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EFFECTS OF CONCENTRATION OF AIR POLLUTANTS IN THE CHANNEL BOUNDED BY POROUS BEDS

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ABSTRACT. A Mathematical model describing the problem of concentration of air pollutants in the atmosphere has been examined. The study unravels the behavior of airborne particles that is aerosols, which play an imperative role in the Earth's environment. The main motive of the paper is to observe the effects of smog and haze through the concentration of aerosols with and without chemical reaction. A two-dimensional schematic geometry is considered which includes a rectangular channel extended to infinity in the x-axis bounded by porous layers. Assumptions are made that electric and magnetic field are transversely applied while there is heavy rain, thunderstorm, and lightning. A general solution of the dimensionless governing partial differential equations is obtained by the perturbation technique with appropriate boundary conditions. In this work velocity, the skin friction coefficient, the concentration of aerosols are numerically computed for different values of various parameters. As a measure to control the effects of air pollutants, significant conclusions are specified for healthier attentive of aerosols concentration properties.

1. INTRODUCTION

Air pollution modeling is a set of a mathematical equation which is used to estimate the concentration of air pollutant discharged from industrial and vehicular emissions. The air pollutants considered here are atmospheric aerosols,

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which are tiniest airborne particles specifically dangerous to the Earth's atmosphere. When aerosols such as dust and smoke are choked by weather conditions, dust particles accumulate and concentrate to form a low-hanging shroud called Haze and aerosols react with smoke to form smog. Haze and smog particles can sometimes upset the heart, lung and can lessen visibility to a greater magnitude.

Both theoretical and experimental work have done by taking aerosols in the account. Dorsey et al. [2] investigated the aerosol number concentrations and size distributions in cities. Eldrige [3] measured aerosol distribution through haze and fog and derived relationship between visibility and liquid water content. Jha et al. [4] examined the cumulative effect of pollution aerosol and dust also analyzed the impact of aerosol pollution and dust. Kumari and Sahoo [5] proposed a fast Fourier transform method to remove Haze from a single input image which improves visibility. Zhou et al [14] dissected the formation mechanisms behind the major smog pollution incidents and proposed principles to eliminate smog pollution. Different analytical approaches are used to study the dispersion and transport of air pollutants under the effects of both electric and magnetic field by Meenapriya et al. [6–8]. Following the classic perturbation technique given by Nayfeh [9], many researchers studied their work using Perturbation method. Nirmala et al. [10] examined the rate of hydrocarbon contaminant in the soil-water environment using perturbation technique. Several researchers [11–13] have investigated the combined effects of the magnetic field, permeable walls, Darcy velocity, electrically conducting fluid.

The above studies are vital for developing a two dimensional model to focus the effects of smog and haze by finding the concentration of aerosols with and without chemical reaction in the atmospheric fluid. Numerical calculations are facilitated by MATHEMATICA and results are portrayed graphically which enunciated the effects of concentration of aerosols.

2. Mathematical Formulation

A rectangular coordinate system (x, y) is formulated to model the flow with x as horizontal coordinate and y as vertical coordinate with an origin at the middle of the channel. The geometry considered $(-h < y < h, -\infty < x < \infty)$ is symmetric about $x - axis$ and is filled with atmospheric fluid bounded by porous layers on both sides. The electric and uniform magnetic field is applied transversely to the system externally through the boundaries. The electric field is applied through electrodes which are electro-conducting impermeable rigid plates with electric potentials $\phi = \frac{V}{h}x$ at $y = -h$ and $\phi = \frac{V}{h}(x - x_0)$ at $y = h$. A strong transverse magnetic field of unit strength is applied externally, neglecting the induced electric and magnetic field.

The flow is assumed to be incompressible, laminar, and presented through the solution of Navier strokes equation for velocity and advection - diffusion equation for the concentration of air pollutants. The governing equations are solved using the perturbation technique and let homogeneous first-order chemical reaction (K) takes place inside the channel with D as the mass diffusivity. Based

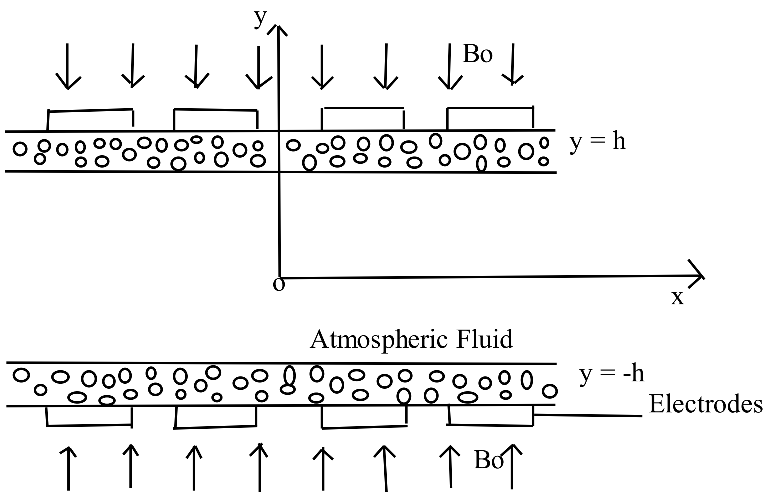

Figure 1. Physical Configuration

on all the assumptions made above, the governing equations are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\rho\left[\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right] = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + \rho_e E_x - \sigma_c B_0^2 u \tag{2.1}$$

$$\rho\left[\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right] = \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \tag{2.2}$$

$$\frac{\partial C_i}{\partial t} + u\frac{\partial C_i}{\partial x} + v\frac{\partial C_i}{\partial y} = D\left(\frac{\partial^2 C_i}{\partial x^2} + \frac{\partial^2 C_i}{\partial y^2}\right) - KC_i, \tag{2.3}$$

where ρ is the density of the fluid, μ is the co-efficient of viscosity, p is the pressure of the fluid, E_x is the electric field, B_0 is applied magnetic field, and ρ_e is the density of charge distribution. The concentration of aerosols is represented by C_i also we assume $\beta \neq 0$ when i=1, and $\beta = 0$ when i=2. To solve the governing equations Beaver Joseph slip conditions [1] are used to sculpt the boundary effects for velocity.

$$u = 0, v = 0, C_i = C_0 \epsilon e^{\mathbf{i}(\alpha x + wt)},\ at\ y = 0 \tag{2.4}$$

$$\frac{\partial u}{\partial y} = \frac{-\alpha_p}{\sqrt{k}}(u - u_p), v = v_1, C_i = C_0\left(1 + \epsilon e^{\mathbf{i}(\alpha x + wt)}\right),\ at\ y = h \tag{2.5}$$

$$\frac{\partial u}{\partial y} = \frac{-\alpha_p}{\sqrt{k}}(u - u_p), v = v_2,\ at\ y = -h, \tag{2.6}$$

where $u_p = -(k/\mu)\frac{\partial p}{\partial x}$ which represents Darcy law, u_p is the Darcy velocity of the porous layer, α_p is the slip parameter, k is the permeability of the porous layer, α is the streamwise wave number, w is the frequency parameter, ϵ is the perturbation parameter, **i** is the imaginary number. To make (2.1) to (2.6) dimensionless the following dimensionless quantities are introduced,

$$x^* = \frac{x}{h}; y^* = \frac{y}{h}; u^* = \frac{u}{\left(\frac{V}{h}\right)}; v^* = \frac{v}{\left(\frac{V}{h}\right)}; {\rho_e}^* = \frac{\rho_e}{\left(\frac{\epsilon_0 V}{h^2}\right)}; p^* = \frac{p}{\rho\left(\frac{V}{h}\right)^2}; t^* = \frac{t}{\left(\frac{h^2}{V}\right)};$$

$${E_x}^* = \frac{E_x}{\left(\frac{V}{h}\right)}; u_p^* = \frac{u_p}{\left(\frac{V}{h}\right)}; \beta^2 = \frac{h^2 K}{D}; C_i^* = \frac{C_i}{C_0}.$$

The chemical reaction rate parameter is β and V is the applied constant electric potential due to embedded electrodes at the boundaries. The value of the porous parameter σ is given by $\sigma = \frac{h}{\sqrt{k}}$, the electric number $W_e = \frac{\epsilon_0 V}{\mu}$, the Hartmann number $M^2 = \frac{\sigma_0 B_0^2 h^2}{\mu}$ and $a_0 = \frac{\rho V}{\mu}$. After non-dimensionalising the above governing equations and boundary conditions neglecting asterisk we obtain,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - a_0\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) - a_0\frac{\partial p}{\partial x} + W_e \rho_e E_x - M^2 u = 0. \tag{2.7}$$

$$\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} - a_0\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = 0. \tag{2.8}$$

$$\left(\frac{\partial^2 C_i}{\partial x^2}+\frac{\partial^2 C_i}{\partial y^2}\right)-\frac{V}{D}\left(\frac{\partial C_i}{\partial t}+u\frac{\partial C_i}{\partial x}+v\frac{\partial C_i}{\partial y}\right)=\beta^2 C_i. \tag{2.9}$$

$$u=0, v=0, C_i=\epsilon e^{\mathbf{i}(\alpha x+wt)},\ at\ y=0. \tag{2.10}$$

$$\frac{\partial u}{\partial y}=-\alpha\sigma(u-u_p), v=v_1, C_i=\left(1+\epsilon e^{\mathbf{i}(\alpha x+wt)}\right)),\ at\ y=1. \tag{2.11}$$

$$\frac{\partial u}{\partial y}=\alpha\sigma(u-u_p), v=v_2,\ at\ y=-1. \tag{2.12}$$

Assuming the atmospheric fluid understudy is poorly conducting using Maxwell's and conservation of charges equation, the value of $\rho_e E_x$ is calculated as $\rho_e E_x = a_2 e^{-\alpha_c y}$ so (2.7) takes the form,

$$\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-a_0\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)-a_0\frac{\partial p}{\partial x}+W_e a_2 e^{-\alpha_c y}-M^2 u=0. \tag{2.13}$$

2.1 Method of Solution

Velocity and concentration of air pollutants are calculated using perturbation technique i.e., decomposing the flow variables into steady base state quantities denoted by upper case and two-dimensional linear perturbed quantities denoted by tilde (˜) symbol as follows,

$$u(x,y)=U_B(y)+\tilde{u}(y)\epsilon e^{\mathbf{i}(\alpha x+wt)}+O(\epsilon^2)$$
$$v(x,y)=\tilde{v}(y)\epsilon e^{\mathbf{i}(\alpha x+wt)}+O(\epsilon^2)$$
$$p(x,y)=p_B(y)+\tilde{p}\epsilon e^{\mathbf{i}(\alpha x+wt)}+O(\epsilon^2)$$
$$C_i(x,y)=C_{B_i}(y)+\tilde{C}_i(y)\epsilon e^{\mathbf{i}(\alpha x+wt)}+O(\epsilon^2).$$

After decomposing the above equation into base and perturbed parts, the solution of the base part are obtained analytically and that of perturbed part are obtained numerically. Assuming the flow to be steady, splitting (2.7) to (2.13) and neglecting the higher orders of perturbation parameter ϵ, we obtain the following set of partial differential equations.

Base Part

$$\frac{\partial^2 U_B}{\partial y^2}-M^2 U_B-P_1+W_e a_2 e^{-\alpha_c y}=0. \tag{2.14}$$

$$\frac{\partial^2 C_{B_i}}{\partial y^2}-\beta^2 C_{B_i}=0. \tag{2.15}$$

Perturbed part

$$\frac{\partial \tilde{v}}{\partial y} - \tilde{u}a_3 = 0.$$

(2.16)
$$\frac{\partial \tilde{u}}{\partial y^2} + \tilde{u}(a_0a_4 - M^2 - \alpha^2 - a_3U_B) - \tilde{v}\frac{\partial U_B}{\partial y} + a_0a_3\tilde{p} = 0.$$

(2.17)
$$\frac{\partial^2 \tilde{v}}{\partial y^2} + \tilde{v}(a_0a_3U_B - \alpha^2 + a_0a_4) = 0.$$

(2.18)
$$\frac{\partial^2 \tilde{C}_i}{\partial y^2} + \tilde{C}_i(a_1a_4 - \alpha^2 - \beta^2 - a_3U_B) + \tilde{v}\frac{\partial C_{B_i}}{\partial y} = 0.$$

where

$$P_1 = a_0\frac{\partial p_B}{\partial x}, a_3 = \alpha tan(\alpha x + wt), a_4 = wtan(\alpha x + wt)$$

with Base part boundary conditions,

(2.19)
$$U_B = 0, C_{B_i} = 0, aty = 0.$$

(2.20)
$$\frac{\partial U_B}{\partial y} = -\alpha\sigma(U_B - U_{p_B}), aty = 1.$$

(2.21)
$$\frac{\partial U_B}{\partial y} = \alpha\sigma(U_B - U_{p_B}), aty = -1.$$

(2.22)
$$C_{B_i} = 1, aty = 1.$$

Perturbed part boundary conditions as follows,

(2.23)
$$\tilde{u} = 0, \tilde{v} = 0, \tilde{C}_i = 1, aty = 0.$$

(2.24)
$$\tilde{v} = 1, \tilde{C}_i = 1, aty = 1.$$

(2.25)
$$\frac{\partial \tilde{u}}{\partial y} = -\alpha\sigma(\tilde{u} - \tilde{u}_p), aty = 1.$$

(2.26)
$$\frac{\partial \tilde{u}}{\partial y} = \alpha\sigma(\tilde{u} - \tilde{u}_p), aty = -1.$$

Base part solutions:

Solution of (2.14) is

(2.27)
$$U_B = C\cosh[My] + D\sinh[My] - \frac{P_1}{M^2} + a_5e^{-\alpha_c y}.$$

The value of the constants C and D are calculated using boundary conditions (2.19),(2.20)and (2.21) gives

$$C = \frac{P_1}{M^2} - a_5, D = \frac{a_5 \alpha_c \cosh[\alpha_c]}{M \cosh[M]}.$$

2.1.1 Case 1: Concentration of air pollutants with chemical reaction ($\beta \neq 0$)
The solution for C_{B_1} in (2.15) is

$$C_{B_1} = E \cosh[\beta y] + F \sinh[\beta y].$$

Using boundary conditions (2.19) and (2.22) we get,

$$E = 0, F = 1/(\sinh[\beta]).$$

Perturbed part solutions.
Let

$$c_1 = a_0 a_4 - M^2 - \alpha^2, c_2 = C a_3, c_3 = D a_3, c_4 = a_3 a_5, c_5 = CM, c_6 = DM, c_7 = a_5 \alpha_c,$$

$$c_8 = a_0 a_3 \tilde{p}, c_9 = \frac{P_1}{M^2}, b_1 = a_0 a_3 C, b_2 = a_0 a_3 D, b_3 = a_0 a_3 a_4,$$

$$b_4 = a_0 a_4 - \frac{p_1}{M^2} - \alpha^2, d_1 = a_1 a_4 - a_3 \frac{p_1}{M^2} - \alpha^2 - M^2, d_5 = \frac{\beta}{\sinh[\beta]}$$

then (2.16),(2.17) and (2.18) take the form,

$$\frac{\partial^2 \tilde{u}}{\partial y^2} + \tilde{u}(c_1 - c_2 \cosh[My] - c_3 \sinh[My] + c_9 - c_4 e^{-\alpha_c y}) + \tilde{v}(c_5 \sinh[My] + c_6 \cosh[My] - c_7 e^{-\alpha_c y}) + c_8 = 0.$$

$$\frac{\partial^2 \tilde{v}}{\partial y^2} + \tilde{v}(b_1 \cosh[My] + b_2 \sinh[My] + b_4 + b_3 e^(- \alpha_c y)) = 0.$$

$$\frac{\partial^2 \tilde{C}_1}{\partial y^2} + \tilde{C}_1(d_1 - c_2 \cosh[My] - c_3 \sinh[My] - c_4 e^{-\alpha_c y}) + \tilde{v}(d_5 \cosh[\beta y]) = 0.$$

These perturbed equations are solved numerically subject to the boundary conditions prescribed in (2.23)to (2.26). Graphs are plotted for the axial velocity and concentration of air pollutants with chemical reaction(C_1) using MATHEMATICA.

2.1.2 Case 2: Concentration of air pollutants without chemical reaction ($\beta = 0$)

If no chemical reaction takes place in the channel then the reaction rate parameter β is zero, the corresponding base part equation is given by,

$$\frac{\partial^2 C_{B_2}}{\partial y^2} = 0.$$

The solution of C_{B_2} is got by integrating the above equation twice,

$$C_{B_2} = e_1 y + e_2.$$

The constants e_1, e_2 are calculated using boundary conditions described in (2.19),(2.22) are

$$e_1 = 0, e_2 = 1.$$

The real perturbed equation of concentration without chemical reaction is,

$$\frac{\partial^2 \tilde{C}_2}{\partial y^2} + \tilde{C}_2(a_1 a_4 - \alpha^2 - a_3 U_B) + \tilde{v}\frac{\partial C_{B_2}}{\partial y} = 0.$$

Let

$$e_3 = a_1 a_4 - \alpha^2 + a_3 \frac{p_1}{M^2}, e_4 = C a_3, e_5 = D a_3, e_6 = a_3 a_5$$

then the above equation can be written as

$$\frac{\partial^2 \tilde{C}_2}{\partial y^2} + \tilde{C}_2(e_3 - e_4 \cosh[My] - e_5 \sinh[My] - e_6 e^{-\alpha_c y}) + e_1 \tilde{v} = 0. \tag{2.28}$$

Equation (2.28) is solved numerically for C_2 and graphical solution is obtained for C_2 using MATHEMATICA.

2.2 Skin Friction coefficient

Friction between atmospheric fluid and the boundary surface passing through it is called skin friction co-efficient (τ) and is given by

$$\tau = \frac{\partial u}{\partial y}.$$

The solution for (τ) is evaluated numerically, and the results satisfying the boundary conditions are depicted as graphs.

3. Results and Discussion

The velocity profile for different values of Hartmann number (M) and electric number (We) is displayed in Fig.2. The effects of skin friction coefficient (τ) are depicted in Fig.3 by varying Hartmann number and electric number. By increasing the values of both Hartmann and electric number, the velocity and skin friction co-efficient increases. The effects of the concentration on various parameters are described in the proceeding sub-sections.

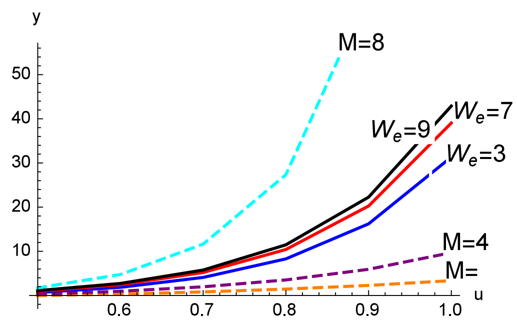

Figure 2. Velocity plots for some values of Hartmann and electric number

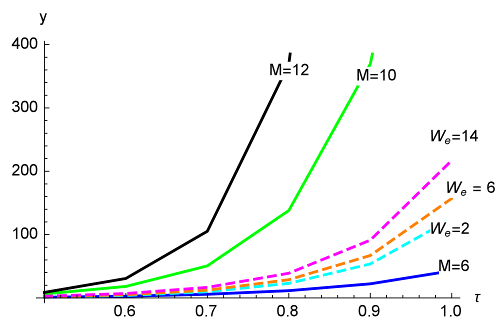

Figure 3. Skin friction coefficient plots for some values of Hartmann and electric number

3.1. Effects of concentration of smog

Figure 4 depicts that increasing the rate of the reaction, smog concentration increases. In the absence of chemical reaction, the concentration decreases. In

Fig.5 plots clearly indicates that increasing electric number, concentration of smog decreases. In the absence of an electric field, the concentration of smog increases. Figure 6 tells that increasing the effects of magnetic field the smog concentration decreases, but in the presence of very low magnetic field smog concentration increases.

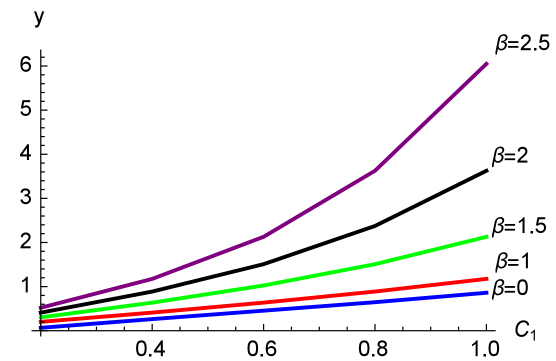

Figure 4. Plots of smog concentration for different values of reaction rate parameter

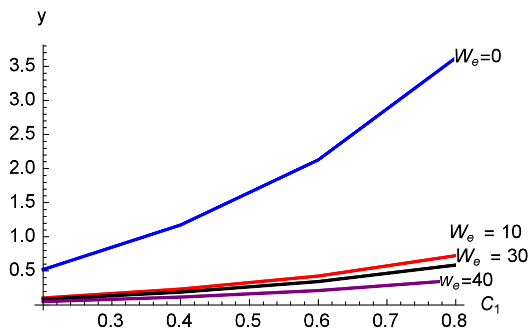

Figure 5. Plots of smog concentration for some values of electric number

3.2. Effects of concentration of Haze

In Fig.7, plots shows that if effects of an electric field increases haze concentration increases. Also in the absence of electric field haze concentration decreases. Figure 8 predicts that while increasing Hartmann number, haze concentration decreases,but in the absence of magnetic field haze concentration increases.

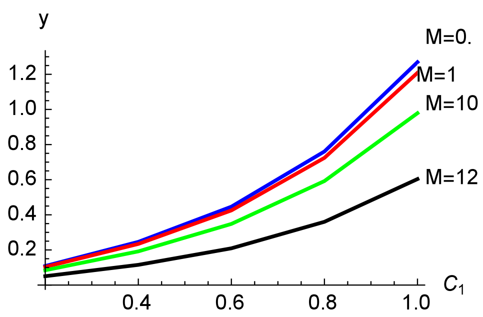

Figure 6. Plots of smog concentration for different values of Hartmann number

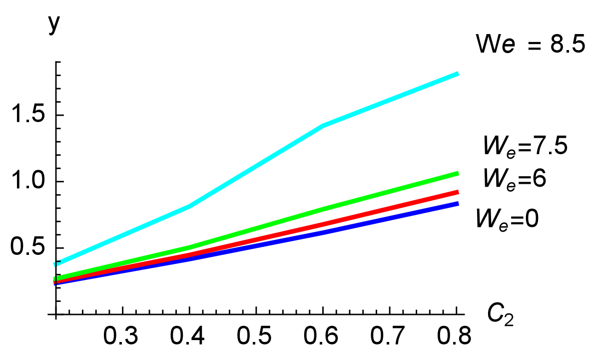

Figure 7. Plots of haze concentration for different values of electric number

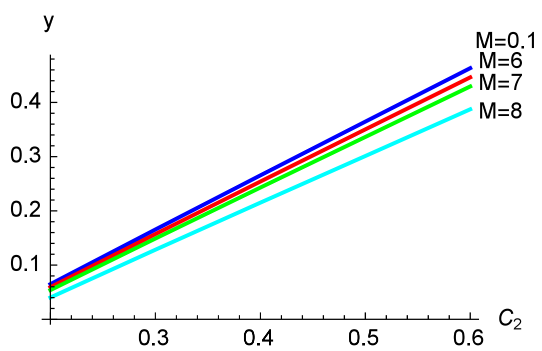

Figure 8. Plots of haze concentration for some values of Hartmann number

4. Conclusion

The concentration of smog is reduced by enhancing both electric and magnetic field in the form of thunderstorm. Haze concentration is minimized when improving the magnetic effect and reducing the effects of an electric field. It is concluded that concentration of air pollutants is minimized when there is a rain together with thunderstorm and lightning.

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