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COMPLEMENTARY NIL G-ECCENTRIC DOMINATION IN FUZZY GRAPHS

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ABSTRACT. A g-eccentric dominating set $D \subseteq V$ of a fuzzy graph $G = (\rho, \phi)$ is said to be a complementary nil g-eccentric dominating set (CNGED-set) if $V - D$ contains no g-eccentric dominating set of $G = (\rho, \phi)$. The least scalar cardinality taken over all CNGED- set of G is called the complementary nil g-eccentric domination number of $G = (\rho, \phi)$. In this article, bounds for complementary nil g-eccentric domination number for a few standard fuzzy graph are given and theorems related to CNGED - sets are discussed. The relation between complementary nil g-eccentric domination number and other well known parameters are analyzed.

1. INTRODUCTION

Tamilchelvam and Robinson Chelladurai [10] defined the approach of complementary nil domination on graph in 2009. In 2020, Mohamed Ismayil and Muthupandiyan [7] presented the concept of g-eccentric domination in fuzzy graph. A fuzzy graph $G = (\rho, \phi)$ characterized with two functions ρ characterized on V and ϕ characterized on $E \subseteq V \times V$, where $\rho : V \to [0, 1]$ and $\phi : E \to [0, 1]$ such that $\phi(x, y) \leq \rho(x) \wedge \rho(y) \forall x, y \in V$. We expect that V is finite and non-empty, ϕ is reflexive and symmetric. We indicate the crisp graph by $G^* = (\rho^*, \phi^*)$ of the fuzzy graph $G(\rho, \phi)$, where $\rho^* = \{x \in V : \rho(x) > 0\}$ and $\phi^* = \{(x, y) \in E : \phi(x, y) > 0\}$. The fuzzy graph $G = (\rho, \phi)$ is called trivial in

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this case $|\rho^*| = 1$. For all the definitions pertinent to fuzzy graph we bring up in [8, 9, 11].

A strong path π from r to s is called geodesics in a fuzzy graph in the event that there is no shorter strong path from r to s and a length of a $r - s$ geodesic is the geodesic distance(g-distance) from r to s denoted by $d_g(r, s)$. The geodesic eccentricity (g-eccentricity) $e_g(x)$ of a node x in a connected fuzzy graph $G = (\rho, \varphi)$ is characterized by $e_g(x) = \max\{d_g(x, y), y \in V\}$. The least g-eccentricity among the vertices of G is called g-radius and indicated by $r_g(G) = \min\{e_g(x), x \in V\}$ and the greatest g-eccentricity among the vertices of G is called g-diameter and indicated by $d_g(G) = \max\{e_g(x), x \in V\}$. A vertex y is said to be a g-central node in case $e_g(y) = r_g(G)$. Moreover, a vertex y in G is said to be a g-peripheral node in case $e_g(y) = d_g(G)$. For all the definitions pertinent to g-eccentricity we bring up in [3, 5–7].

A subset D of V is called a dominating set of G in the event that for each $y \in V - D$ there exists $x \in D$ such that x dominates y. The least cardinality taken over all the minimal dominating set is called the domination number of G and it is indicated by $\gamma(G)$. A dominating set $D \subseteq V(G)$ in a fuzzy graph $G = (\rho, \phi)$ is said to be a g-eccentric dominating set each vertex $y \notin D$, there exists at least one g-eccentric vertex $x \in D$. The least scalar cardinality taken over all g-eccentric dominating set is called g-eccentric domination number and is indicated by $\gamma_{ged}(G)$. For all the definitions pertinent to eccentric domination we bring up in [1, 2, 4, 6, 7].

In this article, the new domination parameter $\gamma_{cnged}(G)$ is presented. complementary nil g-eccentric domination set and its number for a few standard fuzzy graph are characterized and a few theorems related to complementary nil g-eccentric domination are expressed and demonstrated.

2. Complementary Nil g-Eccentric Domination in Fuzzy Graph

In this chapter, a modern g-eccentric dominating parameter known as complementary nil g-eccentric domination number is characterized. The relationship between $\gamma_{cnged}(G)$ and other well known parameters are obtained.

Definition 2.1. *A g-eccentric dominating set $D \subseteq V(G)$ of a fuzzy graph $G(\rho, \phi)$ is called complementary nil g-eccentric dominating set(CNGED -set) in the event that $V - D$ is not a g-eccentric dominating set. A CNGED-set D is minimal if no proper*

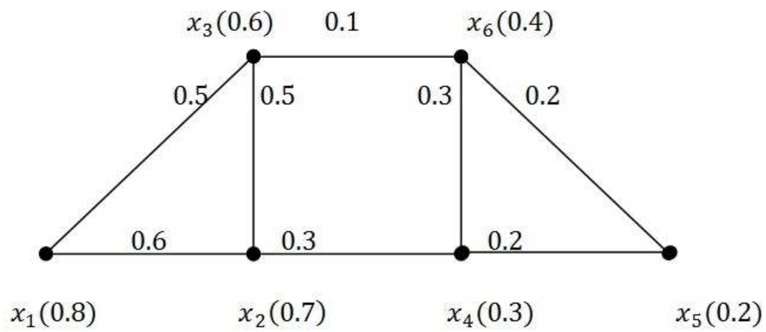

FIGURE 1

subset $D^{'} \subset D$ be a CNGED-set. The Complementary nil g-eccentric domination number of G is the least scalar cardinality taken over all the minimal CNGED-set and is indicated by $\gamma_{cnged}(G)$. The most prominent scalar cardinality taken over all the minimal CNGED -set is called a upper complementary nil g-eccentric domination number and is implied by $\Gamma_{cnged}(G)$.

Example 1. *In the Figure 1, a minimum dominating set is $D_1 = \{x_3, x_5\}$, a minimum g-eccentric dominating set is $D_2 = \{x_3, x_5\}$, a minimum CNGED-set is $D_3 = \{x_3, x_5, x_6\}$ and minimal CNGED-sets are $D_4 = \{x_1, x_3, x_5, x_6\}$ and $D_5 = \{x_2, x_3, x_4, x_5, x_6\}$. The minimum complementary nil dominating set is $D_6 = \{x_3, x_4, x_5, x_6\}$. Therefore $\gamma(G) = 0.8$, $\gamma_{ged}(G) = 0.8$, $\gamma_{cnged}(G) = 1.2$, $\gamma_{cnd}(G) = 1.5$ and $\Gamma_{cnged}(G) = 2.2$. Here, $\gamma_{cnd}(G) > \gamma_{cnged}(G)$.*

Note 2.1. *In a fuzzy graph $G = (\rho, \phi)$, there is no relation between $\gamma_{cnged}(G)$ and $\gamma_{cnd}(G)$.*

Note 2.2. *The minimum CNGED-set is denoted by $\gamma_{cnged}(G)$.*

Observation 2.1. *For any fuzzy graph $G = (\rho, \phi)$*

(1) $\gamma(G) \leq \gamma_{ged}(G) \leq \gamma_{cnged}(G)$.
(2) *Every super set of a CNGED-set is also a CNGED-set.*
(3) *Complement of a CNGED-set is not a CNGED -set.*
(4) *Complement of a γ-set is need not a CNGED-set.*
(5) *γ_{cnged}-set need not be unique.*
(6) $\gamma_{cnged}(G) \leq \Gamma_{cnged}(G)$.
(7) *Complete fuzzy graph has no CNGED -set.*

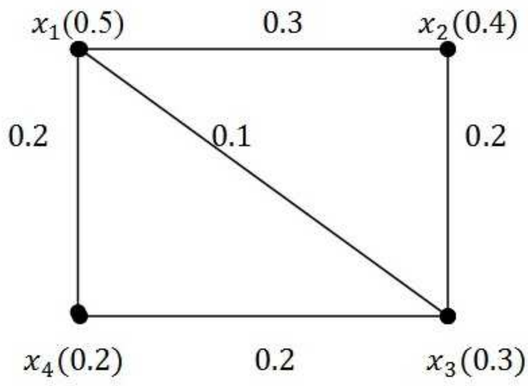

FIGURE 2

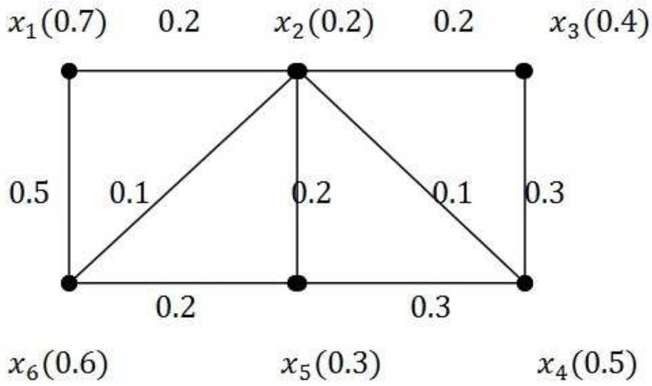

FIGURE 3

Example 2. *In the figure 2, we have the minimum dominating set is $D_1 = \{x_3, x_4\}$, minimum g-eccentric dominating set is $D_2 = \{x_3, x_4\}$, minimum complementary nil dominating set is $D_3 = \{x_2, x_3, x_4\}$ and minimum complementary nil g-eccentric dominating set is $D_4 = \{x_2, x_3, x_4\}$. Hence, $\gamma(G) = 0.5$, $\gamma_{ged}(G) = 0.5$, $\gamma_{cnd}(G) = 0.9$ and $\gamma_{cnged}(G) = 0.9$. Here, $\gamma_{cnd}(G) = \gamma_{cnged}(G)$.*

Example 3. *In the figure 3, we have the minimum dominating set is $D_1 = \{x_2, x_5\}$, minimum g-eccentric dominating set is $D_2 = \{x_3, x_4, x_6\}$, minimum complementary nil dominating set is $D_3 = \{x_2, x_3, x_4, x_5\}$ and minimum complementary nil g-eccentric dominating set is $D_4 = \{x_3, x_4, x_6\}$. Hence, $\gamma(G) = 0.5$, $\gamma_{ged}(G) = 1.5$, $\gamma_{cnd}(G) = 1.4$ and $\gamma_{cnged}(G) = 1.5$. Here, $\gamma_{cnd}(G) < \gamma_{cnged}(G)$.*

Definition 2.2. *Let $S \subseteq V(G)$ in a fuzzy graph $G(\rho, \phi)$. A vertex $x \in S$ is said to be an enclave of S if $\phi(x, y) < \rho(x) \wedge \rho(y)$ for all $\in V - S$, that is $N_s[x] \subseteq S$.*

Definition 2.3. *Let $S \subseteq V$ be a subset in a fuzzy graph $G(\rho, \phi)$. A vertex $x \in S$ is said to be a g- eccentric enclave of S if $E_g(x) \subseteq S$.*

Example 4. *In the fuzzy graph given in figure 3.1 a vertex x_5 is an enclave of D_5 and the vertex x_3 is g-eccentric enclave of D_3 since $E_g(x_3) = \{x_5, x_6\} \subseteq D_3$.*

3. Theorems Related to Complementary Nil g-Eccentric Domination in Fuzzy Graph

In this chapter, some theorems related to complementary nil g-eccentric domination in fuzzy graphs are stated and proved.

Theorem 3.1. *Let $G(\rho, \phi)$ be a fuzzy graph and $D \subseteq G$ be a CNGED-set. Then*

(i) *D contains at least one enclave or*
(ii) *D has at least one vertex y such that $e_g(y) = d_g(x, y), \forall x \in D$.*

Proof. Let D be a CNGED - set of a fuzzy graph $G(\rho, \phi)$. By the definition of CNGED -set, $V - D$ is not a g-eccentric dominating set. At that point there exists a vertex $x \in D$ such that (i) x is not strong neighbours to any of the vertices in $V - D$. That is $N_s[x] \subseteq D$. In this manner, D contains at least one enclave or (ii) x has no g-eccentric vertex in $V - D$, Hence x has all its g-eccentric vertices are in D only. □

Theorem 3.2. *Let D be a CNGED-set of a fuzzy graph $G(\rho, \phi)$. At that point D is minimal $\Leftrightarrow$ for each $x \in D$ one of the given conditions satisfied.*

(i) *x contains no strong neighbors in D or g-eccentric vertex of u is not in D.*
(ii) *There exists a few $y \in V - D$ such that $N_s(y) \cap D = \{x\}$ or $E_g(y) \cap D = \{x\}$.*
(iii) *$[V - D] \cup \{x\}$ could be a g-eccentric dominating set.*

Proof. Assume D is a minimal CNGED-set. If there exists a vertex $x \in D$ such that x does not fulfill any of the given conditions (i), (ii) and (iii). **Case(i):** Assume x contains all strong neighbors at that point D is not negligible. **Case (ii):** Suppose g-eccentric vertex of x is in D then D is not minimal, by (ii) $N_s(y) \cap D = \varphi$, D is not dominating set and $E_g(y) \cap D = \varphi$ at that point D is not g-eccentric dominating set **Case(iii):** $[V - D] \cup \{x\}$ is not a g-eccentric dominating set. This infers that $D-\{x\}$ may be a CNGED-set of $G(\rho,\phi)$, which is a contradiction to the minimality of D.

Conversly, Let D may be a CNGED-set and for all $x \in D$, one of the three conditions hold. we claim that D must be a minimal CNGED-set. Assume that D is not a minimal CNGED-set, then there exists a vertex $x \in D$ such that $D-\{x\}$ is a CNGED-set. Thus, x is strong neighbours to at least one vertex in $D-\{x\}$ and x has g-eccentric vertex in $D-\{x\}$, which implies (i) does not hold.

Moreover, if $D-\{x\}$ is a CNGED- set, each vertex x in $[V-D]\cup\{x\}$ is strong neighbors to at least one vertex in $D-\{x\}$ and x has a g-eccentric vertex in $D-\{x\}$. Subsequently condition (ii) does not hold. Since $D-\{x\}$ is a CNGED-set, at that point by observation 2.1(3) $[V-D]\cup\{x\}$ is not a CNGED-set, therefore condition (iii) does not hold.

Hence, there exists $x \in D$ such that x does not fulfill the conditions (i), (ii) and (iii), which is a contradiction to our assumption. □

Theorem 3.3. *Let D be a CNGED-set of a connected fuzzy graph $G(\rho,\phi)$, and g-eccentric point of x in $D-\{x\}$, then there exists a vertex $x \in D$ such that $D-\{x\}$ is g-eccentric dominating set.*

Proof. Let D be a CNGED-set of a fuzzy graph $G(\rho,\phi)$. By a theorem 3.1, each CNGED -set has at least one enclave in D. Let $x \in D$ be an enclave of D. Implies that $\phi(x,y) < \rho(x)\wedge\rho(y)$ for all $y \in V-D$, that is $N_s[x] \subseteq D$. Since $G(\rho,\phi)$ is a connected fuzzy graph, at that point there exists at least one vertex $z \in D$ such that $\phi(x,z) \leq \rho(x)\wedge\rho(z)$ and g-eccentric vertex of x is in $D-\{x\}$. Hence $D-\{x\}$ is a g-eccentric dominating set. □

Theorem 3.4. *A CNGED- set in a connected fuzzy graph $G(\rho,\phi)$ is not singleton.*

Proof. Let D be a CNGED-set of a connected fuzzy graph $G(\rho,\phi)$. By a Theorem 3.1, each CNGED-set has at least one enclave in D or D has at least one vertex whose g-eccentric vertices in D. Let $x \in D$ be an enclave of D. Implies that $\phi(x,y) < \rho(x)\wedge\rho(y)$ for all $y \in V-D$, that is

$$N_s[x] \subseteq D\,. \tag{3.1}$$

Suppose D contains only a vertex x, (3.1) does not exists or isolated in $G(\rho,\phi)$, which is a contradiction to our connectedness. Thus CNGED-set has more than one vertex in a connected fuzzy graph $G(\rho,\phi)$. □

Corollary 3.1. *Let D be a γ_{cnged}-set of a fuzzy graph (ρ, ϕ). If x and y are two enclave of D, then*

(i) $N_s[x] \cap N_s[y] \neq \varphi$ *and*

(ii) x *and* y *are strong neighbors.*

Example 5. *In figure 1 x_5, x_6 are two enclave of D_6.*

Theorem 3.5. *A γ_{cnged}- set in a fuzzy graph $G(\rho, \phi)$ is not independent.*

Proof. Let $G(\rho, \phi)$ be a fuzzy graph and D be a CNGED-set which is independent. Then D is a minimal g-eccentric dominating set which infers that $V - D$ is additionally a g-eccentric dominating set. By the definition, D is not CNGED-set, which is a contradistinction. □

Observation 3.1. *For any fuzzy graph $G(\rho, \phi)$, each γ_{cnged}-set intersects with each γ_{ged}-set of $G(\rho, \phi)$.*

Theorem 3.6. *In a star fuzzy grap S_ρ, $\gamma_{cnged}(K_{\rho_1,\rho_2}) = \rho_1 0 + \rho_{20}$, where $\rho_{10} = \min\{\rho(x), x \in \rho_1\}$ and $\rho_{20} = \min\{\rho(y), y \in \rho_2\}$ and $|\rho_1^*| = 1$ and $|\rho_2^*| \geq 1$.*

Proof. Let K_{ρ_1,ρ_2} be a star fuzzy graph. Let $D = \{x, y\}$, where x may be a central vertex which dominates all the vertices in $V - D$ and y is pendent vertex, that is y is the g-eccentric vertices of $V - D$, $V - D$ is not g-eccentric dominating set. In this manner, D is γ_{cnged}-set. Clearly, g-eccentric dominating set is also CNGED-set. That is, $\gamma_{ged}(K_{\rho_1,\rho_2}) = \gamma_{cnged}(K_{\rho_1,\rho_2}) = \rho_{10} + \rho_{20}$. □

Theorem 3.7. *Let (K_{ρ_1,ρ_2}) be a complete bipartite fuzzy graph, then $\gamma_{cnged}(K_{\rho_1,\rho_2}) \leq \min(|\rho_1|, |\rho_2|) + \rho_n$, where $\rho_n = \max\{\rho(x), x \in V\}$.*

Proof. (K_{ρ_1,ρ_2}) be a complete bipartite fuzzy graph, $\rho = \rho_1 \cup \rho_2$ where $m = |\rho_1^*|$ and $n = |\rho_2^*|$ such that each vertex of V_1 is strong neighbors of a vertex in V_2 and vice versa. Let $D = \rho_1 \cup \{y\}, y \in V_2$. Since $V - D$ is not g-eccentric dominating set, at that point D is CNGED-set.

Subsequently, $\gamma_{cnged}(K_{\rho_1,\rho_2}) \leq \min(|\rho_1|, |\rho_2|) + \rho_n$. □

Corollary 3.2.

(i) *Let (K_{ρ_1,ρ_2}) be the complete bipartite fuzzy graph, then*

$$\gamma_{cnged}(K_{\rho_1,\rho_2}) = \begin{cases} |\rho_1| + \rho_{20}, \ if \ |\rho_1| \leq |\rho_2| \\ |\rho_2| + \rho_{10}, \ if \ |\rho_1| > |\rho_2| \end{cases}$$

where ρ_{10} is the minimum membership value of ρ_1 and ρ_{20} are the minimum membership value of ρ_2.

(ii) *Let T_ρ be a fuzzy tree. Then $\gamma_{cnged}(T_\rho) \leq \gamma(T_\rho) + \rho_n$.*

Corollary 3.3. *In a fuzzy wheel graph $W_\rho, |\rho^*| = 4$ has no CNGED-set.*

Proof. By observation 2.1(7), complete fuzzy graph has no CNGED-set. Hence, $W_\rho, |\rho^*| = 4$ is a complete fuzzy graph with 4 vertices has no CNGED-set. □

4. Bounds for Complementary Nil g-Eccentric Domination in Fuzzy Graph

In this chapter, we talk about theorem related to bounds for few fuzzy standard graphs.

Observation 4.1. *For any fuzzy graph $G = (\rho, \phi)$*

(1) $2\rho_0 \leq \gamma_{cnged} \leq p - \rho_0$.

Observation 4.2.

(1) *For a complete fuzzy graph $K_\rho, \gamma_{cnged}(K_\rho - e) \leq p - \rho_0$ where $\rho_0 = \min_{x \in V} \rho(x)$.*

(2) *For a complete fuzzy graph $K_\rho, \gamma_{cnged}(K_\rho - e) = p - \rho(x)$, where $\rho(x)$ is obtained from $\phi(e) = \rho(x) \wedge \rho(y) = \rho(y)$.*

(3) *For a path fuzzy graph $P_\rho, \gamma_{cnged}(P_\rho) = \gamma_{ged}(P_\rho) \leq \frac{p}{3} + 1$.*

Theorem 4.1. *Let $G(\rho, \phi)$ be a fuzzy graph with pendent vertex, then $\gamma_{cnged}(G) = \gamma_{ged}(G)$ or $\gamma_{ged}(G) + 1$.*

Proof. Let $G(\rho, \phi)$ be a fuzzy graph and D be a γ_{ged}-set. Let x be a pendent vertex in G. If x and its support vertex y is in D, at that point $V - D$ is not a g-eccentric dominating set. Subsequently, $\gamma_{cnged}(G) = \gamma_{ged}(G)$. Assume x or its support vertex y is in D, at that point $D_1 = D \cup \{y\}$ or $D_1 = D \cup \{x\}$ may be a g-eccentric dominating set and $V - D_1$ is not a g-eccentric dominating set. Consequently $\gamma_{cnged}(G) = \gamma_{ged}(G) + 1$. □

Theorem 4.2. *In the event that $G(\rho, \phi)$ be a fuzzy graph with $d_g(G) = 2$, at that point $\gamma_{cnged}(G) \leq \delta_s(G) + 1$.*

Proof. Let $G(\rho, \phi)$ be a fuzzy graph with $d_g(G) = 2$. At that point we have $x \in V(G)$ be such that $d_s(x) = \delta_s(G)$. Presently, let us take $D = \{x\} \cup N_s(x) = N_s[x]$. In this manner, each vertex in $V - D$ is strong neighbors to a few vertices of $N_s(x)$ and are g-eccentric to x. Subsequently D is g-eccentric dominating set and $V - D$ is not g-eccentric dominating set, since x can not dominated by any vertex in $V - D$. Hence, $\gamma_{cnged}(G) \leq \delta_s(G) + 1$. □

Theorem 4.3. *W_ρ be a fuzzy wheel graph, at that point $\gamma_{cnged}(W_\rho) \leq 4$, $|\rho^*| = n \geq 5$.*

Proof. Let $W_\rho, |\rho^*| = n, n \geq 5$ be a fuzzy wheel graph. Let $D = \{x, y, z, w\}$, where w may be a central vertex, z may be a enclave vertex and x, y are any two pheriperal vertices which are no strong neighbors. D is a least g-eccentric dominating set. Hence, $V - D$ is not a g-eccentric dominating set. Subsequently, D is a CNGED-set. Hence, $\gamma_{cnged}(W_\rho) \leq 4$. □

Theorem 4.4. *Let T be a fuzzy tree graph $G(\rho, \phi)$ such that each support vertex is strong neighbor of at least one pendent vertex. Then $\gamma_{cnged}(T) \leq U + 2$, where U is the number of support vertices.*

Proof. Let U be the set of all support vertices of $G(\rho, \phi)$. Here, all the non end vertices frame a dominating set. In this manner, to create a g-eccentric dominating set we have to be include at most two pendent vertices. Subsequently, the CNGED-set contains all the non conclusion vertices and at most two pendent vertices. Therefore, $V - D$ is not a g-eccentric dominating set. Hence, $\gamma_{cnged}(T) \leq U + 2$. □

Theorem 4.5. *Let K_ρ be a completel fuzzy graph, $|\rho^*| = n$, n is even. Let $G(\rho, \phi)$ be a fuzzy graph obtained from the complete fuzzy graph K_ρ by deleting edges of linear factor. Then $\gamma_{cnged}(G) \leq \frac{p}{2} + 1$.*

Proof. Let $G(\rho, \phi)$ be a fuzzy graph obtained from a non-trivial complete fuzzy graph. let $V = \{x_1, x_2, ..., x_n\}$ be the vertices of $G(\rho, \phi)$ and $G(\rho, \phi) = K_\sigma - \{x_1x_2, x_3x_4, ..., x_{n-1}x_n\}$. Then $D = \{x_1, x_3...x_{n-1}\}$ and $V - D = \{x_2, x_4...x_n\}$ are g-eccentric dominating sets and we know that $\gamma_{ged}(G) \leq \frac{p}{2}$. Hence, when we include one more vertex in D, then D is CNGED-set and $V - D$ is not g-eccentric dominating set. Thus, $\gamma_{cnged}(G) \leq \frac{p}{2} + 1$. □

5. Conclusion

In this paper, we examined the new domination parameter complementary nil g-eccentric domination number for a few standard fuzzy graphs and theorems related to bounds for CNGED-set for fuzzy graph.

References

[1] M. Bhanumathi, S. Muthammai: *Eccentric domination in trees*, International Journal of Engineering Science, Advanced Computing and Bio-Technology, **2**(1) (2011), 38–46.

[2] M. Bhanumathi, Sudhasenthil: *Complementary Nil Eccentric Domination number of a graph*, IJMSC, **6** (2016), 183–192.

[3] K. R. Bhuttani, A. Rosenfeld: *Geodesics in Fuzzy Graphs*, Elsevier, 2003.

[4] T. N. Janakiraman, M. Bhanumathi, S. Muthumani: *Eccentric domination in Graphs*, International journal of Engineering Science, Advanced Computing and Bio-Techonology, **2** (2010), 33–44.

[5] J. P. Linda, M. S. Sunitha: *On g-eccentric nodes g-boundary nodes and interior nodes of a fuzzy graph*, International Journal of science and Application, **2**(2) (2012), 107–123.

[6] M. Ismayil, I. Mohideen: *Complementary nil domination in fuzzy graph*, Annals of fuzzy mathematics and Informatics, ISSN:209-39310, 2014.

[7] M. Ismayil, S. Muthupandiyan: *g-Eccentric Domination in Fuzzy graphs*, our Heritage(ISSN:0474-9030), **68**(4)(2020), 234–245.

[8] A. Rosenfeld: *Fuzzy graphs*, In: L. A. Zadeh, K. S.Fu and M. Shimura, Eds, I, Academic Press, New York, (1975), 77–95.

[9] M. S. Sunitha, S. Mathew: *Fuzzy graph theory: Survey*, Annals of pure and Applied Mathematics, **4**(1) (2013), 92–110.

[10] T. Chelvam, S. R. Chellathurai: *Complementary nil domination number of a graph*, Tamkang Journal of Mathematics, **40** (2009), 165–172.

[11] L. A. Zadeh: *Fuzzy sets*, Information and Control, **8** (1965), 338–353.

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