&\quad = \vee_{q_i \in Q}\{\alpha_{N_Q}(q_i) \wedge \{\vee_{q_k \in Q}\{\alpha_{N^*}(q_i, y, q_k) \wedge \alpha_N(q_k, a, q_j)\}\}\} \\
&\quad = \vee_{q_k \in Q}\{\vee_{q_i \in Q}\{\alpha_{N_Q}(q_i) \wedge \alpha_{N^*}(q_i, y, q_k) \wedge \alpha_N(q_k, a, q_j)\}\} \\
&\quad \leq \vee_{q_k \in Q}\{\alpha_{N_Q}(q_k) \wedge \alpha_N(q_k, a, q_j)\} \\
&\quad \leq \alpha_{N_Q}(q_j). \\
&\vee_{q_i \in Q}\{\alpha_{N_Q}(q_i) \wedge \alpha_{N^*}(q_i, x, q_j)\} \leq \alpha_{N_Q}(q_j).
\end{aligned}$$

Thus,

$$\begin{aligned}
&\alpha_{N_Q}(q_j) \geq \vee_{q_i \in Q}\{\alpha_{N_Q}(q_i) \wedge \alpha_{N^*}(q_i, x, q_j)\} \\
&\wedge_{q_i \in Q}\{\beta_{N_Q}(q_i) \vee \beta_{N^*}(q_i, x, q_j)\} = \wedge_{q_i \in Q}\{\beta_{N_Q}(q_i) \vee \beta_{N^*}(q_i, ya, q_j)\} \\
&\quad = \wedge_{q_i \in Q}\{\beta_{N_Q}(q_i) \vee \{\wedge_{q_k \in Q}\{\beta_{N^*}(q_i, y, q_k) \vee \beta_N(q_k, a, q_j)\}\}\} \\
&\quad = \wedge_{q_k \in Q}\{\wedge_{q_i \in Q}\{\beta_{N_Q}(q_i) \vee \beta_{N^*}(q_i, y, q_k) \vee \beta_N(q_k, a, q_j)\}\} \\
&\quad \geq \wedge_{q_k \in Q}\{\beta_{N_Q}(q_k) \vee \beta_N(q_k, a, q_j)\} \\
&\quad \geq \beta_{N_Q}(q_j). \\
&\wedge_{q_i \in Q}\{\beta_{N_Q}(q_i) \vee \beta_N(q_i, x, q_j)\} \geq \beta_{N_Q}(q_j)
\end{aligned}$$

Thus,

$$\begin{aligned}
&\beta_{N_Q}(q_j) \leq \wedge_{q_i \in Q}\{\beta_{N_Q}(q_i) \vee \beta_{N^*}(q_i, x, q_j)\} \\
&\wedge_{q_i \in Q}\{\gamma_{N_Q}(q_i) \vee \gamma_{N^*}(q_i, x, q_j)\} = \wedge_{q_i \in Q}\{\gamma_{N_Q}(q_i) \vee \gamma_{N^*}(q_i, ya, q_j)\} \\
&\quad = \wedge_{q_i \in Q}\{\gamma_{N_Q}(q_i) \vee \{\wedge_{q_k \in Q}\{\gamma_{N^*}(q_i, y, q_k) \vee \gamma_N(q_k, a, q_j)\}\}\} \\
&\quad = \wedge_{q_k \in Q}\{\wedge_{q_i \in Q}\{\gamma_{N_Q}(q_i) \vee \gamma_{N^*}(q_i, y, q_k) \vee \gamma_N(q_k, a, q_j)\}\} \\
&\quad \geq \wedge_{q_k \in Q}\{\gamma_{N_Q}(q_k) \vee \gamma_N(q_k, a, q_j)\} \\
&\quad \geq \gamma_{N_Q}(q_j). \\
&\wedge_{q_i \in Q}\{\gamma_{N_Q}(q_i) \vee \gamma_{N^*}(q_i, x, q_j)\} \geq \gamma_{N_Q}(q_j).
\end{aligned}$$

Thus, $\gamma_{N_Q}(q_j) \leq \wedge_{q_i \in Q}\{\gamma_{N_Q}(q_i) \vee \gamma_{N^*}(q_i, x, q_j)\}$.

The converse is obvious. □

Theorem 4.2. *Let $M = (Q, \Sigma, N)$ be an interval neutrosophic automaton. Let $N_Q = \{\langle \alpha_{N_Q}, \beta_{N_Q}, \gamma_{N_Q} \rangle\}$ be an interval neutrosophic subset of Q. Then N_Q is a subsystem of M if and only if $\alpha_{N_Q} x \subseteq \alpha_{N_Q}$, $\beta_{N_Q} x \supseteq \beta_{N_Q}$, and $\gamma_{N_Q} x \supseteq \gamma_{N_Q}$ $\forall x \in \Sigma^*$.*

Proof. Let N_Q be a subsystem of M. Let $x \in \Sigma^*$ and $q_j \in Q$. Then

$$(\alpha_{N_Q} x)(q_j) = \vee\{\alpha_{N_Q}(q_i) \wedge \alpha_{N^*}(q_i, x, q_j) \mid q_i \in Q\} \leq \alpha_{N_Q}(q_j)$$

$(\beta_{N_Q}x)(q_j) = \wedge \left\{\beta_{N_Q}(q_i) \vee \beta_{N^*}(q_i, x, q_j) \mid q_i \in Q\right\} \geq \beta_{N_Q}(q_j),$
$(\gamma_{N_Q}x)(q_j) = \wedge \left\{\gamma_{N_Q}(q_i) \vee \gamma_{N^*}(q_i, x, q_j) \mid q_i \in Q\right\} \geq \gamma_{N_Q}(q_j).$

Hence, $\alpha_{N_Q}x \subseteq \alpha_{N_Q},\ \beta_{N_Q}x \supseteq \beta_{N_Q}$, and $\gamma_{N_Q}x \supseteq \gamma_{N_Q}\ \ \forall x \in \Sigma^*$.

Conversely, suppose $\alpha_{N_Q}x \subseteq \alpha_{N_Q},\ \beta_{N_Q}x \supseteq \beta_{N_Q}$, and $\gamma_{N_Q}x \supseteq \gamma_{N_Q}\ \ \forall x \in \Sigma^*$. Let $q_j \in Q$ and $x \in \Sigma^*$. Now,

$\alpha_{N_Q}(q_j) \geq (\alpha_{N_Q}x)(q_j) = \vee_{q_i \in Q} \left\{\alpha_{N_Q}(q_i) \wedge \alpha_{N^*}(q_i, x, q_j)\right\}.$

Thus,

$\alpha_{N_Q}(q_j) \geq \vee_{q_i \in Q}\{\alpha_{N_Q}(q_i) \wedge \alpha_{N^*}(q_i, x, q_j)\}$
$\beta_{N_Q}(q_j) \leq (\beta_{N_Q}x)(q_j) = \wedge_{q_i \in Q} \left\{\beta_{N_Q}(q_i) \vee \beta_{N^*}(q_i, x, q_j)\right\}.$

Thus,

$\beta_{N_Q}(q_j) \leq \wedge_{q_i \in Q}\{\beta_{N_Q}(q_i) \vee \beta_{N^*}(q_i, x, q_j)\},$
$\gamma_{N_Q}(q_j) \leq (\gamma_{N_Q}x)(q_j) = \wedge_{q_i \in Q} \left\{\gamma_{N_Q}(q_i) \vee \gamma_{N^*}(q_i, x, q_j)\right\},$
$\gamma_{N_Q}(q_j) \leq \wedge_{q_i \in Q}\{\gamma_{N_Q}(q_i) \vee \gamma_{N^*}(q_i, x, q_j)\}.$

Hence N_Q is a subsystem of M. □

Theorem 4.3. *Let $M = (Q,\ \Sigma,\ N)$ be an interval neutrosophic automaton. Let N_{Q_1}, and N_{Q_2} be subsystems of M. Then the following conditions hold:*

(i) *$N_{Q_1} \wedge N_{Q_2}$ is a subsystem of M;*
(ii) *$N_{Q_1} \vee N_{Q_2}$ is a subsystem of M.*

Proof. Here, N_{Q_1} and N_{Q_2} are subsystems of an interval neutrosophic automaton M.

(i) Now we have to prove $N_{Q_1} \wedge N_{Q_2}$ is a subsystem of M. That is
$(\alpha_{N_{Q_1}} \wedge \alpha_{N_{Q_2}})(q_j) \geq \vee_{q_i \in Q}\{(\alpha_{N_{Q_1}} \wedge \alpha_{N_{Q_2}})(q_i) \wedge \alpha_N(q_i, x, q_j)\},$
$(\beta_{N_{Q_1}} \wedge \beta_{N_{Q_2}})(q_j) \leq \wedge_{q_i \in Q}\{(\beta_{N_{Q_1}} \wedge \beta_{N_{Q_2}})(q_i) \vee \beta_N(q_i, x, q_j)\}$, and
$(\gamma_{N_{Q_1}} \wedge \gamma_{N_{Q_2}})(q_j) \leq \wedge_{q_i \in Q}\{(\gamma_{N_{Q_1}} \wedge \gamma_{N_{Q_2}})(q_i) \vee \gamma_N(q_i, x, q_j)\}.$

Now,

$$\begin{aligned}
(\alpha_{N_{Q_1}} \wedge \alpha_{N_{Q_2}})(q_j) &= (\alpha_{N_{Q_1}}(q_j) \wedge \alpha_{N_{Q_2}}(q_j)) \\
&\geq \{\vee_{q_i \in Q}\{\alpha_{N_{Q_1}}(q_i) \wedge \alpha_N(q_i, x, q_j)\}\} \wedge \{\vee_{q_i \in Q}\{\alpha_{N_{Q_2}}(q_i) \wedge \alpha_N(q_i, x, q_j)\}\} \\
&= \{\vee_{q_i \in Q}\{\alpha_{N_{Q_1}}(q_i) \wedge \alpha_{N_{Q_2}}(q_i) \wedge \alpha_N(q_i, x, q_j)\}\} \\
&= \{\vee_{q_i \in Q}\{(\alpha_{N_{Q_1}} \wedge \alpha_{N_{Q_2}})(q_i) \wedge \alpha_N(q_i, x, q_j)\}\},
\end{aligned}$$

Thus,

$$(4.1)\qquad (\alpha_{N_{Q_1}} \wedge \alpha_{N_{Q_2}})(q_j) \geq \{\vee_{q_i \in Q}\{(\alpha_{N_{Q_1}} \wedge \alpha_{N_{Q_2}})(q_i) \wedge \alpha_N(q_i, x, q_j)\}\}$$

$$\begin{aligned} (\beta_{N_{Q_1}} \wedge \beta_{N_{Q_2}})(q_j) &= (\beta_{N_{Q_1}}(q_j) \wedge \beta_{N_{Q_2}}(q_j)) \\ &\leq \{\wedge_{q_i \in Q}\{\beta_{N_{Q_1}}(q_i) \vee \beta_N(q_i, x, q_j)\}\} \wedge \{\wedge_{q_i \in Q}\{\beta_{N_{Q_2}}(q_i) \vee \beta_N(q_i, x, q_j)\}\} \\ &= \{\wedge_{q_i \in Q}\{\beta_{N_{Q_1}}(q_i) \wedge \beta_{N_{Q_2}}(q_i) \vee \beta_N(q_i, x, q_j)\}\} \\ &= \{\wedge_{q_i \in Q}\{(\beta_{N_{Q_1}} \wedge \beta_{N_{Q_2}})(q_i) \vee \beta_N(q_i, x, q_j)\}\}. \end{aligned}$$

Thus,

$$(4.2)\qquad (\beta_{N_{Q_1}} \wedge \beta_{N_{Q_2}})(q_j) \leq \{\wedge_{q_i \in Q}\{(\beta_{N_{Q_1}} \wedge \beta_{N_{Q_2}})(q_i) \vee \beta_N(q_i, x, q_j)\}\}$$

$$\begin{aligned} \gamma_{N_{Q_1}} \wedge \gamma_{N_{Q_2}})(q_j) &= (\gamma_{N_{Q_1}}(q_j) \wedge \gamma_{N_{Q_2}}(q_j)) \\ &\leq \{\wedge_{q_i \in Q}\{\gamma_{N_{Q_1}}(q_i) \vee \gamma_N(q_i, x, q_j)\}\} \wedge \{\wedge_{q_i \in Q}\{\gamma_{N_{Q_2}}(q_i) \vee \gamma_N(q_i, x, q_j)\}\} \\ &= \{\wedge_{q_i \in Q}\{\gamma_{N_{Q_1}}(q_i) \wedge \gamma_{N_{Q_2}}(q_i) \vee \gamma_N(q_i, x, q_j)\}\} \\ &= \{\wedge_{q_i \in Q}\{(\gamma_{N_{Q_1}} \wedge \gamma_{N_{Q_2}})(q_i) \vee \gamma_N(q_i, x, q_j)\}\}. \end{aligned}$$

Thus,

$$(4.3)\qquad (\gamma_{N_{Q_1}} \wedge \gamma_{N_{Q_2}})(q_j) \leq \{\wedge_{q_i \in Q}\{(\gamma_{N_{Q_1}} \wedge \gamma_{N_{Q_2}})(q_i) \vee \gamma_N(q_i, x, q_j)\}\}.$$

From (4.1), (4.2) and (4.3) $N_{Q_1} \wedge N_{Q_2}$ is a subsystem of an interval neutrosophic automaton M.

(ii) Now to prove $N_{Q_1} \vee N_{Q_2}$ is a subsystem of interval neutrosophic automaton M.

Now,

$$\begin{aligned} (\alpha_{N_{Q_1}} \vee \alpha_{N_{Q_2}})(q_j) &= (\alpha_{N_{Q_1}}(q_j) \vee \alpha_{N_{Q_2}}(q_j)) \\ &\geq \{\vee_{q_i \in Q}\{\alpha_{N_{Q_1}}(q_i) \wedge \alpha_N(q_i, x, q_j)\}\} \vee \{\vee_{q_i \in Q}\{\alpha_{N_{Q_2}}(q_i) \wedge \alpha_N(q_i, x, q_j)\}\} \\ &= \{\vee_{q_i \in Q}\{\alpha_{N_{Q_1}}(q_i) \vee \alpha_{N_{Q_2}}(q_i) \wedge \alpha_N(q_i, x, q_j)\}\} \\ &= \{\vee_{q_i \in Q}\{(\alpha_{N_{Q_1}} \vee \alpha_{N_{Q_2}})(q_i) \wedge \alpha_N(q_i, x, q_j)\}\}, \end{aligned}$$

Thus,

$$(4.4)\qquad (\alpha_{N_{Q_1}} \vee \alpha_{N_{Q_2}})(q_j) \geq \{\vee_{q_i \in Q}\{(\alpha_{N_{Q_1}} \vee \alpha_{N_{Q_2}})(q_i) \wedge \alpha_N(q_i, x, q_j)\}\}$$

$$\begin{aligned} (\beta_{N_{Q_1}} \vee \beta_{N_{Q_2}})(q_j) &= (\beta_{N_{Q_1}}(q_j) \vee \beta_{N_{Q_2}}(q_j)) \\ leq\ &\{\wedge_{q_i \in Q}\{\beta_{N_{Q_1}}(q_i) \vee \beta_N(q_i, x, q_j)\}\} \vee \{\wedge_{q_i \in Q}\{\beta_{N_{Q_2}}(q_i) \vee \beta_N(q_i, x, q_j)\}\} \\ &= \{\wedge_{q_i \in Q}\{\beta_{N_{Q_1}}(q_i) \vee \beta_{N_{Q_2}}(q_i) \vee \beta_N(q_i, x, q_j)\}\} \\ &= \{\wedge_{q_i \in Q}\{(\beta_{N_{Q_1}} \vee \beta_{N_{Q_2}})(q_i) \vee \beta_N(q_i, x, q_j)\}\}. \end{aligned}$$

Thus,

$$(\beta_{N_{Q_1}} \vee \beta_{N_{Q_2}})(q_j) \leq \{\wedge_{q_i \in Q}\{(\beta_{N_{Q_1}} \vee \beta_{N_{Q_2}})(q_i) \vee \beta_N(q_i, x, q_j)\}\} \tag{4.5}$$

$$\begin{aligned} (\gamma_{N_{Q_1}} \vee \gamma_{N_{Q_2}})(q_j) &= (\gamma_{N_{Q_1}}(q_j) \vee \gamma_{N_{Q_2}}(q_j)) \\ &\leq \{\wedge_{q_i \in Q}\{\gamma_{N_{Q_1}}(q_i) \vee \gamma_N(q_i, x, q_j)\}\} \vee \{\wedge_{q_i \in Q}\{\gamma_{N_{Q_2}}(q_i) \vee \gamma_N(q_i, x, q_j)\}\} \\ &= \{\wedge_{q_i \in Q}\{\gamma_{N_{Q_1}}(q_i) \vee \gamma_{N_{Q_2}}(q_i) \vee \gamma_N(q_i, x, q_j)\}\} \\ &= \{\wedge_{q_i \in Q}\{(\gamma_{N_{Q_1}} \vee \gamma_{N_{Q_2}})(q_i) \vee \gamma_N(q_i, x, q_j)\}\}. \end{aligned}$$

Thus,

$$(\gamma_{N_{Q_1}} \vee \gamma_{N_{Q_2}})(q_j) \leq \{\wedge_{q_i \in Q}\{(\gamma_{N_{Q_1}} \vee \gamma_{N_{Q_2}})(q_i) \vee \gamma_N(q_i, x, q_j)\}\}. \tag{4.6}$$

From (4.4), (4.5), and (4.6), $N_{Q_1} \vee N_{Q_2}$ is a subsystem of interval neutrosophic automata M. □

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