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ECCENTRIC DOMINATION POLYNOMIAL OF GRAPHS

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ABSTRACT. In this paper, the concept of eccentric domination polynomial

$$ED(G,k) = \sum_{k=\gamma_{ed}(G)}^{V(G)} ed(G,k)x^k$$

is introduced. Here $\gamma_{ed}(G)$ is the eccentric domination number of a graph G and $ed(G,k)$ is the number of eccentric dominating sets of G of size k. Theorems related to eccentric domination polynomials are stated and proved. The eccentric domination polynomials of some standard graphs are computed.

1. INTRODUCTION

Let $G = (V,E)$ be a simple, connected and undirected graph. Let $V(G)$ and $E(G)$ be the set of vertices and edges of a graph G respectively. The open neighborhood and the closed neighborhood of a vertex $v \in V$ are defined by $N(v) = \{u \in V(G) : \forall\ (u,v) \in E\}$ and $N[v] = N(v)\bigcup\{v\}$ respectively. If $(u,v) \in E$ then (u,v) is incident with u and v. The degree of a vertex is the total number of edges incident on it. The minimum and maximum degree of a vertex is defined by $\delta(G) = min_{u\in V}|N(u)|$ and $\Delta(G) = max_{u\in V}|N(u)|$ respectively. A graph is connected if every pair of vertices in a graph are joined by a path. The distance $d(u,v)$ between two points u and v in G is the length of a shortest path

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between them. The eccentricity $e(v) = max\{d(u,v) : u \in V\}$. The eccentric set of a vertex v is defined by $E(v) = \{u \in V(G)/d(u,v) = e(v)$. Undefined terminology and notations related to the graph the reader is refered to the book [1,4].

T. N. Janakiraman et.al [2], introduced eccentric domination in graphs in 2010. If a set D is an eccentric point set as well as a dominating set then it is said to be an eccentric dominating set of G. A. M. Ismayil and R. Priyadharshini [6] introduced detour eccentric domination in graphs in 2019. A. M. Ismayil and A. Riyaz Ur Rehman [7,8] introduced equal eccentric domination in 2019 and the same authors introduced accurate eccentric domination in graphs in 2020. For more details about eccentric domination the reader can refer article [2].

S. Alikhani and Y.H. Peng [3] introduced domination polynomial of a graph in the year 2009. They also determined the domination polynomials of some standard graphs and proved theorems related to them. S. S. Kahat et al, [5] introduced dominating sets and domination polynomial of star graphs in 2014. In this paper, eccentric domination polynomials are introduced. Theorems related to eccentric domination polynomial of some standard graphs are discussed.

2. Eccentric Domination Polynomial of a graph

In this section, we define eccentric domination polynomial of a graph, we find the eccentric domination polynomial of a complete graph and bipartite graph. We prove the isomorphism of eccentric domination polynomial.

Definition 2.1. *Let $\mathcal{E}d(G,k)$ be the set of all dominating sets of a graph G with cardinality k and let $ed(G,k) = |\mathcal{E}d(G,k)|$. Thus the eccentric dominating polynomial $ED(G,x)$ of G is defined by $ED(G,x) = \sum\limits_{k=\gamma_{ed}(G)}^{n} ed(G,k)x^k$, where $\gamma_{ed}(G)$ is the eccentric domination number of G.*

Example 1. *For a graph G given in figure-2.1. $\{v_1,v_2,v_3,v_4\}$ is the only eccentric dominating set with cardinality 4, $\{v_1,v_2,v_3\}$, $\{v_1,v_2,v_4\}$, $\{v_1,v_3,v_4\}$, $\{v_2,v_3,v_4\}$ are the eccentric dominating sets with cardinality 3. There are three eccentric dominating sets $\{v_1,v_2\}$, $\{v_1,v_3\}$, $\{v_1,v_4\}$ containing two vertices. There is no eccentric dominating set with cardinality one. Therefore the eccentric domination polynomial is given by $ED(G,x) = x^4 + 4x^3 + 3x^2$.*

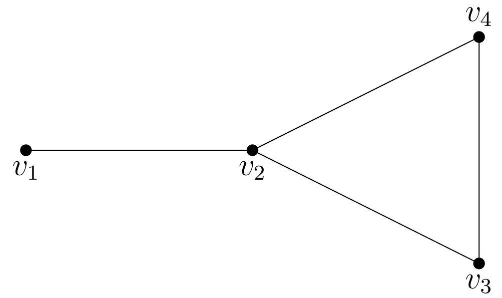

FIGURE 1

Theorem 2.1. *The eccentric dominating polynomial of a complete graph K_n is given by $ED(K_n, x) = (1+x)^n - 1$.*

Proof. Since, every vertex $u \in V$ is connected to every other vertex $v \in V$ in a complete graph K_n, every vertex $u \in V$ is an eccentric vertex of $v \in V - \{u\}$. Therefore there are $\binom{n}{k}$ combinations of eccentric dominating sets in a complete graph K_n. Then, the dominating sets and eccentric dominating sets of K_n are equal. The eccentric dominating polynomial of a complete graph K_n is given by

$$\begin{aligned} ED(K_n, x) &= \tbinom{n}{1}x + \tbinom{n}{2}x^2 + \tbinom{n}{3}x^3 + \tbinom{n}{4}x^4 + \tbinom{n}{5}x^5 + \tbinom{n}{6}x^6 + \tbinom{n}{7}x^7 + \tbinom{n}{8}x^8 \\ &\quad + \tbinom{n}{9}x^9 + \tbinom{n}{10}x^{10} + \tbinom{n}{11}x^{11} + \cdots + \tbinom{n}{n}x^n \\ &= (1+x)^n - 1. \end{aligned}$$

□

Theorem 2.2. *If $G_1 \cong G_2$, then $ED(G_1, x) = ED(G_2, x)$.*

Proof. Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be any two isomorphic graphs. Since $G_1 \cong G_2$, there exists a bijection $f : V_1 \to V_2$ such that v_i and v_j are eccentric vertices in G_1 if and only if $f(v_i)$ and $f(v_j)$ are eccentric vertices in G_2. Hence, there is a one to one correspondence between the eccentric dominating sets of G_1 and the eccentric dominating sets of G_2. Therefore, $ed(G_1, k) = ed(G_2, k)\ \forall\, k$. If $ED(G_1, x)$ and $ED(G_2, x)$ are eccentric domination polynomials of G_1 and G_2 respectively, then $ED(G_1, x) = ED(G_2, x)$. □

Example 2. *In figure 2 (i) we have the following eccentric dominating sets of different cardinalities,*

$\mathcal{E}d(G_1, 1) = \emptyset \implies ed(G_1, 1) = 0$

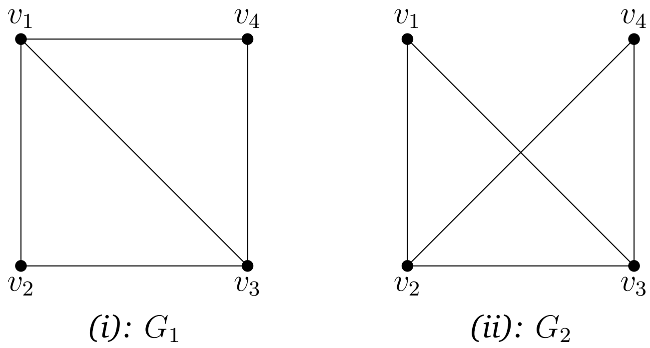

FIGURE 2. Isomorphic graphs

$\{v_1, v_2\}, \{v_1, v_4\}, \{v_2, v_3\}, \{v_2, v_4\}, \{v_3, v_4\} = \mathcal{E}d(G_1, 2) \implies ed(G_1, 2) = 5.$
$\{v_1, v_2, v_3\}, \{v_1, v_2, v_4\}, \{v_1, v_3, v_4\}, \{v_2, v_3, v_4\} = \mathcal{E}d(G_1, 3) \implies ed(G_1, 3) = 4.$
$\{v_1, v_2, v_3, v_4\} = \mathcal{E}d(G_1, 4) = ed(G_1, 4) = 1.$
Therefore

$$ED(G_1, x) = x^4 + 4x^3 + 5x^2 \tag{2.1}$$

Similarly in figure 2 (ii), we have $\mathcal{E}d(G_2, 1) = \emptyset \implies \mathcal{E}d(G_2, 1) = 0.$
$\{v_1, v_2\}, \{v_1, v_3\}, \{v_1, v_4\}, \{v_2, v_4\}, \{v_3, v_4\} \implies \mathcal{E}d(G_2, 2) = ed(G_2, 2) = 5.$
$\{v_1, v_2, v_3\}, \{v_1, v_2, v_4\}, \{v_1, v_3, v_4\}, \{v_2, v_3, v_4\} \implies \mathcal{E}d(G_2, 3) = ed(G_2, 3) = 4.$
$\{v_1, v_2, v_3, v_4\} \implies \mathcal{E}d(G_2, 4) = ed(G_2, 4) = 1.$
Therefore

$$ED(G_2, x) = x^4 + 4x^3 + 5x^2 \tag{2.2}$$

From (2.1) and (2.2) $ED(G_1, x) = ED(G_2, x)$.

Theorem 2.3. *The eccentric domination polynomial of a bipartite graph* $K_{n,n}$ *is given by* $ED(K_{n,n}, x) = [(1 + x)^n - 1]^2$.

Proof. Let $K_{n,n}$ be a complete bipartite graph. Let X and Y be two partite sets of $K_{n,n}$. Let $v \in X, u \in Y$ every vertex of $X - \{v\}$ is an eccentric vertex of v and always the eccentricity is two. Similarly, for every vertex $u \in Y$, all the vertices in $Y - \{u\}$ are the eccentric vertices and there exists no eccentric dominating set D such that $D \in X$ or $D \in Y$. There has to be atleast one vertex from both the sets to form an eccentric dominating set.

$$ED(K_{n,n},x)=\sum_{K=\gamma_{ed}(G)}^{|V(K_{n,n})|} ed(K_{n,n},k)x^k=[(1+x)^n-1]^2.$$

□

3. Eccentric domination polynomial of star graph

In this section, we discuss the eccentric domination polynomial of star graph, we also study the properties of eccentric domination polynomial of star graphs.

Observation 3.1. *In a star graph S_n*

(i) $|\mathcal{E}d(S_1,1)|=ed(S_1,1)=1$,
(ii) $|\mathcal{E}d(S_2,1)|=ed(S_2,1)=2$,
(iii) $|\mathcal{E}d(S_2,2)|=ed(S_2,2)=1$.

Theorem 3.1. *There exist no eccentric dominating set of cardinality one for a star graph S_n i.e, $|\mathcal{E}d(S_n,1)|=0$ for $n>2$.*

Proof. Let S_n be a star graph with vertices $\{v_1,v_2,\ldots,v_n\}=V$. Let us assume v_1 is the central vertex of star graph S_n then the central vertex dominates all the other vertices in the star graph, $\gamma(S_n)=1$. But the eccentric vertex of a central vertex is $V-\{v_1\}$ and the eccentric vertices of the pendent vertices are all the vertices in the graph excluding the central vertex and itself. Hence the central vertex alone forms the dominating set which is not an eccentric dominating set. Therefore the eccentric dominating set contains atleast two vertices. Hence $|\mathcal{E}d(S_n,1)|=0,\,\forall\,n>2$. □

Theorem 3.2. *For a star graph S_n, $|\mathcal{E}d(S_n,k)|={}^{n-1}C_{k-1}+{}^{n-1}C_k$ for $n=3,4$ and $K\leq n$.*

Proof. Let $\{v_1,v_2,v_3\}$ be the vertices of the star graph S_3, from Theorem 3.1, we know that there exists no eccentric dominating set of cardinality one. Therefore $\mathcal{E}d(S_3,1)=0$ and $\forall\ n-1$ we have n combinations. It is obvious to have one eccentric dominating set of cardinality n. Similar proof follows for $n=4$. □

Theorem 3.3. *For a star graph S_n where $n > 4$,*

$$|\mathcal{E}(S_n,k)| = \begin{cases} |\mathcal{E}d(S_{n-1},k)| + 1, \; k = 2 \\ |\mathcal{E}d(S_{n-1},k-1)| + |\mathcal{E}d(S_{n-1},k)| - 1, \; k = n-2 \\ |\mathcal{E}d(S_{n-1},k-1)| + |\mathcal{E}d(S_{n-1},k)|, \; otherwise\,. \end{cases}$$

Proof.

Case(i): If $k = 2$, let $\{v_1, v_2, v_3, v_4, \dots, v_n\}$ be the vertex set of the star graph S_n. Let v_1 be the central vertex of star graph and all others are pendent vertices. Then $E(v_1) = \{v_2, v_3, v_4, \dots, v_n\}$ is an eccentric set of vertices of v_1 and $e(v_1) = 1$. Therefore $\{v_1\}$ is a dominating set but not a eccentric dominating set. The possible number of eccentric dominating sets of cardinality two in S_5 is $|\mathcal{E}d(S_5,2)| = |\mathcal{E}d\,(S_4,2)| + 1 = 4$. The possible number of eccentric dominating sets of cardinality two in S_6 is $|\mathcal{E}d(S_6,2)| = |\mathcal{E}d\,(S_5,2)|+1 = 5$. Proceeding like this the possible number of eccentric dominating set of cardinality two in S_{n-1} is $|\mathcal{E}d(S_{n-1},2)| = |\mathcal{E}d\,(S_{n-2},2)|+1 = n-2$ and the possible number of eccentric dominating set of cardinality two in S_n is $|\mathcal{E}d(S_n,2)| = |\mathcal{E}d\,(S_{n-1},2)| + 1 = n-1$.

Case(ii): If $k = n-2$, for a star S_5, the possible number of eccentric dominating set of cardinality three in S_5 is $|\mathcal{E}d(S_5,3)| = |\mathcal{E}d\,(S_4,2)|+|\mathcal{E}d\,(S_4,3)|-1 = 3+4-1 = 6$. The possible number of eccentric dominating set of cardinality four in S_6 is $|\mathcal{E}d(S_6,4)| = |\mathcal{E}d\,(S_5,3)| + |\mathcal{E}d\,(S_5,4)| - 1 = 6+5-1 = 10$. Similarly, for a star S_{n-1}, the possible number of eccentric dominating set of cardinality $n-3$ in S_{n-1} is $|\mathcal{E}d(S_{n-1},n-3)| = |\mathcal{E}d\,(S_{n-2},n-2)| + |\mathcal{E}d\,(S_{n-2},n-3)| - 1$. Proceeding like this the possible number of eccentric dominating set of cardinality $n-2$ in S_n is $|\mathcal{E}d(S_n,n-2)| = |\mathcal{E}d\,(S_{n-1},n-1)| + |\mathcal{E}d\,(S_{n-1},n-2)| - 1$. Since $k = n-2$, we obtain $|\mathcal{E}d(S_n,k)| = |\mathcal{E}d\,(S_{n-1},k-1)| + |\mathcal{E}d\,(S_{n-1},k)| - 1$.

Case(iii): If $k = 3, 4, \dots, n-3, n-1, n$, here we have $\binom{n-2}{k-2}$ eccentric dominating sets of cardinality k. Therefore $|\mathcal{E}d(S_n,k)| = \binom{n-2}{k-2}$. $|\mathcal{E}d(S_{n-1},k-1)| = \binom{n-3}{k-3}$. Similarly $|\mathcal{E}d(S_{n-1},k)| = \binom{n-3}{k-2}$. Then we have, $\binom{n-2}{k-2} = \binom{n-3}{k-3} + \binom{n-3}{k-2}$.

Therefore $|\mathcal{E}d(S_n,k)| = |\mathcal{E}d(S_{n-1},k-1)| + |\mathcal{E}d(S_{n-1},k)|$. □

Theorem 3.4. *For a star graph $n \geq 4$,*

$$ED(S_n,x) = x\,ED(S_n,x) + ED(S_n,x) - x^{n-2} + x^2\,.$$

Proof. We prove the following theorem by taking the summation of eccentric dominating sets of every possible cardinality.
When $k=2$, $|\mathcal{E}d(S_n,2)| = |\mathcal{E}d(S_{n-1},2)|+1 \implies x^2\, ed(S_n,2) = x^2\, ed(S_{n-1},2)+x^2$.
By the Theorem 3.3 we have $|\mathcal{E}d(S_n,k)| = |\mathcal{E}d(S_{n-1},k-1)| + |\mathcal{E}d(S_{n-1},k)|$.
When $k=3$, $|\mathcal{E}d(S_n,3)| = |\mathcal{E}d(S_{n-1},2)| + |\mathcal{E}d(S_{n-1},3)|$
$\implies x^3\, ed(S_n,3) = x^3\, ed(S_{n-1},2) + x^3\, ed(S_{n-1},3)$
When $k=n-2$, by Theorem 3.3 $|\mathcal{E}d(S_n,n)| = |\mathcal{E}d(S_{n-1},k-1)|+|\mathcal{E}d(S_{n-1},k)|-1$.
$\implies |\mathcal{E}d(S_n,n-2)| = |\mathcal{E}d(S_{n-1},n-3)| + |\mathcal{E}d(S_{n-1},n-2)| - 1$.
$\implies x^{n-2}\, ed(S_n,n-2) = x^{n-2}\, ed(S_{n-1},n-3) + x^{n-2}\, ed(S_{n-1},n-2) - x^{n-2}$.
When $k=n-1$, $|\mathcal{E}d(S_n,n-1)| = |\mathcal{E}d(S_{n-1},n-2)| + |\mathcal{E}d(S_{n-1},n-1)|$.
$\implies x^{n-1}\, ed(S_n,n-1) = x^{n-1}\, ed(S_{n-1},n-2) + x^{n-1}\, ed(S_{n-1},n-1)$.
When $k=n$, $|\mathcal{E}d(S_n,n)| = |\mathcal{E}d(S_n,n-1)| + |\mathcal{E}d(S_n,n)|$.
$\implies x^n\, ed(S_n,n) = x^n\, ed(S_n,n-1) + x^n\, ed(S_n,n)$.

$$
\begin{aligned}
&x^2\, ed(S_n,2) + x^3\, ed(S_n,3) + \cdots + x^{n-2}\, ed(S_n,n-2) + \\
&x^{n-1}\, ed(S_n,n-1) + x^n\, ed(S_n,n) = \\
&x^2\, ed(S_{n-1},2) + x^2 + x^3\, ed(S_{n-1},2) + x^3\, ed(S_{n-1},3) + x^4\, ed(S_{n-1},3) + \\
&x^4\, ed(S_{n-1},4) + \cdots + x^{n-2}\, ed(S_{n-1},n-3) + x^{n-2}\, ed(S_{n-1}n-2) - x^{n-2} + \\
&x^{n-1}\, ed(S_{n-1},n-2) + x^{n-1}\, ed(S_{n-1},n-1) + \\
&x^n\, ed(S_{n-1},n-1) + x^n\, ed(S_{n-1},n)
\end{aligned}
$$
(3.1)

By rearranging the terms of equation(3.1),
$x^2\, ed(S_n,2) + x^3\, ed(S_n,3) + \cdots + x^{n-2}\, ed(S_n,n-2) + x^{n-1}\, ed(S_n,n-1) + x^n\, ed(S_n,n) = x[x^2\, ed(S_{n-1},2) + x^3\, ed(S_{n-1},3) + \cdots + x^{n-2}\, ed(S_{n-1},n-2) + x^{n-1}\, ed(S_{n-1},n-1) + x^n\, ed(S_{n-1},n)] + [x^2\, ed(S_{n-1},2) + x^3\, ed(S_{n-1},3) + \cdots + x^{n-2}\, ed(S_{n-1},n-2) + x^{n-1}\, ed(S_{n-1},n-1) + x^n\, ed(S_{n-1},n)] + x^2 - x^{n-2}$.
Since, $ed(S_{n-1},1) = ed(S_{n-1},n) = 0$, we get
$\sum\limits_{k=2}^{n} ed(S_n,k)x^k = x \sum\limits_{k=2}^{n} ed(S_{n-1},k)x^k + \sum\limits_{k=2}^{n} ed(S_{n-1},k)x^k - x^{n-2} + x^2$.
$\implies ED(S_n,x) = x\, ED(S_n,x) + ED(S_n,x) - x^{n-2} + x^2$.
Using the Theorem 3.1, Theorem 3.2 and Theorem 3.3 we get $\mathcal{E}d(S_n,k)$ for $2 < n \leq 15$ as shown in the table below.

□

TABLE 1. Construction of coefficients of eccentric domination polynomial of S_n

n \ k	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	1													
2	2	1												
3	0	3	1											
4	0	3	4	1										
5	0	4	6	5	1									
6	0	5	10	10	6	1								
7	0	6	15	20	15	7	1							
8	0	7	21	35	35	21	8	1						
9	0	8	28	56	70	56	28	9	1					
10	0	9	36	84	126	126	84	36	10	1				
11	0	10	45	120	210	252	210	120	45	11	1			
12	0	11	55	165	330	462	462	330	165	55	12	1		
13	0	12	66	220	495	702	924	792	495	220	66	13	1	
14	0	13	78	286	715	1287	1716	1716	1287	715	286	78	14	1

Theorem 3.5. *The following properties for the coefficients of $ED(S_n, x)$ hold.*

(i) $ed(S_n, n) = 1, \ \forall n \geq 2$.

(ii) $ed(S_n, n-1) = n, \ \forall n \geq 2$.

(iii) $ed(S_n, 1) = 0, \ \forall n > 2$.

(iv) $ed(S_n, n-3) = \dfrac{(n-1)(n-2)(n-3)}{6}, \ \forall n \geq 5$.

(v) $ed(S_n, n-4) = \dfrac{(n-1)(n-2)(n-3)(n-4)}{24}, \ \forall n \geq 6$.

(vi) $\sum\limits_{k=2}^{n} ed(S_n, k) = 2\left[\sum\limits_{k=2}^{n-1} ed(S_{n-1}, k)\right] \ \forall n \geq 4$.

(vii) *Total number of eccentric dominating sets in S_n is 2^{n-1} $\forall n \geq 3$.*

Proof.

(i) The whole vertex set of a graph G is an eccentric dominating set. Therefore $ed(S_n, n) = 1 \ \forall n \geq 2$.

(ii) Every set of cardinality $n-1$ has a singleton set in its complement. The eccentric vertex of the singleton vertex lies in the set of cardinality n-1. Therefore it must be an eccentric dominating set and there are n

combinations of eccentric dominating sets with cardinality $k = |n-1|$. Therefore $ed(S_n, n-1) = n \; \forall \, n \geq 2$.

(iii) The proof follows from the Theorem 3.1.

(iv) By induction on n. The result is true for $n = 5$. Since $ed(S_5, 2) = 4$. Assume the result is true for all natural numbers less than n. Now we prove it for n. By the theorem,

$$\begin{aligned}
ed(S_n, n-3) &= ed(S_{n-1}, n-4) + ed(S_{n-1}, n-3). \\
&= \frac{(n-2)(n-3)(n-4)}{6} + \frac{(n-2)(n-3)}{2} \\
&= \frac{(n-2)(n-3)(n-4) + 3(n-2)(n-3)}{6} \\
&= \frac{(n-2)(n-3)[n-4+3]}{6} \\
&= \frac{(n-1)(n-2)(n-3)}{6}.
\end{aligned}$$

The result is true for all n.

(v) By induction on n. The result is true for $n = 6$, since $ed(S_6, 2) = 5$. Assume that the result is true for all natural numbers less than n. Now we prove it for n. By the theorem

$$\begin{aligned}
ed(S_n, n-4) &= ed(S_{n-1}, n-5) + ed(S_{n-1}, n-4) \\
&= \frac{(n-2)(n-3)(n-4)(n-5)}{24} + \frac{(n-2)(n-3)(n-4)}{6} \\
&= \frac{(n-2)(n-3)(n-4)[(n-5)-4]}{24} \\
&= \frac{(n-1)(n-2)(n-3)(n-4)}{24}
\end{aligned}$$

The result is true for all n.

(vi) From Theorem 3.3, we have

$$\begin{aligned}
|\mathcal{E}d(S_n, n)| &= |\mathcal{E}d(S_{n-1}, n-1)| + |\mathcal{E}d(S_{n-1}, n)| \\
\implies ed(S_n, n) &= ed(S_{n-1}, n-1) + ed(S_{n-1}, n) \\
\sum_{k=2}^{n} ed(S_n, k) &= \sum_{k=2}^{n} ed(S_{n-1}, k-1) + \sum_{k=2}^{n-1} ed(S_{n-1}, k) \\
&= \sum_{k=2}^{n-1} ed(S_{n-1}, k) + \sum_{k=2}^{n-1} ed(S_{n-1}, k) \\
\sum_{k=2}^{n} ed(S_n, k) &= 2\left(\sum_{k=2}^{n-1} ed(S_{n-1}, k)\right)
\end{aligned}$$

(vii) By induction on n. When $n = 3$, $\sum\limits_{k=2}^{3} ed(S_3, k) = 2^{3-1} = 2^2 = 4$. Therefore this is true for $n = 3$. Let us assume, this result is true for all natural numbers less than n. Similarly, when $n = n-1$, $\sum\limits_{k=2}^{n-1} ed(S_{n-1}k) = 2^{n-1-1} = 2^{n-2}$. Proceeding like this for n, we get $\sum\limits_{k=2}^{n} ed(S_n, k) = 2^{n-1}$. Therefore total number of eccentric dominating sets in S_n is $2^{n-1}\ \forall n \geq 3$.

□

Theorem 3.6. *The eccentric dominating polynomial of a star graph is given by*

$$ED(S_n, x) = x(1+x)^{n-1} + x^{n-1} - x\,.$$

Proof. The theorem is a direct consequence of Theorem 3.1, Theorem 3.2, Theorem 3.3 and Theorem 3.4. □

Example 3. *Let S_n be the star graph then $ED(S_n, x) = x(x+1)^{n-1} + x^{n-1} - x$. For $n = 7$,*

$$\begin{aligned}
ED(S_7, x) &= x(x+1)^{7-1} + x^{7-1} - x. \\
&= x(x+1)^6 + x^6 - x. \\
&= x(x^6 + 6x^5 + 15x^4 + 20x^3 + 15x^2 + 6x + 1) + x^6 - x. \\
&= x^7 + 6x^6 + 15x^5 + 20x^4 + 15x^3 + 6x^2 + x + x^6 - x. \\
&= x^7 + 7x^6 + 15x^5 + 20x^4 + 15x^3 + 6x^2.
\end{aligned}$$

Refer the coefficients of $ED(S_7, x)$ in the table 1.

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