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SOME BISTAR RELATED EDGE BIMAGIC HARMONIOUS GRAPHS

L. MERRIT ANISHA[1], M. REGEES, AND T. NICHOLAS

ABSTRACT. A graph $G = (V, E)$ with p vertices and q edges is said to be edge bimagic harmonious if there exists a bijection $f : V \cup E \to \{1, 2, 3, ..., p + q\}$ such that for each edge xy in $E(G)$, the value of $[(f(x) + f(y))(mod\, q) + f(xy)]$ is equal to k_1 or k_2, where k_1 and k_2 are two distinct magic constants. In this paper we prove that the $\langle B_{m,n} : 2\rangle$, restricted square graph of $B_{n,n}$ and duplication of apex vertex of $B_{n,n}$ are edge bimagic harmonious graphs.

1. INTRODUCTION

Here we consider a simple, finite, connected and undirected graph with p vertices and q edges. Graph labeling was introduced by Rosa in 1960 [6]. Magic labeling was defined by Sedlacek [10]. In 1970, Kotzig and Rosa [7] magic valuation of a graph. In 1996, Ringel and Llado [3] initiated this labeling as edge magic. Edge bimagic labeling was defined by Babujee [1] in 2004. Graham and Sloane introduced harmonious labeling [5]. Dushyant Tanna [2] defined some harmonious graph labeling techniques. For more detailed, we use dynamic survey of graph labeling by Gallian [6]. $B_{m,n}$ is a (m, n) bistar obtained from two disjoint copies of $K_{1,m}$ and $K_{1,n}$ by joining the central vertices by an edge [8]. The graph $\langle B_{m,n} : 2\rangle$ obtained from the graph $B_{m,n}$ by subdividing the middle edge with a new vertex [8]. The restricted square [4] of $B_{n,n}$ is a graph G with vertex set $V(G) = V(B_{n,n})$ and edge set $E(G) = E(B_{n,n}) \cup \{uv_i, vu_i/1 \le i \le n\}$.

[1] *corresponding author*

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Duplication [4] of a vertex v of a graph G produces a new graph $G^{'}$ by adding a vertex $v^{'}$ with $N(v^{'}) = N(v)$. In other words a vertex $v^{'}$ is said to be a duplication of v if all the vertices which are adjacent to v are now adjacent to $v^{'}$. A graph $G = (V, E)$ with p vertices and q edges is said to be edge bimagic harmonious if there exists a bijection $f : V(G) \cup E(G) \to \{1, 2, 3, ..., p + q\}$ such that for each edge xy in $E(G)$, the value of $[(f(x) + f(y))(mod\ q) + f(xy)] = k_1$ or k_2, called two distinct magic constants [9]. In this paper we prove that the $\langle B_{m,n} : 2\rangle$, restricted square graph of $B_{n,n}$ and duplication of apex vertex of $B_{n,n}$ are edge bimagic harmonious graphs.

2. Main Results

Theorem 2.1. *The graph $\langle B_{m,n} : 2\rangle$ admits an edge bimagic harmonious labeling for all m and n.*

Proof. Let $V = \{u, v, w, u_i, v_j/1 \leq i \leq m, 1 \leq j \leq n\}$ and

$$E = \{uu_i, vv_j, uw, wv/1 \leq i \leq m, 1 \leq j \leq n\}$$

be the vertex set and the edge set of the graph $\langle B_{m,n} : 2\rangle$. The graph $\langle B_{m,n} : 2\rangle$ has $(m + n + 3)$ vertices and $(m + n + 2)$ edges. Define a bijection $f : V \cup E \to \{1, 2, 3, ..., 2m + 2n + 5\}$ such that

$$f(u) = 1$$
$$f(u_i) = i + 1, 1 \leq i \leq m$$
$$f(v) = m + n + 2$$
$$f(v_j) = m + j + 1, 1 \leq j \leq n$$
$$f(w) = m + n + 3$$
$$f(uu_i) = 2m + 2n - i + 4, 1 \leq i \leq m$$
$$f(uw) = 2m + 2n + 4$$
$$f(wv) = 2m + 2n + 5$$
$$f(vv_j) = m + 2n - j + 4, 1 \leq j \leq n.$$

Using these labelings, there exist two magic constants for each edge $xy \in E$, $[(f(x) + f(y))(mod\ q) + f(xy)]$ yields any one of the magic constants $k_1 = 2(m + n + 3)$ and $k_2 = 2m + 2n + 5$. Therefore, the graph $\langle B_{m,n} : 2\rangle$ admits an edge bimagic harmonious labeling for all m and n. □

Example 1. *Bimagic harmonious labeling of $\langle B_{5,6} : 2\rangle$ is given in figure 1.*

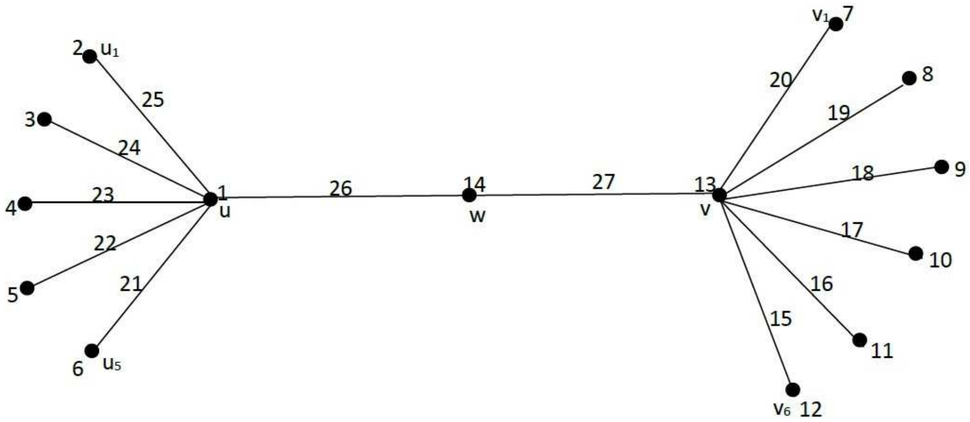

Figure 1: $\langle B_{5,6} : 2\rangle$ with $k_1 = 28$ and $k_2 = 27$.

Theorem 2.2. *The restricted square of bistar $B_{n,n}$ admits an edge bimagic harmonious labeling for all n.*

Proof. Let $V = \{u, v, u_i, v_i/1 \leq i \leq n\}$ and $E = \{uu_i, vv_i, uv, uv_i, vu_i/1 \leq i \leq n\}$ be the vertex set and the edge set of the restricted square of bistar $B_{n,n}$. The restricted square graph of $B_{n,n}$ has $2n + 2$ vertices and $4n + 1$ edges.

Define a bijection $f : V \cup E \to \{1, 2, 3, ..., 6n + 3\}$ such that

$$f(u) = 1$$
$$f(u_i) = i + 1, 1 \leq i \leq n$$
$$f(v) = 2n + 2$$
$$f(v_i) = n + i + 1, 1 \leq i \leq n$$
$$f(uu_i) = 4n - i + 3, 1 \leq i \leq n$$
$$f(uv_i) = 3n - i + 3, 1 \leq i \leq n$$
$$f(uv) = 6n + 3$$
$$f(vv_i) = 5n - i + 3, 1 \leq i \leq n$$
$$f(vu_i) = 6n - i + 3, 1 \leq i \leq n$$

Using these labelings, there exist two magic constants for each edge $xy \in E$, $[(f(x) + f(y))(mod\ q) + f(xy)]$ yields any one of the magic constants $k_1 = 4n + 5$ and $k_2 = 8n + 6$. Therefore, the restricted square of bistar $B_{n,n}$ admits an edge bimagic harmonious labeling for all n. □

Example 2. *Bimagic harmonious labeling of restricted square of bistar $B_{5,5}$ is given in figure 3.*

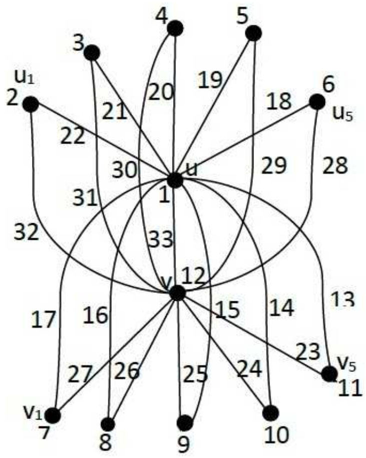

Figure 3: Restricted square of bistar $B_{5,5}$ with $k_1 = 25$ and $k_2 = 46$.

Theorem 2.3. *Duplication of apex vertex of $B_{n,n}$ admits an edge bimagic harmonious labeling for all n.*

Proof. Let $V = \{u, v, u_i, v_i, v^{'}/1 \leq i \leq n\}$ and $E = \{uu_i, vv_i, uv, uv^{'}, v_iv^{'}/1 \leq i \leq n\}$ be the vertex set and the edge set of the duplication of apex vertex of $B_{n,n}$. The duplication of apex vertex of $B_{n,n}$ has $2n + 3$ vertices and $3n + 2$ edges.

Define a bijection $f : V \cup E \rightarrow \{1, 2, 3, ..., 5n + 5\}$ such that

$$f(u) = 1$$
$$f(u_i) = i + 1, 1 \leq i \leq n$$
$$f(v) = n + 2$$
$$f(v_i) = n + i + 2, 1 \leq i \leq n$$
$$f(v^{'}) = 2n + 3$$
$$f(uu_i) = 5n - i + 3, 1 \leq i \leq n$$
$$f(uv) = 4n + 2$$
$$f(uv^{'}) = 3n + 1$$
$$f(uv^{'}) = 5n + 5, forn = 2$$
$$f(vv_i) = 3n - i + 1, 1 \leq i \leq n - 3$$

$$f(vvi_{)} = 6n - i + 3, n - 2 \leq i \leq n$$

$$f(v^{'}v_i) = 4n - i + 2, 1 \leq i \leq n$$

Using these labelings, there exist two magic constants for each edge $xy \in E$, $[(f(x) + f(y))(mod\ q) + f(xy)]$ yields any one of the magic constants $k_1 = 5n + 5$ and $k_2 = 4n + 5$. Therefore, duplication of apex vertex of $B_{n,n}$ admits an edge bimagic harmonious labeling for all n. □

Example 3. *Bimagic harmonious labeling of duplication of apex vertex of bistar $B_{5,5}$ is given in figure 4.*

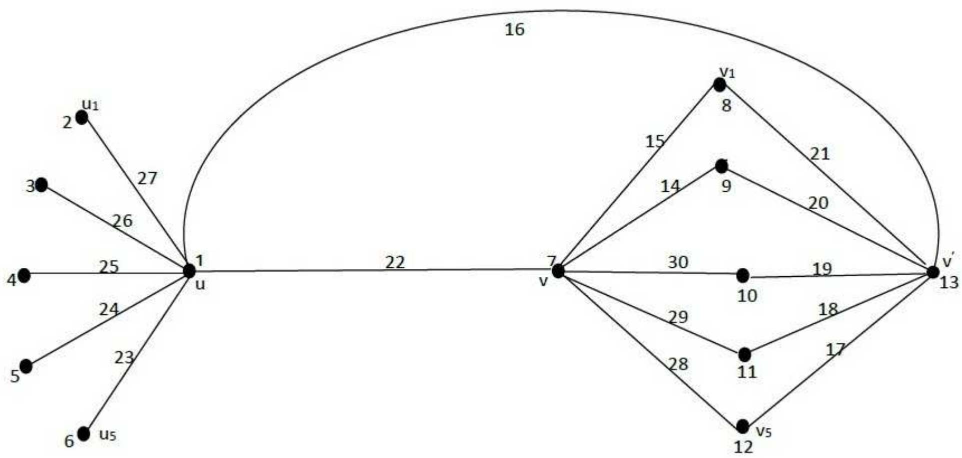

Figure 4: Duplication of apex vertex of bistar $B_{5,5}$ with $k_1 = 30$ and $k_2 = 25$.

3. Conclusion

In this paper, we proved that the $\langle B_{m,n} : 2\rangle$, restricted square graph of $B_{n,n}$ and duplication of apex vertex of $B_{n,n}$are edge bimagic harmonious graphs.

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DEPARTMENT OF MATHEMATICS
ST.JUDE'S COLLEGE, THOOTHOOR-629176
TAMIL NADU, INDIA
E-mail address: merritanisha@gmail.com

DEPARTMENT OF MATHEMATICS
MALANKARA CATHOLIC COLLEGE, MARIAGIRI-629153
TAMIL NADU, INDIA
E-mail address: regeesregees@gmail.com

DEPARTMENT OF MATHEMATICS
ST.JUDE'S COLLEGE, THOOTHOOR-629176
TAMIL NADU, INDIA
E-mail address: nicholas4560@gmail.com