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ECO- CONSCIOUS CUSTOMER CENTRIC INVENTORY MODEL WITH FRACTIONAL ORDER APPROACH

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ABSTRACT. Eco friendly inventory models are gaining momentum as the human race is marching towards environmental conservation by battling against waste generation and pollutants. As the global market also revolves with customer as its prime focus, the product production becomes customer centered. To set a balance between eco - consciousness and customer centeredness the environmental costing and customer acquisition costing is incorporated with the costs of inventory. Environmental oriented inventory models are being formulated by the researchers with different solution procedure but less focus of customer centric inventory model prevails. This stimulated us to frame inventory models by integrating the facets of eco consciousness and customer centric with the goal of profit maximization and costs minimization using fractional calculus approach as the application of fractional calculus in optimizing the inventory models is being explored by several scholars and this research work is a contribution to it. This model will aid the decision makers at managerial level.

1. INTRODUCTION

In term of business, inventory is a source of products like material, staff, budget for the project. Product which are used once by the customer, concentrate only on the prime product not the waste generate by them (after usage). To conserve the environment that hostility human race producing pollution, can be collected and make useful product again in the market.i.e Reverse logistics. It's

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a method of rehashing the essential item for the product. Reverse logistics is used to reprocess the production supply and component to recover the product. As an organization it provides solutions for the customers according to their problems in terms of products satisfying their demands. To fulfil the requirements of the customers, their expectations have to be given life for which the customers have to be approached with new and innovative ideasto build relationships. Offering new services to customer made them stay tuned in the same organization. Both customer and customer centric organizations are benefited with profit categories like product, service provided for complex problem to customer satisfaction, and goal achievement (target value) of the organization. An organization has to minimize the costs by maximizing the profit. The customer's needs are instantaneously accomplished by balancing the surplus and shortage conditions of inventory. To make optimal decisions of inventory levels, mathematical modelling of the business scenario is very essential with certain prerequisites.

Economic Order Quantity (EOQ) is the ideal order quantity a management should purchase to minimize inventory costs such as ordering costs, holding costs and purchase costs. This inventory model was developed in 1913 by W.Harris and has been refined overtime. The classical EOQ determines the company's optimal order quantity that minimizes its total costs related to ordering, receiving and holding inventory. The classical EOQ formula is best applied in situations where demand, ordering, and holding costs remain over time. This is an important cash flow tool. If classical EOQ helps to minimize the level of inventory, the cash savings can be used for some other business purpose.

Environmental oriented inventory models are being formulated by the researchers with different solution procedure and less focus on customer centric inventory model prevails. In this paper the conventional model is extended to eco-conscious customer centric model using fractional calculus with the incorporation of the related costs to bear out the contemporary requisites of the customers. The ultimate aim of this inventory model is to integrate the facets of eco consciousness and customer centric with the goal of profit maximization and costs minimization.

Nivetha and W.Ritha [6] introduced inventory models based on Maurice Bonney's formulation of socially responsible inventory models. An eco-friendly inventory model with the inclusion of incineration as waste disposal method was discussed by them. Customer centric inventory models were also formulated by them, but these models are deterministic and the solution procedure was analytical in nature. As an extension, the solution procedure is modified with the application of fractional calculus. Asim Kumar Das and Tapan Kumar Roy [1] introduced a generalized economic order quantity and economic production quantity inventory model with the application of fractional calculus. A detailed description of the fractional calculus approach is also presented by them. The solution to the generalized inventory model is constituted by using geometric programming technique. Based on this fractional calculus approach, an eco-conscious customer centric inventory model is formulated catering to the needs of the customers and the stainability of the environment. For more details, see for instance [2–5, 7–9].

2. Model Development

The following notations are used Notations and Assumptions:

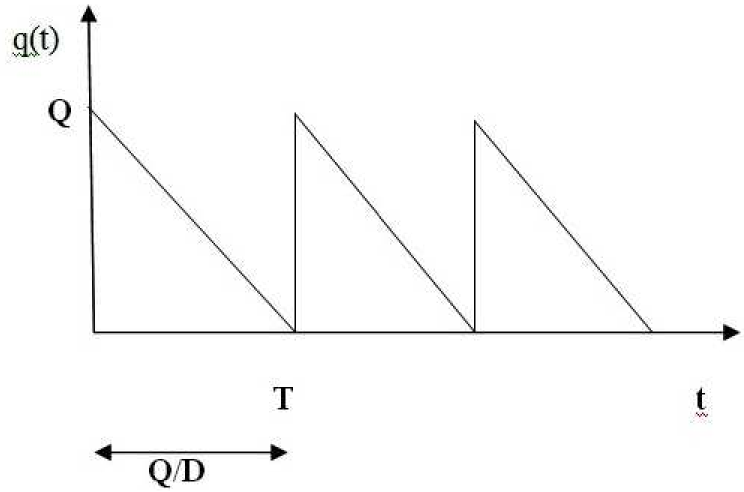
D - Demand rate.
Q - Order quantity.
U - Per unit cost.
C_1 - Holding cost per unit.
E_c - environmental cost per unit.
M_c - Marketing consultation costs.
M_w - Management of website.
E_n - Establishing networks.
M_t - Marketing automation tools.
A_σ - Analytics.
P_e - Product exposure costs.
P_r - Product and customer relation building costs.
C_3 - Set up cost.
$q(t)$ - Stock level.
T - Ordering interval.

2.1. **Eco-Conscious Customer Centric Inventory Model.** The formulation of eco-conscious customer centric inventory model is as follows based on Asim

Kumar Das and Tapan Kumar Roy.

(2.1) $$\frac{dq(t)}{dt} = -D, \quad \text{for} \quad 0 \le t \le T = 0, \quad \text{otherwise}.$$

With the initial condition $q(0) = Q$ and with the boundary condition $q(T) = 0$.



By solving the equation (2.1), we have

$$q(t) = Q - DT, \quad \text{for} \quad 0 \le t \le T.$$

And on using the boundary condition $q(T) = 0$, we have

$$Q = DT.$$

Holding cost,

$$HC(T) = C_1 \int_{t=0}^{T} q(t)dt = C_1 \int_{t=0}^{T} (Q - Dt)dt = C_1 \Big[QT - \frac{Dt^2}{2}\Big] T_{t=0} = \frac{C_1 DT^2}{2}.$$

Total cost,

$$\begin{aligned} TC(T) &= \text{Purchasing cost} + \text{Holding cost} + \text{Set up Cost} \\ &= UQ + \frac{C_1 DT^2}{2} + C_3 + M_c + M_w + E_n + M_t + A_\sigma + P_e + P_r + E_c. \end{aligned}$$

Total average cost over $[0, T]$ is given by

$$\begin{aligned} TAC(T) &= \frac{1}{T}\Big[UQ + \frac{C_1 DT^2}{2} + C_3\Big] \\ &= \frac{UQ}{T} + \frac{C_1 DT}{2} + \frac{C_3}{T} + \frac{M_c + M_w + E_n + M_t + A_\sigma + P_e + P_r + E_c}{T}. \end{aligned}$$

Then the eco-conscious customer centric inventory model is

$$\begin{aligned} \min TAC(T) &= UD \\ &+ \frac{C_1 DT}{2} + \frac{C_3}{T} + \frac{M_c + M_w + E_n + M_t + A_\sigma + P_e + P_r + E_c}{T}. \end{aligned} \tag{2.2}$$

Subject to, $T > 0$.

Solving (2.2) we can show that TAC(T) will be minimum for

$$T^* = \sqrt{\frac{2c_3 * M_c * M_w * E_n * M_t * A_\sigma * P_e * P_r * E_c D}{c_1}}$$

$$TAC^*(T^*) = UD + \sqrt{2 * D * c_1 * c_3 * M_c * M_w * E_n * M_t * A_\sigma * P_e * P_r * E_c}$$

Generalized EOQ model

$$\frac{d^\alpha q(t)}{dt^\alpha} = -D \; for \; 0 \leq t \leq T = 0, \; otherwise.$$

$$Q = \frac{DT^\alpha}{\Gamma(\alpha + 1)} .$$

Generalized eco-conscious customer centric inventory model

Now the holding cost of fractional order, say β, i.e,

$$HC_\beta(T) = C_1 D^{-\beta} q(t).$$

Case (1) : For $\alpha = 1$ and $\beta = 1$, holding cost is

$$HC_{1,1}(T) = C_1 D^{-1} q(t) = \frac{C_1 QT}{2} = \frac{C_1 DT^2}{2}.$$

Case (2) : For $\beta = 1$, holding cost is

$$HC_{1,\alpha}(T) = C_1 \int_0^T q_\alpha(t)dt = \frac{c_1 \alpha}{\alpha + 1} QT.$$

Case (3) : For $\alpha = 1$, holding cost of order β is

$$\begin{aligned} HC_{1,\beta}(T) &= C_1 D^{-\beta} q(t) \\ &= C_1 \frac{1}{\Gamma(\beta)} \int_0^t (t - x)^{\beta - 1} q(x) dx \\ &= C_1 DT^{\beta+1} \left(\frac{1}{\Gamma(\beta + 1)} - \frac{1}{\Gamma(\beta + 2)} \right) \\ D^{-\beta} q(t) &= \left(\frac{Qt^\beta}{\Gamma(\beta + 1)} - \frac{Dt^{\beta+1}}{\Gamma(\beta + 2)} \right). \end{aligned}$$

Case (4) : For any α and β, holding cost is

$$\begin{aligned} HC_{\alpha,\beta}(T) &= C_1 D^{-\beta} q_\alpha(t) \\ &= C_1 D T^{\alpha+\beta} \left(\frac{1}{\Gamma(\alpha+1)\Gamma(\beta+1)} - \frac{1}{\Gamma(\alpha+\beta+1)} \right). \end{aligned}$$

Generalized Total Average cost

Case (1): For $\alpha = 1$ and $\beta = 1$, the model is being as our classical EOQ problem where the optimum

Total Average cost

$$TAC^*_{1,1}(T^*) = UD + \sqrt{2 * D * c_1 * c_3 * M_c * M_w * E_n * M_t * A_\sigma * P_e * P_r * E_c}$$

Case(2): For any $\alpha > 0$ and $\beta = 1$,

Total Cost = set up cost +holding cost + purchasing cost +marketing consultation costs + Management of website + Establishing networks + Marketing automation tools + Analytics+ Product exposure costs + Product and customer relation building costs + Environmental cost

Here

$$TC_{\alpha,1}(T) = UQ + \frac{C_1\alpha}{\alpha+1} QT + C_3 + M_c + M_w + E_n + M_t + A_\sigma + P_e + P_r + E_c$$

Total Average Cost

$$\begin{aligned} TAC_{\alpha,1}(T) &= \frac{1}{T}\Big(UQ + \frac{C_1\alpha}{\alpha+1} QT + C_3 + M_c + M_w + E_n + M_t \\ &\quad + A_\sigma + P_e + P_r + E_c \Big) \\ &= \frac{1}{T}\Big(\frac{UQT^\alpha}{\Gamma(\alpha+1)} + \frac{C_1\alpha}{\Gamma(\beta+1)} DT^{\alpha+1} + C_3 + M_c + M_w \\ &\quad + E_n + M_t + A_\sigma + P_e + P_r + E_c \Big) \\ &= \frac{UQT^{\alpha-1}}{\Gamma(\alpha+1)} + \frac{C_1\alpha}{\Gamma(\beta+1)} DT^\alpha + C_3 T^{-1} + M_c T^{-1} + M_w T^{-1} \\ &\quad + E_n T^{-1} + M_t T^{-1} + A_\sigma T^{-1} + P_e T^{-1} + P_r T^{-1} + E_c T^{-1}. \end{aligned}$$

Here generalized EOQ model is

$$\begin{aligned} \min TAC_{\alpha,1}(T) &= AT^{\alpha-1} + BT^\alpha + CT^{-1} + DT^{-1} + ET^{-1} + FT^{-1} \\ &\quad + GT^{-1} + HT^{-1} + IT^{-1} + JT^{-1} + KT^{-1}. \end{aligned}$$

$$A = \frac{UQ}{\Gamma(\alpha+1)}, B = \frac{C_1\alpha}{\Gamma(\beta+1)}D, C = C_3, D = M_c, E = M_w,$$
$$F = E_n, G = M_t, H = A_\sigma, I = P_e, J = P_r, K = E_c.$$

Primal Geometric Programming Method

$$\max d(w) = \left(\frac{A}{w_1}\right)^{w_1}\left(\frac{B}{w_2}\right)^{w_2}\left(\frac{C}{w_3}\right)^{w_3}\left(\frac{D}{w_4}\right)^{w_4}\left(\frac{E}{w_5}\right)^{w_5}$$
$$\left(\frac{F}{w_6}\right)^{w_6}\left(\frac{G}{w_7}\right)^{w_7}\left(\frac{H}{w_8}\right)^{w_8}\left(\frac{I}{w_9}\right)^{w_9}\left(\frac{J}{w_10}\right)^{w_10}\left(\frac{K}{w_11}\right)^{w_11}.$$

Subject to, with the orthogonal and normal conditions

$$W_1 + W_2 + W_3 + W_4 + W_5 + W_6 + W_7 + W_8 + W_9 + W_{10} + W_{11} = 1$$
$$(\alpha-1)W_1 + \alpha W_2 - W_3 - W_4 - W_5 - W_6 - W_7 - W_8 - W_9 - W_{10} - W_{11} = 0$$
$$W_1, W_2, W_3, W_4, W_5, W_6, W_7, W_8, W_9, W_{10}, W_{11} \geq 0$$

Corresponding primal - dual relations are given below as,

$$AT^{\alpha-1} = W_1 d(w), \quad BT^\alpha = W_2 d(w), \quad \frac{C}{T} = W_3 d(w), \quad \frac{D}{T} = W_4 d(w),$$
$$\frac{E}{T} = W_5 d(w), \quad \frac{F}{T} = W_6 d(w), \quad \frac{G}{T} = W_7 d(w), \quad \frac{H}{T} = W_8 d(w),$$
$$\frac{I}{T} = W_9 d(w), \quad \frac{J}{T} = W_{10} d(w), \quad \frac{K}{T} = W_{11} d(w)$$

We have $\frac{A}{BT} = \frac{W_1}{W_2}, T = \frac{AW_2}{BW_1}$.

$$\left(\frac{AW_2}{BW_1}\right)^{\alpha+1} = \frac{CW_2}{BW_3}, \quad \left(\frac{AW_2}{BW_1}\right)^{\alpha+1} = \frac{DW_2}{BW_4}, \quad \left(\frac{AW_2}{BW_1}\right)^{\alpha+1} = \frac{EW_2}{BW_5},$$
$$\left(\frac{AW_2}{BW_1}\right)^{\alpha+1} = \frac{FW_2}{BW_6}, \quad \left(\frac{AW_2}{BW_1}\right)^{\alpha+1} = \frac{GW_2}{BW_7}, \quad \left(\frac{AW_2}{BW_1}\right)^{\alpha+1} = \frac{HW_2}{BW_8},$$
$$\left(\frac{AW_2}{BW_1}\right)^{\alpha+1} = \frac{IW_2}{BW_9}, \quad \left(\frac{AW_2}{BW_1}\right)^{\alpha+1} = \frac{JW_2}{BW_10}, \quad \left(\frac{AW_2}{BW_1}\right)^{\alpha+1} = \frac{KW_2}{BW_11}$$

along with $T = \frac{AW_2}{BW_1}$.

There are eleven non-linear equations, and the nine equations with eleven unknown $W_1, W_2, W_3, W_4, W_5, W_6, W_7, W_8, W_9, W_{10}, W_{11}$. Solving these eleven nonlinear equations.

We shall get the optimal values W_1^*, W_2^*, W_3^*, W_4^*, W_5^*, W_6^*, W_7^*, W_8^*, W_9^*, W_{10}^*, W_{11}^* and hence optimal ordering interval $T_{\alpha,\beta}^*$ and minimized total average cost can be obtained by substituting W_1^*, W_2^* and then $TOC_{\alpha,\beta}$.

In this case $\alpha > 0, \beta = 1$.

The total average cost is as,

$$TAC_{\alpha,1}(T) = \frac{UDT^{\alpha-1}}{\Gamma(\alpha+1)} + \frac{C_1\alpha}{\Gamma(\beta+1)}DT^{\alpha} + C_3T^{-1} + M_cT^{-1} + M_wT^{-1} + E_nT^{-1} + M_tT^{-1} + A_\sigma T^{-1} + P_eT^{-1} + P_rT^{-1} + E_cT^{-1}.$$

Therefore, the fractional order inventory model

$$\min TAC_{\alpha,1}(T) = AT^{\alpha-1} + BT^{\alpha} + CT^{-1} + DT^{-1} + ET^{-1} + FT^{-1} + GT^{-1} + HT^{-1} + IT^{-1} + JT^{-1} + KT^{-1}$$

$$A = \frac{UD}{\Gamma(\alpha+1)}, B = \frac{C_1\alpha}{\Gamma(\beta)}D, C = C_3, D = M_c, E = M_w,$$
$$F = E_n, G = M_t, H = A_\sigma, I = P_e, J = P_r, K = E_c.$$

Primal geometric programming algoritham can provide the minimized total average cost $TAC^*_{\alpha,1}(T)$ and optimal ordering interval $T^*_{\alpha,1}$. Interesting to note that the analytical results of this model coincides with the results of our classical model where $\alpha > 0, \beta = 1$.

Case(3): For $\alpha = 1$ and for any β, we have the holding cost,

$$HC_{1,\beta}(T) = C_1DT^{\beta+1}\Big[\frac{1}{\Gamma(\beta+1)} - \frac{1}{\Gamma(\beta+2)}\Big].$$

The total cost (TC)= set up cost +holding cost + purchasing cost +marketing consultation costs + Management of website + Establishing networks + Marketing automation tools + Analytics+ Product exposure costs + Product and customer relation building costs + Environmental cost $= UQ + C_1DT^{\beta+1}\Big[\frac{1}{\Gamma(\beta+1)} - \frac{1}{\Gamma(\beta+2)}\Big] + C_3 + M_c + M_w + E_n + M_t + A_\sigma + P_e + P_r + E_c$.

Total Average Cost

$$\begin{aligned} TAC_{1,\beta}(T) &= \frac{1}{T}\Big[UQ + C_1DT^{\beta+1}\Big[\frac{1}{\Gamma(\beta+1)} - \frac{1}{\Gamma(\beta+2)}\Big] + C_3 \\ &\quad + M_c + M_w + E_n + M_t + A_\sigma + P_e + P_r + E_c\Big] \\ &= UD + C_1DT^{\beta}\Big[\frac{1}{\Gamma(\beta+1)} - \frac{1}{\Gamma(\beta+2)}\Big] + \frac{C_3}{T} \\ &\quad + \frac{M_c}{T} + \frac{M_w}{T} + \frac{E_n}{T} + \frac{M_t}{T} + \frac{A_\sigma}{T} + \frac{P_e}{T} + \frac{P_r}{T} + \frac{E_c}{T}. \end{aligned}$$

Since for $\alpha = 1$, we know that $Q = DT$

$$= A + BT^{\beta} + \frac{C}{T} + \frac{D}{T} + \frac{E}{T} + \frac{F}{T} + \frac{G}{T} + \frac{H}{T} + \frac{I}{T} + \frac{J}{T} + \frac{K}{T},$$

where

$$A = UD, \quad B = C_1 D\Big[\frac{1}{\Gamma(\beta+1)} - \frac{1}{\Gamma(\beta+2)}\Big], \quad C = C_3, \quad D = M_c,$$

$$E = M_w, \quad F = E_n, \quad G = M_t \quad H = A_{\sigma}, \quad I = P_e, \quad J = P_r, \quad K = E_c.$$

To minimize $TAC_{1,\beta}(T)$, Primal Geometric Programming Method.
Let us suppose that $M(T) = BT^{\beta} + \frac{C}{T}$.

$$\max d(w) = \left(\frac{B}{w_1}\right)^{w_1} \left(\frac{C}{w_2}\right)^{w_2} \left(\frac{D}{w_3}\right)^{w_3} \left(\frac{E}{w_4}\right)^{w_4} \left(\frac{F}{w_5}\right)^{w_5}$$
$$\left(\frac{G}{w_6}\right)^{w_6} \left(\frac{H}{w_7}\right)^{w_7} \left(\frac{I}{w_8}\right)^{w_8} \left(\frac{J}{w_9}\right)^{w_9} \left(\frac{K}{w_{10}}\right)^{w_{10}}.$$

Subject to, with the orthogonal and normal conditions

$$W_1 + W_2 + W_3 + W_4 + W_5 + W_6 + W_7 + W_8 + W_9 + W_{10} = 1,$$

$$\beta W_1 - W_2 - W_3 - W_4 - W_5 - W_6 - W_7 - W_8 - W_9 - W_{10} = 0,$$

$$W_1, W_2, W_3, W_4, W_5, W_6, W_7, W_8, W_9, W_{10} \geq 0.$$

Then solving for W_1, We get $W_1 = \frac{1}{\beta+1}$.
Corresponding primal - dual relations are given below as,

$$\begin{aligned}
BT^{\beta} &= W_1 d(w), \quad \frac{C}{T} = W_2 d(w), \quad \frac{D}{T} = W_3 d(w), \quad \frac{E}{T} = W_4 d(w),\\
\frac{F}{T} &= W_5 d(w), \quad \frac{G}{T} = W_6 d(w), \quad \frac{H}{T} = W_7 d(w), \quad \frac{I}{T} = W_8 d(w),\\
\frac{J}{T} &= W_9 d(w), \quad \frac{K}{T} = W_{10} d(w)\\
T &= \left(\frac{CW_1}{BW_2}\right)^{\frac{1}{\beta+1}}, \quad T = \left(\frac{DW_1}{BW_2}\right)^{\frac{1}{\beta+1}}, \quad T = \left(\frac{EW_1}{BW_2}\right)^{\frac{1}{\beta+1}},\\
T &= T = \left(\frac{FW_1}{BW_2}\right)^{\frac{1}{\beta+1}}, \quad \left(\frac{GW_1}{BW_2}\right)^{\frac{1}{\beta+1}}, \quad T = \left(\frac{HW_1}{BW_2}\right)^{\frac{1}{\beta+1}},\\
T &= T = \left(\frac{IW_1}{BW_2}\right)^{\frac{1}{\beta+1}} quad \left(\frac{JW_1}{BW_2}\right)^{\frac{1}{\beta+1}}, \quad T = \left(\frac{KW_1}{BW_2}\right)^{\frac{1}{\beta+1}}.
\end{aligned}$$

There are Ten non-linear equations, and the nine equations with eleven unknown $W_1, W_2, W_3, W_4, W_5, W_6, W_7, W_8, W_9, W_{10}$. Solving these eleven nonlinear equations.

We shall get the optimal values $W_1^*, W_2^*, W_3^*, W_4^*, W_5^*, W_6^*, W_7^*, W_8^*, W_9^*, W_{10}^*$, and

Hence optimal ordering interval $T_{1,\beta}^*$ and minimized total average cost can be obtained by substituting W_1^*, W_2^* and then $TAC_{1,\beta}$.

In this case $\alpha = 1$, and for any β.

The total average cost is as,

$$TAC_{1,\beta}(T) = UD + C_1DT^{\beta}\Big[\frac{1}{\Gamma(\beta+1)} - \frac{1}{\Gamma(\beta+2)}\Big] + \frac{C_3}{T} + \frac{M_c}{T} + \frac{M_w}{T} + \frac{E_n}{T} + \frac{M_t}{T} + \frac{A_\sigma}{T} + \frac{P_e}{T} + \frac{P_r}{T} + \frac{E_c}{T}.$$

Therefore, the fractional order inventory model

$$\min TAC_{1,\beta}(T) = A + BT^{\beta} + \frac{C}{T} + \frac{D}{T} + \frac{E}{T} + \frac{F}{T} + \frac{G}{T} + \frac{H}{T} + \frac{I}{T} + \frac{J}{T} + \frac{K}{T}.$$

Subject to $T \geq 0$, where

$$A = UD, \quad B = C_1D\Big[\frac{1}{\Gamma(\beta+1)} - \frac{1}{\Gamma(\beta+2)}\Big], \quad C = C_3, \quad D = M_c,$$
$$E = M_w, \quad F = E_n, \quad G = M_t \quad H = A_\sigma, \quad I = P_e, \quad J = P_r, \quad K = E_c.$$

Primal geometric programming algoritham can provide the minimized total average cost $TAC_{1,\beta}(T)$ and optimal ordering interval $T_{1,\beta}^*$. Interesting to note that the analytical results of this model coincides with the results of our classical model where $\alpha = 1$, and for any β.

Case(4): For any $\alpha > 0$ and any $\beta > 0$.

Holding cost = $HC_{\alpha,\beta}(T) = C_1DT^{\alpha+\beta}\left(\frac{1}{\Gamma(\alpha+1)\Gamma(\beta+1)} - \frac{1}{\Gamma(\alpha+\beta+1)}\right)$.

Total Cost = set up cost +holding cost + purchasing cost +marketing consultation costs + Management of website + Establishing networks + Marketing automation tools + Analytics+ Product exposure costs + Product and customer relation building costs + Environmental cost

$$= UQ + C_1DT^{\alpha+\beta}\Big[\frac{1}{\Gamma(\alpha+1)\Gamma(\beta+1)} - \frac{1}{\Gamma(\alpha+\beta+1)}\Big] + C_3 + M_c + M_w + E_n + M_t + A_\sigma + P_e + P_r + E_c$$

$$\begin{aligned}TAC_{1,\beta}(T) &= \frac{1}{T}\Big[UQ + C_1DT^{\alpha+\beta}\Big[\frac{1}{\Gamma(\alpha+1)\Gamma(\beta+1)} - \frac{1}{\Gamma(\alpha+\beta+1)}\Big] + C_3 \\ &\quad + M_c + M_w + E_n + M_t + A_\sigma + P_e + P_r + E_c\Big] \\ &= \frac{UQ}{T} + \frac{C_1DT^{\alpha+\beta}}{T}\Big[\frac{1}{\Gamma(\alpha+1)\Gamma(\beta+1)} - \frac{1}{\Gamma(\alpha+\beta+1)}\Big] + \frac{C_3}{T} \\ &\quad + \frac{M_c}{T} + \frac{M_w}{T} + \frac{E_n}{T} + \frac{M_t}{T} + \frac{A_\sigma}{T} + \frac{P_e}{T} + \frac{P_r}{T} + \frac{E_c}{T} \\ &= \frac{UQ}{\Gamma(\alpha+1)}T^{\alpha-1} + C_1D\Big[\frac{1}{\Gamma(\alpha+1)\Gamma(\beta+1)} - \frac{1}{\Gamma(\alpha+\beta+1)}\Big]T^{\alpha+\beta-1} \\ &\quad + \frac{C_3}{T} + \frac{M_c}{T} + \frac{M_w}{T} + \frac{E_n}{T} + \frac{M_t}{T} + \frac{A_\sigma}{T} + \frac{P_e}{T} + \frac{P_r}{T} + \frac{E_c}{T} \\ &\qquad\qquad \left(Q = \frac{DT^\alpha}{\Gamma(\alpha+1)}\right) \\ &= AT^{\alpha-1} + BT^{\alpha+\beta-1} + \frac{C}{T} + \frac{D}{T} + \frac{E}{T} + \frac{F}{T} + \frac{G}{T} + \frac{H}{T} + \frac{I}{T} + \frac{J}{T} + \frac{K}{T}\end{aligned}$$

$$A = \frac{UQ}{\Gamma(\alpha+1)}, B = C_1D\Big[\frac{1}{\Gamma(\alpha+1)\Gamma(\beta+1)} - \frac{1}{\Gamma(\alpha+\beta+1)}\Big], C = C_3, D = M_c,$$
$$E = M_w, \quad F = E_n, \quad G = M_t \quad H = A_\sigma, \quad I = P_e, \quad J = P_r, \quad K = E_c$$

Primal Geometric Programming Method

$$\max d(w) = \left(\frac{A}{w_1}\right)^{w_1}\left(\frac{B}{w_2}\right)^{w_2}\left(\frac{C}{w_3}\right)^{w_3}\left(\frac{D}{w_4}\right)^{w_4}\left(\frac{E}{w_5}\right)^{w_5}$$
$$\left(\frac{F}{w_6}\right)^{w_6}\left(\frac{G}{w_7}\right)^{w_7}\left(\frac{H}{w_8}\right)^{w_8}\left(\frac{I}{w_9}\right)^{w_9}\left(\frac{J}{w_{10}}\right)^{w_10}\left(\frac{K}{w_{11}}\right)^{w_11}.$$

Subject to, with the orthogonal and normal conditions

$$W_1 + W_2 + W_3 + W_4 + W_5 + W_6 + W_7 + W_8 + W_9 + W_{10} + W_{11} = 1$$

$$(\alpha-1)W_1 + (\alpha+\beta-1)W_2 - W_3 - W_4 - W_5 - W_6 - W_7 - W_8 - W_9 - W_{10} - W_{11} = 0.$$

Corresponding primal - dual relations are given below as,

$$AT^{\alpha-1} = W_1d(w), \quad BT^{\alpha+\beta-1} = W_2d(w), \quad \frac{C}{T} = W_3d(w), \quad \frac{D}{T} = W_4d(w),$$
$$\frac{E}{T} = W_5d(w), \quad \frac{F}{T} = W_6d(w), \quad \frac{G}{T} = W_7d(w), \quad \frac{H}{T} = W_8d(w),$$
$$\frac{I}{T} = W_9d(w), \quad \frac{J}{T} = W_{10}d(w), \quad \frac{K}{T} = W_{11}d(w).$$

With the help of the above primal-dual relations, we obtain

$$\frac{BT^{\alpha+\beta-1}}{AT^{\alpha-1}} = \frac{W_2}{W_1} \quad \text{and} \quad T = \left(\frac{AW_2}{BW_1}\right)^{\frac{1}{\beta}}.$$

We have $\frac{AT^{\alpha-1}}{CT^{-1}} = \frac{W_1}{W_3}$

$$\left(\frac{AW_2}{BW_1}\right)^{\alpha} = \left(\frac{CW_1}{AW_3}\right)^{\beta}, \quad \left(\frac{AW_2}{BW_1}\right)^{\alpha} = \left(\frac{DW_1}{AW_4}\right)^{\beta}, \quad \left(\frac{AW_2}{BW_1}\right)^{\alpha} = \left(\frac{EW_1}{AW_5}\right)^{\beta},$$
$$\left(\frac{AW_2}{BW_1}\right)^{\alpha} = \left(\frac{FW_1}{AW_6}\right)^{\beta}, \quad \left(\frac{AW_2}{BW_1}\right)^{\alpha} = \left(\frac{GW_1}{AW_7}\right)^{\beta}, \quad \left(\frac{AW_2}{BW_1}\right)^{\alpha} = \left(\frac{HW_1}{AW_8}\right)^{\beta},$$
$$\left(\frac{AW_2}{BW_1}\right)^{\alpha} = \left(\frac{IW_1}{AW_9}\right)^{\beta}, \quad \left(\frac{AW_2}{BW_1}\right)^{\alpha} = \left(\frac{JW_1}{AW_{10}}\right)^{\beta}, \quad \left(\frac{AW_2}{BW_1}\right)^{\alpha} = \left(\frac{KW_1}{AW_{11}}\right)^{\beta}$$

along with

$$T = \left(\frac{AW_2}{BW_1}\right)^{\frac{1}{\beta}}.$$

There are eleven non-linear equations, and the nine equations with eleven unknown $W_1, W_2, W_3, W_4, W_5, W_6, W_7, W_8, W_9, W_{10}, W_{11}$. Solving these eleven non-linear equations.

We shall get the optimal values $W_1^*, W_2^*, W_3^*, W_4^*, W_5^*, W_6^*, W_7^*, W_8^*, W_9^*, W_{10}^*, W_{11}^*$.

Hence optimal ordering interval $T^*_{\alpha,\beta}$ and minimized total average cost can be obtained by substituting W_1^*, W_2^* and then $TAC_{\alpha,\beta}$.

In this case $\alpha = 1, \beta = 1.$

The total average cost is as,

$$TAC_{1,1}(T) = \frac{UQ}{\Gamma(2)}T^{(0)} + C_1D\Big[\frac{1}{\Gamma(2)\Gamma(2)} - \frac{1}{\Gamma(3)}\Big]T^{(1)} + \frac{C_3}{T}$$
$$+ \frac{M_c}{T} + \frac{M_w}{T} + \frac{E_n}{T} + \frac{M_t}{T} + \frac{A_\sigma}{T} + \frac{P_e}{T} + \frac{P_r}{T} + \frac{E_c}{T}.$$

Therefore, the fractional order inventory model

$$\min TAC_{1,1}(T) = AT^{(0)} + BT^{(1)} + \frac{C}{T} + \frac{D}{T} + \frac{E}{T} + \frac{F}{T} + \frac{G}{T} + \frac{H}{T} + \frac{I}{T} + \frac{J}{T} + \frac{K}{T}.$$

Subject to $T \geq 0$, where

$$A = \frac{UQ}{\Gamma(2)}, B = C_1D\Big[\frac{1}{\Gamma(2)\Gamma(2)} - \frac{1}{\Gamma(3)}\Big], C = C_3, D = M_c,$$
$$E = M_w, \quad F = E_n, \quad G = M_t \quad H = A_\sigma, \quad I = P_e, \quad J = P_r, \quad K = E_c.$$

Primal geometric programming algorithm can provide the minimized total average cost $TAC^*_{1,1}(T)$ and optimal ordering interval $T^*_{1,1}$. The initially formulated eco conscious customer centric model is obtained when $\alpha = 1, \beta = 1$.

3. Conclusion

In this paper we have formulated eco conscious customer centric model with the incorporation of the costs related to customer centeredness and environmental sustainability. The proposed model is extended to generalized inventory model and the approach of fractional calculus is applied to enlarge the frontiers of solution procedure. This model will duly address the problems of economic and environmental management. The proposed model can be validated with the numerical examples by collecting primary data from the business firms and the solutions can be obtained after manual or online computations.

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