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# ON PROPERTIES OF BOOLEAN GRAPH $BG_2(G)$

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ABSTRACT. Let $G$ be a simple graph with vertex set $V$ and edge set $E$. Then $B_{G,INC,\overline{L}(G)}(G)$ is a graph with vertex set $V \cup E$ and two vertices are adjacent if and only if they correspond to adjacent vertices of $G$, a vertex and an edge incident to it in $G$ or two non-adjacent edges of $G$. We denote this graph by $BG_2(G)$, Boolean graph of $G$-second kind. In this paper, some properties of $BG_2(G)$ are studied.

## 1. INTRODUCTION

Let $G$ be a graph with vertex set $V$ and edge set $E$ . We can construct many graphs from $G$ with vertex set $V \cup E$ and defining different adjacency relations, such graphs are called Boolean graphs. It is proved that about 32 different Boolean graphs can be constructed from $G$. Expansion of networks has been happening in almost all networks. Some of them are happening in a fixed manner likewise how Boolean graph is constructed from a given graph. The graphs like Line graph , total graph, have many good applications in computer network. So any study of Boolean structure is relevant.

In this paper some properties of a Boolean graph, known as Boolean graph of second kind, is studied. The symbol $B_{G,INC,\overline{L}(G)}(G)$ explains the adjacency, two vertices are adjacent if and only if they correspond to adjacent vertices of $G$, a vertex and an edge incident to it in $G$(INC) or two non-adjacent edges of

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$G$ ($\overline{L(G)}$). This new graph is simply denoted as $BG_2(G)$. T.N. Janakiramam et al. contributed much in the study of the graph $BG2(G)$ [2–4].

All graphs discussed in this paper are simple connected undirected graphs unless it is specified. A cut edge of $G$is an edge, its removal will increase the number of components of $G$. Number of edges incident with a vertex $v$ is its degree and is denoted by $deg(v)$. Vertices of degree one are known as extreme vertices. A tree is a connected acyclic graph. Tree with $n$ vertices and exactly two extreme vertices is called a path which is notated as $P_n$. Length of a path is the number of edges in it. The distance between two vertices $u$ and $v, d(u, v)$, is the length of the shortest path from $u$ to $v$. Distance to the farthest vertex from a vertex $v$ is called its eccentricity, denoted by $e(v)$. The highest eccentricity among vertices of a Graph is its diameter, denoted $diam(G)$.

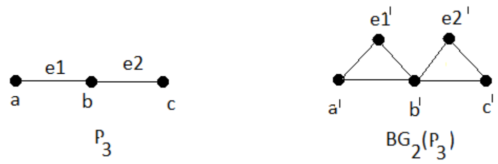
Let $u$ and $v$ be vertices of a connected graph $G$ then we denote the closed interval as

$$I[u, v] = \{w : w \text{ is a vertex in a shortest path from } u \text{ to } v\}.$$

Let $S$ is a set of vertices then,

$$I[S] = \{w : w \text{ is a vertex in a shortest path connecting two points of} S\}.$$

If $I[S] = V$ then $S$ is called a Geodesic set. The smallest cardinality of geodesic sets is defined as the geodesic number of the graph $G$, denoted by $g(G)$. More graph terminologies are from [1, 5, 6].




Vertices of $BG_2(G)$ corresponding to vertices of $G$ are called 'point vertices' and vertices corresponds to edges of $G$ are called 'line vertices'. If $x$ is vertex or edge of $G$ the $x'$ denote the corresponding point or line vertex of $BG_2(G)$. $e_x$ denote an edge adjacent to a vertex $x$.

## 2. Main Results

**Theorem 2.1.** *If $G$ is connected, $BG_2(G)$ has no cut edges.*

*Proof.* Let $f$ be any edge of $BG_2(G)$. Then $f$ has the following 3 possibilities

Case 1: $f$ joins two point vertices:

Here let $f = u'v'$ then there exist an edge $e = uv$ in $G$ since two point vertices are adjacent iff the corresponding vertices in $G$ are adjacent, then $u'e'v'$ is a different path from $u'$ to $v'$.

Case 2: $f$ joins a point vertex and a line vertex:

Here let, $f = u'e'$ where $u$ is a vertex and $e$ is an edge incident to it in $G$. Then there must be an edge $v$ in $G, e = uv$ in $G$. So by definition, $u'v'e'$ is another path from $u'$ to $e'$. ie, $f$ is not a cut edge.

Case 3: $f$ joins two line vertices:

Let $f = e'h'$ ie, $e$ and $h$ are two non adjacent edges of $G$. let $e = ab$ and $h = cd$ in $G$. Since $G$ is connected, there exist a path $P$ from $a$ to $d$. Now $G$ is sub graph of $BG_2(G)$ so a copy of $P$ say $P'$ will be there from $a'$ to $d'$. Thus $e'P'h'$ is a second path from $e'$ to $h'$.

□

**Theorem 2.2.** *For any graph $G$, $BG_2(G)$ is not a tree.*

*Proof.* Two cases are considered for the proof.

Case 1: $G$ has an isolated vertex:

Definition of adjacency in $BG_2(G)$ implies that 'An isolated vertex of $G$ is an isolated vertex of $BG_2(G)$'. Hence $BG_2(G)$ is disconnected.

Case 2: $G$ has no isolated vertex:

In this case there must be at least two vertices $u$ and $v$ in $G$ and an edge connecting them say $e = uv$. Then $e'u'v'e'$ is a cycle in $BG_2(G)$. So in any case $BG_2(G)$ is not a tree.

□

**Theorem 2.3.** *$BG_2(G)$ has no extreme vertex.*

*Proof.* Let $x'$ be a vertex of $BG_2(G)$. No line vertex $e'$ can be an extreme vertex of $BG_2(G)$, since $deg(e') \geq 2$. In case of point vertices, we have the following cases:

Case 1: $x$ is not an extreme vertex of $G$.

Here $deg(x) > 1$. The method of constructing $BG2(G)$ from $G$ implies $deg(x') \geq deg(x)$. Therefore $deg(x') > 1$ i.e, $x'$ is not an extreme point.

Case 2: $x$ is an extreme point of $G$.

Let $deg(x) = 1$ in $G$ and $e = xy$ be the one and only one edge incident with $x$. then $e'x'$ and $x'y'$ are edges in $BG2(G)$ and they are the only edges incident with $x$. Therefore $deg(x') = 2$. i.e $x'$ is not an extreme one.

□

**Theorem 2.4.** *If $G$ is a graph without an isolated vertex,* $diam(BG_2(G)) \leq 3$.

*Proof.* let $x'$ and $y'$ be any two vertices of $BG_2(G)$.

Case 1: $G$ is connected;

Subcase 1.1: Denote $x'$ and $y'$ are point vertices of $BG_2(G)$.
$If d(x, y) \leq 3$ in $G$, then $d(x', y') \leq 3$ in $BG_2(G)$. Assume that $d(x, y) > 3$ in $G$, let $e_x$ and $e_y$ are the edges incident with $x$ and $y$ in the shortest path from $x$ to $y$. Here $e_x$ and $e_y$ are non-adjacent in $G$. Then $x'e_{x'}e_{y'}y'$ is the shortest path from $x'$ to $y'$ in $BG_2(G)$. Therefore $d(x', y') = 3$.

Subcase 1.2: Denote $x'$ and $y'$ are line vertices of $BG_2(G)$.
If $x$ and $y$ are incident with a same vertex $v$ in $G$ then $x'v'y'$ is the shortest path from $x'$ to $y'$ in $BG_2(G)$. Thus $d(x', y') = 2$. If $x$ and $y$ are not incident with same vertex then they are two non adjacent edges of $G$. So $x'$ and $y'$ are adjacent in $BG_2(G)$. Hence $d(x', y') = 1$.

Subcase 1.3: $x'$ is a line vertex and $y'$ is a point vertex:
If $x$ is incident with $y$ in $G$ then $d(x', y') = 1$. Assume $x$ is not incident with $y$. If there is an edge $ey$ incident with $y$ and $u$ where $x$ is also adjacent to $u$ then $x'u'e_{y'}y'$ is the shortest path from $x'$ to $y'$ then $d(x', y') = 3$. If there is no such vertex $u$ then $x$ and every edge incident with $y$ are non adjacent. Hence $x'z'y'$ is a shortest path from $x'$ to $y'$ in $BG_2(G)$ where $z$ is any edge meeting $y$. Thus $d(x', y') = 2$.

Case 2: $G$ is disconnected.

If $x$ and $y$ lies in the same component of $G$ then from case 1 $d(x', y') \leq 3$. Assume $x$ and $y$ lies in different components of $G$.

Subcase 2.1: Denote $x'$ and $y'$ are point vertices of $BG_2(G)$ . $x'e'_xe'_yy'$ is a shortest path where $e_x$ and $e_y$ are any edges meeting $x$ and $y$ respectively then $d(x', y') = 3$.

Subcase 2.2: Denote $x'$ and $y'$ are line vertices of $BG_2(G)$. Here $x$ and $y$ are non adjacent edges in $G$ then $x'y'$ is an edge in $BG_2(G)$. So $d(x', y') = 1$.

Subcase 2.3: $x'$ is a line vertex and $y'$ is a point vertex Let $ey$ be any edge meeting with $y$ in $G$ then $x'e'_y y'$ is a shortest path. So $d(x', y') = 2$. Hence in every case $d(x', y') \leq 3$. Therefore diam$(BG_2(G)) \leq 3$.

□

**Theorem 2.5.** *Let $X$ be the set of all extreme vertices of a graph $G(V, E)$ where $V$ is the vertex set and $E$ is the edge set then $X' \cup E'$ is a geodesic set of $BG_2(G)$. $X'$ and $E'$ are the sets vertices of $BG_2(G)$ corresponding to vertices in $V$ and edges in $E$.*

*Proof.* Let $S' = V' \cup E' - X' \cup E' = V' - X'$, where $V'$ is the set of all point vertices. Let $v'_i \epsilon S'$. Here $deg(v'_i) \geq 2$. Let $e_1$ and $e_2$ are any two edges in $E$ incident $v_i$ in $G$ then $e'_1 v'_i e'_2$ is a shortest path from $e'_1$ to $e'_2$. There is no edge $e'_1 e'_2$ in $BG_2(G)$, since $e_1$ and $e_2$ are adjacent in $G$. Thus $v'_i \epsilon I(X' \cup E') \Rightarrow S' \subset I(X' \cup E')$. Also we have $X' \cup E' \subset I(X' \cup E')$.

Therefore

$$
\begin{aligned}
S' \cup (X' \cup E') \subset I(X' \cup E')(V' - X') \cup (X' \cup E') \subset I(X' \cup E') \\
\Rightarrow V' \cup E' \subset I(X' \cup E') \Rightarrow I(X' \cup E') = V' \cup E' \\
\Rightarrow X' \cup E'
\end{aligned}
$$

is a geodesic set. □

**Corollary 2.1.** $2 \leq g(BG_2(G)) \leq \mid X \mid + \mid E \mid .$

The result follows immediately from the fact that $X'$ and $E'$ are disjoint and $X' \cup E'$ is a geodesic set.

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