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A NOTE ON PSEUDO SP-ALGEBRA

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ABSTRACT. As a generalization of SP-Algebra, the concept of pseudo SP-Algebra is imported and some of their properties are researched in this paper. Pseudo SP-Ideal, identity pseudo SP-Algebra and characterization of their properties are defined and proved. Decomposition of any pseudo SP-Ideal of a pseudo SP-Algebra is also introduced in this paper.

1. INTRODUCTION

Two branches of abstract algebra were established in [1–4]. Following this, many authors mounted various algebras which have been acted as a generalization or subclasses of these two algebras. In all these papers, they investigated the algebraic structures in special algebras and made an assessment between those algebras. In 2016, K. Shanmuga Priya and M. Mullai established a new notion of algebra known as SP-Algebra [5]. They used the concept of SP-Ideal in SP-Algebra to assemble quotient SP-Algebra. Homomorphism and kernal of SP-Algebra were additionally described through them. They extended SP-Algebra to SP-Ring [6] and discussed its properties. The idea of SP-Ring was applied to polyomials named SP-polyomials in [7]. Young Bae Jun and Sun Shin Ahn introduced the notion of Pseudo BH-algebra [9] and Pseudo BCH- Algebra [8] as a generalization of BH-algebra and BCH-Algebra in 2015. They also investigated some of its properties. In this paper, the perception of pseudo SP-Algebra, as a

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generalization of SP-Algebra is introduced and the notion of pseudo SP-Ideal in pseudo SP-Algebra is defined. Moreover characterization of their properties in pseudo SP-Algebra are also established.

2. PSEUDO SP-ALGEBRA

Definition 2.1. *[5] An Algebra (S_P, $*$, e_{S_P}) of type (2, 0) is said to be SP-Algebra if*

(SP_1) : *c $*$ c = e_{S_P},*

(SP_2) : *c $*$ e_{S_P} = c,*

(SP_3) : *if c $*$ f = e_{S_P} and f $*$ c = e_{S_P}, then c = f,*

(SP_4) : *(c $*$ f) $*$ (f $*$ h) = (c $*$ h),*

where $c, f, h \in S_P$ and '$*$' is any binary operation and 'e_{S_P}' is any constant.

Example 1. *[5] If S_P is any non-void finite set and P(S_P) denotes the power set of S_P, then (P(S_P), Δ, ϕ) is a SP-Algebra, where Δ = (A-B) $\cup$ (B-A), ϕ is an empty set and A, B $\in$ P(S_P).*

Definition 2.2. *[5] A non-void subset I_{S_P} of SP-Algebra S_P is said to be SP-Ideal if*

(1) *$e_{S_P} \in I_{S_P}$;*

(2) *for every c, f $\in I_{S_P}$, c$*$ f $\in I_{S_P}$;*

(3) *if f$*$ c $\in I_{S_P}$ and $c \in I_{S_P}$, then $f \in I_{S_P}$, where $c, f \in S_P$.*

Example 2. *[5] I_{S_P} = {ϕ, {a}, {b}, {a, b}} is a SP-Ideal of (P(S_P), Δ, ϕ), where S_P = {a, b, c}.*

Definition 2.3. *A pseudo SP-Algebra is a non empty set S_P with a constant 'e_{S_P}' and two binary operations '$*$' and 'δ' satisfying the following axioms:*

(1) *$c*$ c = c δ c = e_{S_P};*

(2) *$c*$ e_{S_P} = c δ $e_{S_P} = c$;*

(3) *if $c * f = f \delta c = e_{S_P}$, then $c = f$;*

(4) *$(c * f)$ δ (f$*$ h) = (c δf) $*$ ($f \delta$ h), then c $*h = c$ δ h, $\forall c, f$ and $h \in S_P$.*

Example 3. *Consider the set $S_P = \{0, 1, 2\}$. Then S_P is a pseudo SP-Algebra with operations '$*$' and 'δ' defined as follows:*

Definition 2.4. *A non empty subset I_{S_P} of the pseudo SP-Algebra (S_P, $*$, δ, e_{S_P}) is called pseudo SP-Ideal of S_P if*

$*$	0	1	2
0	0	1	2
1	1	0	2
2	2	2	0

δ	0	1	2
0	0	1	2
1	1	0	1
2	2	2	0

(1) $e_{S_P} \in S_P$;
(2) *c* $*f, c\delta f \in I_{S_P}$, *whenever c, f* $\in I_{S_P}$;
(3) *if c* $*f, c\delta f \in I_{S_P}$ *and f* $\in I_{S_P} \Rightarrow$ *c* $\in I_{S_P}$, $\forall c, f \in S_P$.

Example 4. I_{S_P} = {0} *and* S_P *are pseudo SP-Ideals of the example 3.*

Note 1. *If the pseudo SP-Algebra* (S_P, $*$, δ, e_{S_P}) *satisfies* $e_{S_P} * (c * f) = (e_{S_P} * c) * (e_{S_P} * f)$ *and* $e_{S_P}\ \delta\ (c\delta f) = (e_{S_P}\ \delta c)\ \delta\ (e_{S_P}\ \delta f)$, $\forall c, f \in S_P$, *then* S_P *is called identity pseudo SP-Algebra.*

Theorem 2.1. *For any identity pseudo SP-Algebra* (S_P, $*$, δ, e_{S_P}), *the set* $K_{S_P} = \{c \in S_P / e_{S_P} * c = e_{S_P}, e_{S_P}\ \delta c = e_{S_P}\}$ *is a pseudo SP-Ideal.*

Proof. Let (S_P, $*$, δ, e_{S_P}) be an identity pseudo SP-Algebra.

To prove K_{S_P} is a pseudo SP-Ideal:

(1) Clearly $e_{S_P} \in K_{S_P}$, since $e_{S_P} * e_{S_P} = e_{S_P}$ and $e_{S_P}\ \delta\ e_{S_P} = e_{S_P}$
(2) Let $c, f \in K_{S_P}$. Then $e_{S_P} * c = e_{S_P}\ \delta c = e_{S_P}$, $e_{S_P} * f = e_{S_P}\ \delta f = e_{S_P}$. To prove $c*$ f $\in K_{S_P}$ and $c\delta$ f $\in K_{S_P}$, i.e., to prove $e_{S_P} *(c * f) = e_{S_P}\ \delta$ (c $\delta f) = e_{S_P}$,

$$\begin{aligned} e_{S_P} * (c * f) &= (e_{S_P} * c) * (e_{S_P} * f) \\ &= (e_{S_P} * c) * e_{S_P} \\ &= e_{S_P} * c \\ &= e_{S_P}. \end{aligned}$$

Similarly, $e_{S_P}\ \delta(c\delta f) = e_{S_P}$. Hence, $(c * f)$ and $(c\delta f) \in K_{S_P}$.

(3) Let $(c * f)$, $(c\delta f) \in K_{S_P}$ and f $\in K_{S_P}$
$\Rightarrow e_{S_P} * (c * f) = e_{S_P}$ and $e_{S_P}\ \delta$ (c $\delta f) = e_{S_P}$

$\Rightarrow (e_{S_P} * c) * (e_{S_P} * f) = e_{S_P}$ and $(e_{S_P} \delta c) \ \delta \ (e_{S_P} \delta f) = e_{S_P}$
$\Rightarrow e_{S_P} * c = e_{S_P}$ and $e_{S_P} \delta c = e_{S_P}$, since $f \in K_{S_P}$
$\Rightarrow$ c $\in K_{S_P}$.

Hence, K_{S_P} is a pseudo SP-Ideal of (S_P, $*$, δ, e_{S_P}). □

Theorem 2.2. *If (S_P, $*$, δ, e_{S_P}) is an identity pseudo SP-Algebra and if* c $\in K_{S_P}$ *and f* $\notin K_{S_P}$*, then* c $*$ *f* $\in S_P$-K_{S_P} *and* c δ *f* $\in S_P$-K_{S_P}*.*

Proof. Let (S_P, $*$, δ, e_{S_P}) be an identity pseudo SP-Algebra. Let $c \in K_{S_P}$ and $f \notin K_{S_P}$. To prove $c * f, c\delta$ f $\in S_P$-K_{S_P}, suppose, $c * f, c\delta$ f $\in K_{S_P}$. Then, $f \in K_{S_P}$, since K_{S_P} is a pseudo SP-Ideal, which is a conflict to our hypothesis. Hence, $c * f, c\delta$ f $\in S_P$-K_{S_P}. □

Theorem 2.3. *If I_{S_P} is a pseudo SP-Ideal of pseudo SP-Algebra (S_P, $*$, δ, e_{S_P}) then*

(i) *$(h * f) \ \delta c = e_{S_P} \Rightarrow$* c $\in I_{S_P}$*, $\forall c, f \in I_{S_P}$ and* h $\in S_P$*;*
(ii) *$(h\delta f) * c = e_{S_P} \Rightarrow$* h $\in I_{S_P}$*, $\forall c, f \in I_{S_P}$ and $h \in S_P$.*

Proof. Let I_{S_P} be a pseudo SP-Ideal of pseudo SP-Algebra (S_P, $*$, δ, e_{S_P}).
(i) Let $(h * f)\delta c = e_{S_P}$, where $c, f \in I_{S_P}$ and $h \in S_P$;
$\Rightarrow (h * f)\delta c \in I_{S_P}$, since $e_{S_P} \in I_{S_P}$;
$\Rightarrow (h * f) \in I_{S_P}$, since $(h * f)\delta c \in I_{S_P}$ and $c \in I_{S_P}$;
$\Rightarrow h \in I_{S_P}$, since $f \in I_{S_P}$.
(ii) Given, $(h\delta f) * c = e_{S_P}$, where $c, f \in I_{S_P}$ and $h \in S_P$.
$\Rightarrow (h\delta f) * c \in I_{S_P}$, since $e_{S_P} \in I_{S_P}$;
$\Rightarrow (h\delta f) \in I_{S_P}$, since $c \in I_{S_P}$;
$\Rightarrow h \in I_{S_P}$, since $f \in I_{S_P}$. □

Theorem 2.4. *Let I_{S_P} be a closure subset of a pseudo SP-Algebra (S_P, $*$, δ, e_{S_P}) and $e_{S_P} \in I_{S_P}$, then $c \in I_{S_P}$, $f \in S_P$-$I_{S_P} \Rightarrow f * c, f\delta c \in S_P$-$I_{S_P}$, $\forall c, f \in S_P$ iff I_{S_P} is a Pseudo SP-Ideal of the Pseudo SP-Algebra S_P.*

Proof. Let f $*c, f\delta c \in S_P$-I_{S_P}, $\forall c \in I_{S_P}$ and $f \in S_P$-I_{S_P}. To prove I_{S_P} is a Pseudo SP-Ideal, it is enough to prove that $f \in I_{S_P}$ if $f * c, f\delta c$ and $c \in I_{S_P}$. Suppose, $f \notin I_{S_P}$, then $f * c, f\delta c \in S_P$-I_{S_P}, which is a contraction to our assumption that, $f * c, f\delta c \in I_{S_P} \Rightarrow f \in I_{S_P}$. Hence, I_{S_P} is a pseudo SP-Ideal of S_P.

In reverse, assume that I_{S_P} is a pseudo SP-Ideal of S_P. Let $c, f \in S_P$, such that $c \in I_{S_P}$ and $f \notin I_{S_P}$. To prove $f * c \in S_P$-I_{S_P}, suppose $f * c \notin S_P$-I_{S_P}. Then

$\Rightarrow f * c \in I_{S_P}$ which implies $\Rightarrow f \in I_{S_P}$, since $c \in I_{S_P}$. This is a contradiction to $f \notin I_{S_P}$. Therefore $f * c \in S_P$-I_{S_P}.

Similarly, $f\delta c \in S_P$-I_{S_P}. □

Proposition 2.1. *Let S_{P1} be a pseudo SP-Ideal of a pseudo SP-Algebra (S_P, $*$, δ, e_{S_P}) and S_{P2} be a pseudo SP-Ideal of S_{P1}, then S_{P2} is a pseudo SP-Ideal of S_P.*

Proof. Given, S_{P1} is a pseudo SP-Ideal of S_P and S_{P2} is a pseudo SP-Ideal of S_{P1}. To prove: S_{P2} is a pseudo SP-Ideal of S_P.

Clearly, $e_{S_P} \in S_{P2}$ and $c * f, c\delta$ f $\in S_{P2}$, $\forall c, f \in S_{P2}$, since S_{P2} itself is a pseudo SP-Ideal of S_{P1}.

It remains to prove that $c \in S_{P2}$, if $f, c * f, c\delta f \in S_{P2}$, for some $c \in S_P$. If $c \in S_{P1}$, then $c \in S_{P1}$, since S_{P2} is a pseudo SP-Ideal of S_{P1}.

Let $f, c * f, c\delta f \in S_{P2} \subseteq S_{P1}$. Then $\Rightarrow c \in S_{P1}$, since S_{P1} is a pseudo SP-Ideal, i.e., $\Rightarrow c \in S_{P1}$. Hence S_{P2} is a pseudo SP-Ideal of S_{P1}. □

Definition 2.5. *Let (S_P, $*$, δ, e_{S_P}) be a pseudo SP-Algebra. For any $c, f \in S_P$, we define $H(c, f) = \{x \in S_P$/(x $*c)\delta f = e_{S_P}\}$.*

Theorem 2.5. *If I_{S_P} is a pseudo SP-Ideal of a pseudo SP-Algebra (S_P, $*$, δ, e_{S_P}), then $I_{S_P} = \cup H(c, f)$, where $c, f \in I_{S_P}$.*

Proof. Let I_{S_P} be a pseudo SP-Ideal of SP-Algebra (S_P, $*$, δ, e_{S_P}).
We will prove that $I_{S_P} = \cup H(c, f)$, where $c, f \in I_{S_P}$. Let $c \in I_{S_P}$. Consider, $(c * c)\delta\, e_{S_P} = e_{S_P}\ \delta\ e_{S_P} = e_{S_P}$. Hence

$c \in H(c, e_{S_P}) \Rightarrow H(c, e_{S_P}) \subseteq H(c, f)$, since $c \in H(c, f)$
$\Rightarrow \cup H(c, e_{S_P}) \subseteq \cup H(c, f)$, where $c, f \in I_{S_P}$
$\Rightarrow c \in \cup H(c, f)$, where $c, f \in I_{S_P}$
$\Rightarrow I_{S_P} \subseteq \cup H(c, f)$.

Conversely, let $x \in \cup H(c, f)$, where $c, f \in I_{S_P}$.
$\Rightarrow x \in H(c, f)$, for some $c, f \in I_{S_P}$
$\Rightarrow (x * c)\delta f = e_{S_P} \in I_{S_P}$, since I_{S_P} is a pseudo SP-Ideal
$\Rightarrow x * c \in I_{S_P}, f \in I_{S_P}$
$\Rightarrow x \in I_{S_P}, c \in I_{S_P}$.

Hence, $\cup H(c, f) \subseteq I_{S_P} \Rightarrow I_{S_P} = \cup H(c, f)$. □

Corollary 2.1. *If I_{S_P} is a pseudo SP-Ideal of pseudo SP-Algebra (S_P, $*$, δ, e_{S_P}), then $I_{S_P} = \cup H(c, e_{S_P})$, $\forall c \in S_P$.*

Proof. We will prove $I_{S_P} = \cup H(c, e_{S_P})$, $\forall c \in S_P$. We have $\cup H(c, f) \subseteq I_{S_P}$, since

$$\cup H(c, e_{S_P}) \subseteq \cup H(c, f) = I_{S_P}. \tag{2.1}$$

Let $c \in I_{S_P}$ be arbitrary (i.e.,) $c \in$ H(c, e_{S_P}). Then c $\in \cup$ H(c, e_{S_P}), i.e.,

$$I_{S_P} \subseteq \cup H(c, e_{S_P}). \tag{2.2}$$

From (2.1) and (2.2), we obtain $I_{S_P} = \cup H(c, e_{S_P})$. □

Proposition 2.2. *Let I_{S_P} be a non void closure subset of SP-Algebra (S_P, $*$, δ, e_{S_P}) with the element e_{S_P}. If $I_{S_P} = \cup H(c, f)$, where $c, f \in I_{S_P}$, then I_{S_P} is a pseudo SP-Ideal of S_P.*

Proof. To prove I_{S_P} is a pseudo SP-Ideal of S_P, it is enough to prove that if $r * s, r\delta s, s \in I_{S_P}$, then $r \in I_{S_P}$.

Let $r * s, r\delta s, s \in I_{S_P}$. Then $r \in H(s, r * s)$, since $(r * s)\delta(r * s) = e_{S_P}$ and further $r \in I_{S_P}$. Hence, I_{S_P} is a pseudo SP-Ideal of S_P. □

Proposition 2.3. *Let I_{S_P} be a subset of a pseudo SP-Algebra (S_P, $*$, δ, e_{S_P}) with the following conditions:*

(i) $e_{S_P} \in I_{S_P}$,

(ii) $c * h, c\delta h, f * h, f\delta h \in I_{S_P}$ *and* $h \in I_{S_P} \Rightarrow c * f, c\delta f \in I_{S_P}$, $\forall c, f, h \in S_P$.

Then I_{S_P} is a Pseudo SP-Ideal of S_P.

Proof. Let, I_{S_P} is a subset of pseudo SP-Algebra(S_P, $*$, δ, e_{S_P}). We will prove I_{S_P} is a Pseudo SP-Ideal of S_P.

Clearly, $e_{S_P} \in I_{S_P}$, by (i). Let $c, f \in I_{S_P}$. Then $\Rightarrow c * f, c\delta f \in I_{S_P}$, since $c * e_{S_P}, c\delta\ e_{S_P}, f * e_{S_P}, f\delta\ e_{S_P} \in I_{S_P}$.
Next, let $c * f, c\delta f, f \in I_{S_P}$. Then $e_{S_P} * f, e_{S_P}\ \delta f, c * f, c\delta$ f $\in I_{S_P}$, i.e., $c \in I_{S_P}$, by (ii). Hence, I_{S_P} is a pseudo SP-Ideal of S_P. □

3. Conclusion

In this paper, the concept of pseudo SP-Algebra is introduced with definitions and suitable examples. Some theorems on pseudo SP-Algebra and pseudo SP-Ideals are also established. Using this concept, the properties of pseudo SP-Algebra will be investigated in future.

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