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IDEMPOTENT INTUITIONISTIC FUZZY MATRIX USING IMPLICATION OPERATOR

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ABSTRACT. In this paper, we discuss some properties of reflexive and transitive Intuitionistic Fuzzy Matrix using implication operator. Also we derive some results of an idempotent Intuitionistic Fuzzy Matrix with Min-Min operator. We illustrate the above results by an example.

1. INTRODUCTION

After introduction of fuzzy set theory by Zadeh [7] in 1965. As a result, a new concept namely intuitionistic fuzzy set (IFS) was introduced by Atanassov [1] in 1983 and represent it as $A = \{x, \mu_A(x), \gamma_A(x)/x \in X\}$, where the functions $\mu_A(x) : X \to [0,1]$ and $\gamma_A(x) : X \to [0,1]$ define the membership function and non-membership function of the element $x \in X$ respectively and for every $x \in X, 0 \leq \mu_A(x)+\gamma_A(x) \leq 1$. Xu, Yager [6] represents $\langle \mu_A(x), \gamma_A(x) \rangle$ as intuitionistic fuzzy values with $\mu_A(x) + \gamma_A(x) \leq 1$.

The notation Intuitionistic Fuzzy Matrix (IFM) was introduced by Atanassov [2] in 1987. Murugadas and Padder [4, 5] for Max-Max operation on Intuitionistic Fuzzy Matrix and we obtain reduction of a nilpotent intuitionistic fuzzy matrix using implication operator. Lalitha in [3] introduced Min-Min operation

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for IFS as well as IFM. The purpose of this paper is to study the idempotent IFM by using implication operator.

2. Preliminaries

If $\langle x, x'\rangle, \langle y, y'\rangle \in$ IFS then $\langle x, x'\rangle \vee \langle y, y'\rangle = \langle \text{Max}\{x, y\}, \text{ Min}\{x', y'\}\rangle$ and $\langle x, x'\rangle \wedge \langle y, y'\rangle = \langle \text{Min}\{x, y\}, \text{ Max}\{x', y'\}\rangle$ $\langle x, x'\rangle \geq \langle y, y'\rangle \Rightarrow x \geq y$ and $x' \leq y'$, therefore in this case we say $\langle x, x'\rangle$ and $\langle y, y'\rangle$ are comparable.

For any two comparable elements $\langle x, x'\rangle \leftarrow \langle y, y'\rangle$ is defined as

$$\langle x, x'\rangle \leftarrow \langle y, y'\rangle = \begin{cases} \langle 1, 0\rangle & \text{if } \langle x, x'\rangle \geq \langle y, y'\rangle \\ \langle x, x'\rangle & \text{if } \langle x, x'\rangle < \langle y, y'\rangle \end{cases}$$

Definition 2.1. *For IFMs Q and R define, the Min-Min product of Q and R as*

$$A \bullet R = \left(\bigwedge_{K=1}^{n} \langle q_{iK} \wedge r_{Kj}\rangle, \bigvee_{K=1}^{n} \langle q'_{iK} \vee r'_{Kj}\rangle \right).$$

Definition 2.2. *Let $X = \{x_1, x_2, \cdots, x_m\}$ be a set of alternatives and let $y = \{y_1, y_2, \cdots, y_n\}$ be the attribute set of each element of X. An Intuitionistic Fuzzy Matrix (IFM) is defined by*

$$A = (\langle (x_i, y_i), \mu_A(x_i, y_i), \gamma_A(x_i, y_i)\rangle)$$

for $i = 1, 2, \cdots, m$ and $j = 1, 2, \cdots, n$ where $\mu_A : X \times Y \to [0, 1]$ and $\gamma_A : X \times Y \to [0, 1]$ satisfies the condition $0 \leq \mu_A(x_i, y_i) + \gamma_A(x_i, y_i) \leq 1$.

For simplifying we denote an Intuitionistic Fuzzy Matrix (IFM) is a matrix of pairs $A = (\langle a_{ij}, a'_{ij}\rangle)$ of a non negative real numbers satisfying $a_{ij} + a'_{ij} \leq 1$ for all i, j. We denote the set of all IFM of order $m \times n$ by $\Im_{mn}$.

For $n \times n$ IFMs $Q = \langle q_{ij}, a'_{ij}\rangle$ and $R = \langle r_{ij}, r'_{ij}\rangle$. Some operations are defined as follows:

- $Q \vee R = (\langle q_{ij} \vee r_{ij}, q'_{ij} \wedge r'_{ij}\rangle)$
- $Q \wedge R = (\langle q_{ij} \wedge r_{ij}, q'_{ij} \vee r'_{ij}\rangle)$
- $Q \overset{C}{\leftarrow} R = \langle q_{ij}, a'_{ij}\rangle \overset{C}{\leftarrow} \langle r_{ij}, r'_{ij}\rangle$ (compound wise)
- $Q \times R = (\langle q_{i1} \wedge r_{1j}, q'_{i1} \vee r'_{1j}\rangle \vee \langle q_{i2} \wedge r_{2j}, q'_{i2} \vee r'_{2j}\rangle \vee \cdots \vee \langle q_{in} \wedge r_{nj}, q'_{in} \vee r'_{nj}\rangle)$
- $Q/R = Q \overset{C}{\leftarrow} (Q \times R)$.
- $Q' = Q$

If $R \geq I_n$, then R is reflexive. If $R^2 \leq R$, then R is transitive. If $R^2 = R$, then R is idempotent. $Q^{K+1} = Q^K \times Q, K = 1, 2, 3, \cdots$, Q^+, $Q \leq S(S \geq Q)$ if and only if $\langle q_{ij}, q'_{ij}\rangle \leq \langle s_{ij}, s'_{ij}\rangle$. A identity matrix is a matrix whose entries are $\langle 1, 0\rangle$. An IFM Q is called transitive if $Q^2 \leq Q$. An IFM Q is said to be reflexive if all of its then Q is idempotent $(Q^2 = Q)$. If a matrix is reflexive and transitive the matrix is idempotent. From definition $Q/Q = \overset{C}{\leftarrow} (Q \times Q)$, the (i, j) entry of Q/Q is either $\langle q_{ij}, q'_{ij}\rangle$ or $\langle 1, 0\rangle$.

3. Idempotent of Intuitionistic Fuzzy Matrix

In this section, we examine the reflexive and transitive IFMs and obtain some theorems. Then we use Min-Min operator and derive some results.

Theorem 3.1. *If Q is an $n \times n$ reflexive and transitive IFM, then*

(1) $(Q/Q)^+ \neq Q$

(2) $S^+ \neq Q$ *for any $n \times n$ IFMs.*

Proof. Since $S = \langle S_{ij}, S'_{ij}\rangle = Q/Q$, clearly $S^K = \langle S^{(K)}_{ij}, S'^{(K)}_{ij}\rangle = (Q/Q)^+$, $(Q/Q)^+ = (Q/Q)\vee, (Q/Q)^2 \vee \cdots \vee (Q/Q)^{n-1} \neq Q$.

We have to prove that

$$Q \neq (Q/Q) \vee (Q/Q)^2 \vee \cdots \vee (Q/Q)^{n-1}.$$

That is $\langle q_{ij}, q'_{ij}\rangle = \langle S^{(K)}_{ij}, S'^{(K)}_{ij}\rangle$ for some $K = (1 \leq K \leq n-1)$ and

$$\langle S_{ij}, S'_{ij}\rangle = \langle q_{ij}, q'_{ij}\rangle \leftarrow \left\langle \bigvee_{K=1}^{n} (q_{iK} \wedge q_{Kj}), \bigwedge_{K=1}^{n} (q'_{iK} \vee q'_{Kj}) \right\rangle = \langle 1, 0\rangle.$$

That is

$$\langle q_{ij}, q'_{ij}\rangle = \left\langle \bigvee_{K=1}^{n} (q_{iK} \wedge q_{Kj}), \bigwedge_{K=1}^{n} (q'_{iK} \vee q'_{Kj}) \right\rangle.$$

Thus, we get $S^+ \neq Q$. □

Example 1. *Let Q be the following reflexive and transitive intuitionistic fuzzy matrix*

$$Q = \begin{pmatrix} \langle 1, 0\rangle & \langle 0.3, 0.4\rangle & \langle 1, 0\rangle \\ \langle 1, 0\rangle & \langle 1, 0\rangle & \langle 1, 0\rangle \\ \langle 0.3, 0.4\rangle & \langle 0.3, 0.4\rangle & \langle 1, 0\rangle \end{pmatrix},$$

$$Q \times Q = \begin{pmatrix} \langle 1,0\rangle & \langle 0.3,0.4\rangle & \langle 1,0\rangle \\ \langle 1,0\rangle & \langle 1,0\rangle & \langle 1,0\rangle \\ \langle 0.3,0.4\rangle & \langle 0.3,0.4\rangle & \langle 1,0\rangle \end{pmatrix} = Q^2.$$

This implies $Q = Q^2$ is idempotent. Now $Q/Q = Q \overset{C}{\leftarrow} (Q \times Q)$,

$$Q/Q = \begin{pmatrix} \langle 1,0\rangle & \langle 1,0\rangle & \langle 1,0\rangle \\ \langle 1,0\rangle & \langle 1,0\rangle & \langle 1,0\rangle \\ \langle 1,0\rangle & \langle 1,0\rangle & \langle 1,0\rangle \end{pmatrix},$$

$(Q/Q)^+ = (Q/Q) \vee (Q/Q)^2 \vee (Q/Q)^3$ *and* $(Q/Q)^+$ *is unit matrix. Therefore,*

$$Q \neq (Q/Q)^+ = (Q/Q) \vee (Q/Q)^2 \vee (Q/Q)^3.$$

Remark 3.1. *Let Q and S as Min-Min product then Q be an $n \times n$ reflexive and transitive IFM then:*

(1) $(Q/Q)^+ \neq Q$;
(2) $S^+ \neq Q$ for any $n \times n$ IFMs.

Example 2. *Let* $Q = \begin{pmatrix} \langle 1,0\rangle & \langle 0.3,0.4\rangle & \langle 1,0\rangle \\ \langle 1,0\rangle & \langle 1,0\rangle & \langle 1,0\rangle \\ \langle 0.3,0.4\rangle & \langle 0.3,0.4\rangle & \langle 1,0\rangle \end{pmatrix}$,

$$Q \bullet Q = \begin{pmatrix} \langle 1,0\rangle & \langle 0.3,0.4\rangle & \langle 1,0\rangle \\ \langle 1,0\rangle & \langle 1,0\rangle & \langle 1,0\rangle \\ \langle 0.3,0.4\rangle & \langle 0.3,0.4\rangle & \langle 1,0\rangle \end{pmatrix} \begin{pmatrix} \langle 1,0\rangle & \langle 0.3,0.4\rangle & \langle 1,0\rangle \\ \langle 1,0\rangle & \langle 1,0\rangle & \langle 1,0\rangle \\ \langle 0.3,0.4\rangle & \langle 0.3,0.4\rangle & \langle 1,0\rangle \end{pmatrix},$$

$$Q^2 = \begin{pmatrix} \langle 1,0\rangle & \langle 0.3,0.4\rangle & \langle 1,0\rangle \\ \langle 1,0\rangle & \langle 1,0\rangle & \langle 1,0\rangle \\ \langle 0.3,0.4\rangle & \langle 0.3,0.4\rangle & \langle 1,0\rangle \end{pmatrix} \leq Q,$$

$Q/Q = Q \overset{C}{\leftarrow} (Q \bullet Q)$,

$$Q/Q = \begin{pmatrix} \langle 1,0\rangle & \langle 0.3,0.4\rangle & \langle 1,0\rangle \\ \langle 1,0\rangle & \langle 1,0\rangle & \langle 1,0\rangle \\ \langle 0.3,0.4\rangle & \langle 0.3,0.4\rangle & \langle 1,0\rangle \end{pmatrix} \overset{C}{\leftarrow} \begin{pmatrix} \langle 0.3,0.4\rangle & \langle 0.3,0.4\rangle & \langle 0.3,0.4\rangle \\ \langle 1,0\rangle & \langle 0.3,0.4\rangle & \langle 1,0\rangle \\ \langle 0.3,0.4\rangle & \langle 0.3,0.4\rangle & \langle 0.3,0.4\rangle \end{pmatrix}$$

$$= \begin{pmatrix} \langle 1,0\rangle & \langle 1,0\rangle & \langle 1,0\rangle \\ \langle 1,0\rangle & \langle 1,0\rangle & \langle 1,0\rangle \\ \langle 1,0\rangle & \langle 1,0\rangle & \langle 1,0\rangle \end{pmatrix},$$

$(Q/Q)^+ = (Q/Q) \vee (Q/Q)^2 \vee (Q/Q)^3$. *So,* $(Q/Q)^+$ *is unit matrix. Therefore,*

$$Q \neq (Q/Q)^+ = (Q/Q) \vee (Q/Q)^2 \vee (Q/Q)^3.$$

Theorem 3.2. *Let* Q *be an* $n \times n$ *reflexive and transitive matrix. Then,*

(1) $Q \neq S = Q/Q$.

(2) $S^+ \neq Q$ *for any* $n \times n$ *IFM* $Q/Q = S/Q$.

Proof. Let $F = \langle f_{ij}, f'_{ij}\rangle = Q/Q$ $G = \langle g_{ij}, g'_{ij}\rangle = S/Q$. Then:

$$\langle f_{ij}, f'_{ij}\rangle = \langle q_{ij}, q'_{ij}\rangle \leftarrow \left(\bigvee_{K=1}^{n} \langle q_{iK} \wedge q_{Kj}\rangle, \bigwedge_{K=1}^{n} \langle q'_{iK} \vee q'_{Kj}\rangle\right),$$

$$\langle g_{ij}, g'_{ij}\rangle = \langle s_{ij}, s'_{ij}\rangle \leftarrow \left(\bigvee_{K=1}^{n} \langle s_{iK} \wedge q_{Kj}\rangle, \bigwedge_{K=1}^{n} \langle s'_{iK} \vee q'_{Kj}\rangle\right).$$

(1) $\Rightarrow$ (2): Suppose that $Q \neq S = Q/Q$. So that $\langle q_{ij}, q'_{ij}\rangle \neq \langle s_{ij}, s'_{ij}\rangle = \langle f_{ij}, f'_{ij}\rangle$

(a) First we show that $\langle f_{ij}, f'_{ij}\rangle \leq \langle g_{ij}, g'_{ij}\rangle$.
Let $\langle f_{ij}, f'_{ij}\rangle < \langle 1, 0\rangle$, then $\langle f_{ij}, f'_{ij}\rangle \neq \langle g_{ij}, g'_{ij}\rangle < \langle 1, 0\rangle$. So that $\langle s_{ij}, s'_{ij}\rangle \neq \langle q_{ij}, q'_{ij}\rangle$ and

$$\left(\bigvee_{K=1}^{n} \langle q_{iK}, q_{Kj}\rangle, \bigwedge_{K=1}^{n} \langle q'_{iK}, q'_{Kj}\rangle\right) = \langle q_{ij}, q'_{ij}\rangle.$$

Since $\langle q_{iK}, q'_{iK}\rangle \geq \langle s_{iK}, s'_{iK}\rangle$ we have

$$\left(\bigvee_{K=1}^{n} \langle s_{iK} \wedge q_{Kj}\rangle, \bigwedge_{K=1}^{n} \langle s'_{iK} \vee q'_{Kj}\rangle\right) = \langle q_{ij}, q'_{ij}\rangle.$$

Thus

$$\langle g_{ij}, g'_{ij}\rangle = \langle s_{ij}, s'_{ij}\rangle \leftarrow \left(\bigvee_{K=1}^{n} s_{iK} \wedge q_{Kj}, \bigwedge_{K=1}^{n} s'_{iK} \vee q'_{Kj}\right).$$

So that $\langle f_{ij}, f'_{ij}\rangle \leq \langle g_{ij}, g'_{ij}\rangle$.

(b) Next we show that $\langle g_{ij}, g'_{ij}\rangle \leq \langle f_{ij}, f'_{ij}\rangle$.
Let $\langle g_{ij}, g'_{ij}\rangle = \langle 1, 0\rangle$, then $\langle g_{ij}, g'_{ij}\rangle = \langle s_{ij}, s'_{ij}\rangle < \langle 1, 0\rangle$ and hence

$$\left(\bigvee_{K=1}^{n} \langle s_{iK} \wedge q_{Kj}\rangle, \bigwedge_{K=1}^{n} \langle s'_{iK} \vee q'_{Kj}\rangle\right) = \langle q_{ij}, q'_{ij}\rangle.$$

Recall that

$$\langle f_{ij}, f'_{ij}\rangle = \langle q_{ij}, q'_{ij}\rangle \leftarrow \left(\bigvee_{K=1}^{n} q_{iK} \wedge q_{Kj},\ \bigwedge_{K=1}^{n} q'_{iK} \vee q'_{Kj}\right)$$
$$= \langle s_{ij}, s'_{ij}\rangle \neq \langle q_{ij}, q'_{ij}\rangle.$$

Since $\langle s_{ij}, s'_{ij}\rangle = \langle 1, 0\rangle$, we have $\langle q_{ij}, q'_{ij}\rangle < \langle 1, 0\rangle$. We shall show that if $\langle f_{ij}, f'_{ij}\rangle > \langle q_{ij}, q'_{ij}\rangle$. Suppose that $\langle f_{ij}, f'_{ij}\rangle > \langle q_{ij}, q'_{ij}\rangle$. Then

$$\left(\bigvee_{K=1}^{n} \langle q_{iK} \wedge q_{Kj}\rangle,\ \bigwedge_{K=1}^{n} \langle q'_{iK} \vee q'_{Kj}\rangle\right) = \langle q_{ij}, q'_{ij}\rangle \leq \langle 1, 0\rangle.$$

So that $\langle q_{iK(1)}, q'_{iK(1)}\rangle = \langle q_{ij}, q'_{ij}\rangle$ and $\langle q^{(2)}_{iK(1)}, q'^{(2)}_{iK(1)}\rangle = \langle q_{ij}, q'_{ij}\rangle$ for some $K(1)$. Then, since $F = S$, we have

$$\langle f_{iK(1)}, f'_{iK(1)}\rangle \neq \langle q_{ij}, q'_{ij}\rangle.$$

Further more $\langle f_{iK(1)}, f'_{iK(1)}\rangle = \langle q_{iK(1)}, q'_{iK(1)}\rangle$, since $\langle q_{iK(1)}, q'_{iK(1)}\rangle = \langle q_{ij}, q'_{ij}\rangle$. Thus,

$$\langle q_{ij}, q'_{ij}\rangle = \left(\bigvee_{K=1}^{n} \langle q_{iK} \wedge q_{KK(1)}\rangle,\ \bigwedge_{K=1}^{n} \langle q'_{iK} \vee q'_{KK(1)}\rangle\right) \leq \langle s_{iK(1)}, s'_{iK(1)}\rangle.$$

Therefore

$$\langle q_{iK(2)}, q'_{iK(2)}\rangle = \langle q_{ij}, q'_{ij}\rangle, \langle q_{K(2)K(1)}, q'_{K(2)K(1)}\rangle = \langle q_{ij}, q'_{ij}\rangle,$$

$\langle q^{(3)}_{ij}, q'^{(3)}_{ij}\rangle = \langle q_{ij}, q'_{ij}\rangle < \langle 1, 0\rangle$ for some $K(2)$. Since $\langle q_{K(1)j}, q'_{K(1)j}\rangle = \langle q_{ij}, q'_{ij}\rangle$ and $\langle q_{K(2)K(1)}, q'_{K(2)K(1)}\rangle = \langle q_{ij}, q'_{ij}\rangle$, we have $\langle q_{K(2)j}, q'_{K(2)j}\rangle = \langle q_{ij}, q'_{ij}\rangle$, so that $\langle s_{iK(2)}, s'_{iK(2)}\rangle > \langle q_{ij}, q'_{ij}\rangle$. Since

$$\left(\bigvee_{K=1}^{n} \langle s_{iK} \wedge r_{Kj}\rangle,\ \bigwedge_{K=1}^{n} \langle s'_{iK} \vee r'_{Kj}\rangle\right) = \langle q_{ij}, q'_{ij}\rangle < \langle s_{ij}, s'_{ij}\rangle.$$

By continuing the same process, we get $\langle q^{(n)}_{ij}, q^{1(n)}_{ij}\rangle < \langle 1, 0\rangle$. That is contradiction. Since Q is idempotent. $\therefore Q/Q = S/Q$.

□

Example 3. $S/Q = S \overset{C}{\leftarrow} (S \times Q)$

$$Q = \begin{pmatrix} \langle 1,0\rangle & \langle 0.3,0.4\rangle & \langle 1,0\rangle \\ \langle 1,0\rangle & \langle 1,0\rangle & \langle 1,0\rangle \\ \langle 0.3,0.4\rangle & \langle 0.3,0.4\rangle & \langle 1,0\rangle \end{pmatrix}$$

$$S = Q/Q = \begin{pmatrix} \langle 1,0\rangle & \langle 1,0\rangle & \langle 1,0\rangle \\ \langle 1,0\rangle & \langle 1,0\rangle & \langle 1,0\rangle \\ \langle 1,0\rangle & \langle 1,0\rangle & \langle 1,0\rangle \end{pmatrix}$$

$$S \times Q = \begin{pmatrix} \langle 1,0\rangle & \langle 1,0\rangle & \langle 1,0\rangle \\ \langle 1,0\rangle & \langle 1,0\rangle & \langle 1,0\rangle \\ \langle 1,0\rangle & \langle 1,0\rangle & \langle 1,0\rangle \end{pmatrix} \begin{pmatrix} \langle 1,0\rangle & \langle 0.3,0.4\rangle & \langle 1,0\rangle \\ \langle 1,0\rangle & \langle 1,0\rangle & \langle 1,0\rangle \\ \langle 0.3,0.4\rangle & \langle 0.3,0.4\rangle & \langle 1,0\rangle \end{pmatrix}$$

$$= \begin{pmatrix} \langle 1,0\rangle & \langle 1,0\rangle & \langle 1,0\rangle \\ \langle 1,0\rangle & \langle 1,0\rangle & \langle 1,0\rangle \\ \langle 1,0\rangle & \langle 1,0\rangle & \langle 1,0\rangle \end{pmatrix}$$

$$S/Q = S \overset{C}{\leftarrow} (S \times Q)$$

$$= \begin{pmatrix} \langle 1,0\rangle & \langle 1,0\rangle & \langle 1,0\rangle \\ \langle 1,0\rangle & \langle 1,0\rangle & \langle 1,0\rangle \\ \langle 1,0\rangle & \langle 1,0\rangle & \langle 1,0\rangle \end{pmatrix} \overset{C}{\leftarrow} \begin{pmatrix} \langle 1,0\rangle & \langle 1,0\rangle & \langle 1,0\rangle \\ \langle 1,0\rangle & \langle 1,0\rangle & \langle 1,0\rangle \\ \langle 1,0\rangle & \langle 1,0\rangle & \langle 1,0\rangle \end{pmatrix}$$

$$S/Q = \begin{pmatrix} \langle 1,0\rangle & \langle 1,0\rangle & \langle 1,0\rangle \\ \langle 1,0\rangle & \langle 1,0\rangle & \langle 1,0\rangle \\ \langle 1,0\rangle & \langle 1,0\rangle & \langle 1,0\rangle \end{pmatrix}$$

Therefore $Q/Q = S/Q$.

Remark 3.2. *Let* Q *and* S *as Min-Min product then* Q *be an* $n \times n$ *reflexive and transitive IFM then*

(1) $Q \neq S = Q/Q$.
(2) $S^{+} \neq Q$ *for any* $n \times n$ *IFM* $Q/Q = S/Q$.

Example 4. $S/Q = S \overset{C}{\leftarrow} (S \bullet Q)$.

Let $Q = \begin{pmatrix} \langle 1,0\rangle & \langle 0.3,0.4\rangle & \langle 1,0\rangle \\ \langle 1,0\rangle & \langle 1,0\rangle & \langle 1,0\rangle \\ \langle 0.3,0.4\rangle & \langle 0.3,0.4\rangle & \langle 1,0\rangle \end{pmatrix}$. *Then*

$$S = Q/Q = \begin{pmatrix} \langle 1,0\rangle & \langle 1,0\rangle & \langle 1,0\rangle \\ \langle 1,0\rangle & \langle 1,0\rangle & \langle 1,0\rangle \\ \langle 1,0\rangle & \langle 1,0\rangle & \langle 1,0\rangle \end{pmatrix}$$

$$S \bullet Q = \begin{pmatrix} \langle 1,0\rangle & \langle 1,0\rangle & \langle 1,0\rangle \\ \langle 1,0\rangle & \langle 1,0\rangle & \langle 1,0\rangle \\ \langle 1,0\rangle & \langle 1,0\rangle & \langle 1,0\rangle \end{pmatrix} \begin{pmatrix} \langle 1,0\rangle & \langle 0.3,0.4\rangle & \langle 1,0\rangle \\ \langle 1,0\rangle & \langle 1,0\rangle & \langle 1,0\rangle \\ \langle 0.3,0.4\rangle & \langle 0.3,0.4\rangle & \langle 1,0\rangle \end{pmatrix}$$

$$= \begin{pmatrix} \langle 0.3, 0.4\rangle & \langle 0.3, 0.4\rangle & \langle 1, 0\rangle \\ \langle 0.3, 0.4\rangle & \langle 0.3, 0.4\rangle & \langle 1, 0\rangle \\ \langle 0.3, 0.4\rangle & \langle 0.3, 0.4\rangle & \langle 1, 0\rangle \end{pmatrix}$$

So, $S/Q = S \overset{C}{\leftarrow} S \bullet Q$

$$= \begin{pmatrix} \langle 1, 0\rangle & \langle 1, 0\rangle & \langle 1, 0\rangle \\ \langle 1, 0\rangle & \langle 1, 0\rangle & \langle 1, 0\rangle \\ \langle 1, 0\rangle & \langle 1, 0\rangle & \langle 1, 0\rangle \end{pmatrix} \overset{C}{\leftarrow} \begin{pmatrix} \langle 0.3, 0.4\rangle & \langle 0.3, 0.4\rangle & \langle 1, 0\rangle \\ \langle 0.3, 0.4\rangle & \langle 0.3, 0.4\rangle & \langle 1, 0\rangle \\ \langle 0.3, 0.4\rangle & \langle 0.3, 0.4\rangle & \langle 1, 0\rangle \end{pmatrix}$$

$$= \begin{pmatrix} \langle 1, 0\rangle & \langle 1, 0\rangle & \langle 1, 0\rangle \\ \langle 1, 0\rangle & \langle 1, 0\rangle & \langle 1, 0\rangle \\ \langle 1, 0\rangle & \langle 1, 0\rangle & \langle 1, 0\rangle \end{pmatrix}$$

Therefore $Q/Q = S/Q$.

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