

**AN EXTENDED TOPSIS METHOD BASED ON GENERALIZED
WEIGHTED DICE SIMILARITY MEASURE AND INTUITIONISTIC
PREFERENCE RELATION WITH INTUITIONISTIC
FUZZY MULTI ATTRIBUTE DECISION MAKING**

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ABSTRACT. The intention of this paper is to utilize the weighted generalized dice similarity measure in TOPSIS method to frame a new algorithm. In order to determine the weight vector, an Intuitionistic preference relation is included in the new algorithm because the concept, weight of the attribute is indispensable to evaluate the alternatives. Too intrinsic the idea of the proposed algorithm a real life problem called prioritizing the branded printers is analyzed and finally arranged/ranked in the non-increasing order.

1. INTRODUCTION

In decision theory, multi-attribute decision making (MADM) plays one of the significant roles. The performance of MADM is to give an optimal solution according to the decision maker (s) preference by valuating and ranking the alternative with respect to the attributes. To make finest decision from the set of alternative is becoming a challenging one now-a-days for the decision maker because of its complexities in the real life problems [4].

The Technique for order preference by similarity to an ideal solution (TOPSIS) method is one of the best approaches in MADM [11]. TOPSIS was developed

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by Hwang and Yoon [6] in 1981. In order to rank the alternatives in TOPSIS method, First considered the Positive Ideal Solution (PIS) and also the Negative Ideal Solution (NIS) simultaneously. Then it will become very helpful to prioritize the alternative not from the shortest distance which is from the PIS, but also calculate the farthest distance from the NIS by using relative closeness degree [5]. Many researchers have worked in Intuitionistic fuzzy TOPSIS. For instance, in [2] they used the normalized distance measure in intuitionistic TOPSIS method to evaluate the supplier selection. In [9], they proposed a new distance measure in intuitionistic fuzzy TOPSIS method and applied in credit risk evaluation to check its effective. In [3], they apply separation measure in TOPSIS method to analysis/select the wind power plant. To select best hotel from the online review, in [8] they used TOPSIS method. In [7], to provide best supplier and a reliable solution for the sustainable by using Intuitionistic fuzzy TOPSIS method.

Lofti A Zadeh [13] introduce the concept of fuzzy set theory to overcome the difficulties of the crisp set. Intuitionistic fuzzy set theory was introduced by Atanassov [1] as an extension of fuzzy set with the condition that is sum of the membership and nonmembership must be present in the interval $[0,1]$. In Intuitionistic fuzzy set theory, there is a membership function, non-membership function along with hesitation function which plays a major role to avoid the vagueness of the problem.

In TOPSIS methodology, many of the researchers done their work by elongate and improved the process of TOPSIS to solve complicated problems [12]. In this paper, TOPSIS method is extended based on weighted generalized Dice similarity measure to construct a new perspective of algorithm along with intuitionistic preference relation to ensure the importance of attribute which is very useful to select the best solution. Finally a real life application which is about prioritizing the branded printers is presented to check the effectiveness of the proposed method in real life problems.

2. PRELIMINARIES

Definition 2.1 (Intuitionistic Fuzzy set). [1] Consider X as a Universal Set. Let A be an Intuitionistic fuzzy set defined by

$$A = \{ \langle x, \mu_A(x), \gamma_A(x) \rangle : x \in E \} ,$$

where $\mu_A(x) : X \rightarrow [0, 1], \gamma_A(x) : X \rightarrow [0, 1]$ with the condition $0 \leq \mu_A + \gamma_A \leq 1$.

Definition 2.2 (Generalized dice similarity measure). [10] Let $a^1 = (\mu^1, \gamma^1)_{jj}$ and $a^2 = (\mu^2, \gamma^2)_{jj} \ j = 1, 2, \dots, n$ be two group of intuitionistic fuzzy number, then a generalized dice similarity measure between a^1 and a^2 is defined as

$$GD_{IFS}(a^1, a^2) = \frac{1}{n} \sum_{j=1}^n \frac{\mu_j^1 \mu_j^2 + \gamma_j^1 \gamma_j^2}{\gamma[(\mu_j^1)^2 + (\gamma_j^1)^2] + (1 - \lambda)[(\mu_j^2)^2 + (\gamma_j^2)^2]},$$

where λ is a positive parameter for $0 \leq \lambda \leq 1$.

Definition 2.3 (Intuitionistic preference relation (IPR)). [11] An Intuitionistic preference relation B on X is represented by a matrix $B = (b_{ij})$, where $B \subset X \times X$ with $b_{ij} = \langle (x_i, x_j), \mu(x_i, x_j), \gamma(x_i, x_j) \rangle$ for all $i, j = 1, 2, \dots, n$.

Definition 2.4 (Consistent and Inconsistent IPR). [11] Let $B = (b_{ij})_{m \times n}$ be an intuitionistic preference relation where $b_{ij} = [\mu_{ij}, 1 - \gamma_{ij}]$ for all $i, j = 1, 2, \dots, n$ if there exist a vector $\leq 0.5(w_i - w_j + 1) \leq 1 - \gamma_{ij}$ for all $i = 1, 2, \dots, n - 1; j = 1, 2, \dots, n$ where w satisfies the condition

$$w_i \geq 0, \ i = 1, 2, \dots, n, \sum_{i=1}^n w_i = 1,$$

then b will be called as consistent IPR otherwise B will be considered as inconsistent IPR.

3. A NEW EXTENDED TOPSIS PROCEDURE FOR INTUITIONISTIC FUZZY MULTI ATTRIBUTE DECISION MAKING

Step 1: Based on [5], obtain the intuitionistic fuzzy positive and negative ideal solution from the equation as follow

$$(3.1) \quad \begin{aligned} A^+ &= (\langle \mu_1^+, \gamma_1^+ \rangle, \langle \mu_2^+, \gamma_2^+ \rangle, \dots, \langle \mu_m^+, \gamma_m^+ \rangle)^T \\ A^- &= (\langle \mu_1^-, \gamma_1^- \rangle, \langle \mu_2^-, \gamma_2^- \rangle, \dots, \langle \mu_m^-, \gamma_m^- \rangle)^T \end{aligned}$$

where $\mu_i^+ = \max_{1 \leq j \leq n} \{\bar{\mu}\}$, $\gamma_i^+ = \min_{1 \leq j \leq n} \{\bar{\gamma}\}$, $\mu_i^- = \min_{1 \leq j \leq n} \{\bar{\mu}\}$, $\gamma_i^- = \max_{1 \leq j \leq n} \{\bar{\gamma}\}$.
($i = 1, 2, \dots, n$).

Step 2: Based on [11], convert the intuitionistic preference relation into the equivalent interval fuzzy preference relation

$$(3.2) \quad (i.e.) \quad B = (b_{ij})_{m \times n}, \text{ where } b_{ij} = [\mu_{ij}, 1 - \gamma_{ij}] \quad i, j = 1, 2, \dots, n.$$

Step 3: Based on [11], construct the linear programming problem by using $(M - 1)$

$$J_1^* = \min \sum_{i=1}^{n-1} \sum_{j=i+1}^n (d_{ij}^- + d_{ij}^+)$$

such that

$$\begin{aligned} 0.5(w_i - w_j + 1) + d_{ij}^- &\geq \mu_{ij}, & i = 1, 2, \dots, n-1; \quad j = i+1, \dots, n \\ 0.5(w_i - w_j + 1) - d_{ij}^+ &\leq 1 - \gamma_{ij}, & i = 1, 2, \dots, n-1; \quad j = i+1, \dots, n \\ w_i &\geq 0, & i = 1, 2, \dots, n; \quad \sum_{i=1}^n w_i = 1 \\ d_{ij}^-, d_{ij}^+ &\geq 0, & i = 1, 2, \dots, n-1; \quad j = i+1, \dots, n \end{aligned}$$

Step 4: Based on [11], Generate the weight vector of the given attribute by solving $(M - 2)$ if $M - l$ gives the consistent IPR (or) otherwise solve $(M - 3)$

$$(3.3) \quad (M - 2)w_i^- = \min w_i \text{ and } w_i^+ = \max w_i$$

such that

$$\begin{aligned} 0.5(w_i - w_j + 1) &\geq \mu_{ij}, & i = 1, 2, \dots, n-1; \quad j = i+1, \dots, n \\ 0.5(w_i - w_j + 1) &\leq 1 - \gamma_{ij}, & i = 1, 2, \dots, n-1; \quad j = i+1, \dots, n \\ w_i &\geq 0, & i = 1, 2, \dots, n; \quad \sum_{i=1}^n w_i = 1 \end{aligned}$$

$$(M - 3)w_i^- = \min w_i \text{ and } w_i^+ = \max w_i$$

such that

$$\begin{aligned} 0.5(w_i - w_j + 1) + d_{ij}^- &\geq \mu_{ij}, & i = 1, 2, \dots, n-1; \quad j = i+1, \dots, n \\ 0.5(w_i - w_j + 1) - d_{ij}^+ &\leq 1 - \gamma_{ij}, & i = 1, 2, \dots, n-1; \quad j = i+1, \dots, n \\ w_i &\geq 0, & i = 1, 2, \dots, n; \quad \sum_{i=1}^n w_i = 1. \end{aligned}$$

Step 5: Based on [10], calculate the weighted generalized dice similarity measure between Intuitionistic fuzzy decision matrix and Intuitionistic fuzzy

positive ideal solution

$$(3.4) \quad WGD_{IFS}(A_i, A^+) = \sum_{j=1}^n w_j \frac{\mu_{ij}u_j^+ + \gamma_{ij}\gamma_j^+}{\lambda[(\mu_{ij})^2 + (\gamma_{ij})^2] + (1 - \lambda)[(\mu_j^+)^2 + (\gamma_j^+)^2]}$$

Step 6: Calculate the weighted generalized dice similarity measure between Intuitionistic fuzzy decision matrix and Intuitionistic fuzzy Negative ideal solution

$$(3.5) \quad WGD_{IFS}(A_i, A^-) = \sum_{j=1}^n w_j \frac{\mu_{ij}u_j^- + \gamma_{ij}\gamma_j^-}{\lambda[(\mu_{ij})^2 + (\gamma_{ij})^2] + (1 - \lambda)[(\mu_j^-)^2 + (\gamma_j^-)^2]}$$

Step 7: Based on [5], calculate the relative closeness degree by the equation

$$(3.6) \quad \phi = \frac{WGD_{IFS}(A_i, A^-)}{WGD_{IFS}(A_i, A^+) + WGD_{IFS}(A_i, A^-)}$$

Step 8: Based on [5], Arrange the relative closeness degree of each alternative in the non-increasing order to find the best solution.

4. PRACTICAL APPLICATION OF THE PROPOSED METHOD

Suppose a working peoples in the administration, i.e in the office circumstance wants to a buy a printer for the administrative purpose they select some of the branded printers namely HP, EPSON, PANASONIC and SAMSUNG and taken those printers as alternative. To select any branded it needs some features, so that the customer also put some features to select the branded printers. They are Image/print Quality, print speed, connectivity facilities, paper handling and stay in budget. The values about the alternative regarding the attributes are collected as a linguistic variable from the decision maker afterwards it converted into intuitionistic fuzzy value which are represented in the table 1 and 2.

TABLE 1. Linguistic variables for Decision matrix

Linguistic Variable	IFV
Strongly agree	[0.8, 0.1]
Agree	[0.6, 0.2]
Undecided	[0.5, 0.3]
Disagree	[0.2, 0.4]
Strongly Disagree	[0.3, 0.2]

TABLE 2. Linguistic variables for Intuitionistic preference relation

Linguistic Variable	IFV
Very Important	[0.7, 0.2]
Important	[0.6, 0.3]
Neutral	[0.5, 0.4]
Slightly Important	[0.3, 0.6]
Low Important	[0.1, 0.4]

TABLE 3. Intuitionistic fuzzy decision matrix

Alternatives	Attributes				
	<i>Image/Print Quality</i>	Print Speed	Connectivity Facilities	Paper Handling	Stay in Budget
HP	[0.8, 0.1]	[0.8, 0.1]	[0.8, 0.1]	[0.6, 0.2]	[0.6, 0.2]
EPSON	[0.8, 0.1]	[0.6, 0.2]	[0.6, 0.2]	[0.5, 0.3]	[0.6, 0.2]
PANASONIC	[0.6, 0.2]	[0.6, 0.2]	[0.2, 0.4]	[0.5, 0.3]	[0.2, 0.4]
SAMSUNG	[0.5, 0.3]	[0.2, 0.4]	[0.3, 0.2]	[0.6, 0.2]	[0.2, 0.4]

TABLE 4. Intuitionistic preference Relation

	<i>Image/Print Quality</i>	Print Speed	Connectivity Facilities	Paper Handling	Stay in Budget
<i>Image/Print Quality</i>	[0.5, 0.5]	[0.7, 0.2]	[0.5, 0.4]	[0.6, 0.3]	[0.7, 0.2]
Print Speed	[0.7, 0.2]	[0.5, 0.5]	[0.6, 0.3]	[0.1, 0.4]	[0.5, 0.4]
Connectivity Facilities	[0.5, 0.4]	[0.6, 0.3]	[0.5, 0.5]	[0.5, 0.4]	[0.3, 0.6]
Paper Handling	[0.6, 0.3]	[0.3, 0.6]	[0.1, 0.4]	[0.5, 0.5]	[0.1, 0.4]
Stay in Budget	[0.7, 0.2]	[0.5, 0.4]	[0.3, 0.6]	[0.1, 0.4]	[0.5, 0.5]

Step 1: Obtain the intuitionistic fuzzy positive and negative ideal solution by

(3.1) Positive Ideal Solution

$(A^+) : < 0.8, 0.1 >, < 0.8, 0.1 >, < 0.8, 0.1 >, < 0.6, 0.2 >, < 0.6, 0.2 >$

Negative ideal solution

$(A^-) : < 0.5, 0.3 >, < 0.2, 0.4 >, < 0.2, 0.4 >, < 0.5, 0.3 >, < 0.2, 0.4 >$

Step 2: Convert the intuitionistic preference relation into the equivalent interval fuzzy preference relation by (3.2)

TABLE 5. Interval fuzzy preference relation

	<i>Image/Print</i> Quality	Print Speed	Connectivity Facilities	Paper Handling	Stay in Budget
<i>Image/Print</i> Quality	[0.5, 0.5]	[0.7, 0.8]	[0.5, 0.6]	[0.6, 0.7]	[0.7, 0.8]
Print Speed	[0.7, 0.8]	[0.5, 0.5]	[0.6, 0.7]	[0.1, 0.6]	[0.5, 0.6]
Connectivity Facilities	[0.5, 0.6]	[0.6, 0.7]	[0.5, 0.5]	[0.5, 0.6]	[0.3, 0.4]
Paper Handling	[0.6, 0.7]	[0.3, 0.4]	[0.1, 0.6]	[0.5, 0.5]	[0.1, 0.6]
Stay in Budget	[0.7, 0.8]	[0.5, 0.6]	[0.3, 0.4]	[0.1, 0.6]	[0.5, 0.5]

Step 3: construct the linear programming problem by using (M-1)

$$J_1^* = 0.20$$

the optimal deviation values :

$$\begin{aligned} d_{12}^+ &= d_{12}^+ = d_{13}^+ = d_{13}^+ = d_{14}^+ = d_{14}^+ = d_{15}^+ = d_{15}^+ = 0, \\ d_{23}^- &= 0.20, \\ d_{23}^+ &= d_{24}^+ = d_{24}^+ = d_{25}^+ = d_{34}^+ = d_{34}^+ = d_{35}^+ = 0.025, \\ d_{45}^+ &= d_{45}^+ = 0 \end{aligned}$$

Step 4: Generating the weight vector of the given attribute by (3.3)

$$w_1 = 0.48, w_2 = 0.08, w_3 = 0.28, w_4 = 0.08 \text{ and } w_5 = 0.08$$

Step 5: Calculate the weighted generalized dice similarity measure between Intuitionistic fuzzy decision matrix and Intuitionistic fuzzy positive ideal solution by (3.4)

$$\begin{aligned} WGD_{IFS}(A_1, A^+) &= 1, WGD_{IFS}(A_2, A^+) = 0.97, \\ WGD_{IFS}(A_3, A^+) &= 0.794 \text{ and } WGD_{IFS}(A_4, A^+) = 0.783 \end{aligned}$$

Step 6: Calculate the weighted generalized dice similarity measure between Intuitionistic fuzzy decision matrix and Intuitionistic fuzzy Negative ideal solution by (3.5)

$$WGD_{IFS}(A_1, A^-) = 0.725, WGD_{IFS}(A_2, A^-) = 0.799,$$

$$WGD_{IFS}(A_3, A^-) = 0.959 \text{ and } WGD_{IFS}(A_4, A^-) = 0.952$$

Step 7: Calculate the relative closeness degree by the equation by (3.6)

$$\phi_1 = 0.420$$

$$\phi_2 = 0.451$$

$$\phi_3 = 0.547$$

$$\phi_4 = 0.543$$

Step 8: Arrange the relative closeness degree of each alternative in the non-increasing order to find the best solution.

$$\phi_1 < \phi_2 < \phi_4 < \phi_3$$

here, the best printer among all the four is HP.

5. SENSITIVE ANALYSIS

A sensitivity analysis is done in this section to ensure that the values of the λ doesn't change the consequences.

λ values	ϕ_1	ϕ_2	ϕ_3	ϕ_4	Rank
0	0.530	0.554	0.621	0.617	$\phi_1 < \phi_2 < \phi_4 < \phi_3$
0.1	0.501	0.528	0.608	0.605	$\phi_1 < \phi_2 < \phi_4 < \phi_3$
0.2	0.476	0.507	0.592	0.596	$\phi_1 < \phi_2 < \phi_3 < \phi_4$
0.3	0.453	0.486	0.579	0.581	$\phi_1 < \phi_2 < \phi_3 < \phi_4$
0.4	0.435	0.463	0.563	0.566	$\phi_1 < \phi_2 < \phi_3 < \phi_4$
0.5	0.415	0.446	0.548	0.552	$\phi_1 < \phi_2 < \phi_3 < \phi_4$
0.6	0.401	0.427	0.530	0.536	$\phi_1 < \phi_2 < \phi_3 < \phi_4$
0.7	0.386	0.402	0.513	0.513	$\phi_1 < \phi_2 < \phi_4 < \phi_3$
0.8	0.371	0.391	0.5	0.49	$\phi_1 < \phi_2 < \phi_4 < \phi_3$
0.9	0.358	0.376	0.469	0.454	$\phi_1 < \phi_2 < \phi_4 < \phi_3$
1	0.350	0.358	0.443	0.419	$\phi_1 < \phi_2 < \phi_4 < \phi_3$

6. CONCLUSION

An algorithm of TOPSIS method is reformulated and proposed a new procedure by aggregate two concepts one is Generalized weighted dice similarity

measure and another one is an intuitionistic preference relation. In this paper, the Intuitionistic preference relation is act as a generating function to find the weight vector of each attribute and generalized weighted dice similarity measure is utilized for finding the similarity between the alternatives. Finally, based on the relative closeness degree the alternatives are arranged in the non-increasing order. A real-life problem which is prioritizing the branded printers for the place of administration is engaged to examine the proposed method. In conclusion, this paper found that HP is the best branded printer among the other three printers.

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