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SEVERAL TYPES OF GENERALIZED DOUBLE FUZZY $\mathcal{Z}$ DISCONNECTED SPACES

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ABSTRACT. In this paper we introduce the concepts of pre generalized double fuzzy $\mathcal{Z}$-open, generalized double fuzzy $\mathcal{Z}$-extremally disconnected spaces. Also we introduce the concepts of generalized double fuzzy $\mathcal{Z}$-basically disconnected space and related sets such as (r,κ)-generalized fuzzy $\mathcal{Z}$ open-F_σ, (r,κ)-generalized fuzzy $\mathcal{Z}$closed-G_δ sets and maps such as generalized double fuzzy $\mathcal{Z}F_\sigma$-open, $\mathcal{Z}G_\delta$-continuous and $\mathcal{Z}F_\sigma$-irresolute function. Some characterizations of the concepts are studied.

1. INTRODUCTION AND PRELIMINARIES

In 1986, Atanassov [1] started 'Intuitionistic fuzzy sets' and Coker [2] in 1997, initiated Intuitionistic fuzzy topological space". The term "double" instead of "intuitionistic" coined by Garcia and Rodabaugh [4] in 2005. In the previous two decades many analysts [7–9, 14] accomplishing more applications on double fuzzy topological spaces. The class of L-fuzzy ω-extremally disconnected spaces is defined by Sudha et al. [13]. From 2011, $\mathcal{Z}$-open sets and maps were introduced in topological spaces by El-Maghrabi and Mubarki [6].

X denotes a non-empty set, $I_1=[0,1),\ I_0=(0,1],\ I=[0,1],\ 0=\underline{0}(X),\ 1=\underline{1}(X),\ r\in I_0\ \&\ \kappa\in I_1$ and always $1\geq r+\kappa$. I^X is a family of all fuzzy sets on

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X. In 2002, Double fuzzy topological spaces (briefly, dfts), (X, η, η^*), (r, κ)-fuzzy open (resp. (r, κ)-fuzzy closed) (briefly (r, κ)-fo (resp. (r, κ)-fc)) set were given by Samanta and Mondal [11].

All other undefined notions are from [3, 5, 6, 8–12] and cited there in.

2. Generalized double fuzzy $\mathcal{Z}$-extremally disconnected spaces

Definition 2.1. *Let (X, ρ_1, ρ_1^*) & (Y, ρ_2, ρ_2^*) be $dfts$. A function $f : (X, \rho_1, \rho_1^*) \to (Y, \rho_2, \rho_2^*)$ is called pre generalized double fuzzy $\mathcal{Z}$-open (briefly, $pgDF\mathcal{Z}O$) if $f(\gamma)$ is an (r, κ)-$gf\mathcal{Z}o$ in I^Y $forall$ (r, κ)-$gf\mathcal{Z}o$ set $\gamma \in I^X$.*

Remark 2.1. *Let (X, ρ_1, ρ_1^*) be $dfts$. $\forall\, \gamma \in I^X$, the following statements hold:*

(i) $G\mathcal{Z}I_{\rho,\rho^*}(\gamma, r, \kappa) = G\mathcal{Z}C_{\rho,\rho^*}(\underline{1} - \gamma, r, \kappa)$.

(ii) $G\mathcal{Z}C_{\rho,\rho^*}(\gamma, r, \kappa) = G\mathcal{Z}I_{\rho,\rho^*}(\underline{1} - \gamma, r, \kappa)$.

Proposition 2.1. *Let (X, ρ_1, ρ_1^*) & (Y, ρ_2, ρ_2^*) be $dfts$. A function $f : (X, \rho_1, \rho_1^*) \to (Y, \rho_2, \rho_2^*)$ is a*

(i) $gDF\mathcal{Z}Irr$ *function iff* $f(G\mathcal{Z}C_{\rho_1,\rho_1^*}(\gamma, r, \kappa)) \leq G\mathcal{Z}C_{\rho_2,\rho_2^*}(f(\gamma), r, \kappa)$ $\forall$ *fs* γ *in* I^X.

(ii) $gDF\mathcal{Z}O$ *surjective function, then* $\forall$ *fs* γ *in* I^Y. $f^{-1}(G\mathcal{Z}C_{\rho_2,\rho_2^*}(\gamma, r, \kappa)) \leq G\mathcal{Z}C_{\rho_1,\rho_1^*}(f^{-1}(\gamma), r, \kappa)$.

Definition 2.2. *A $dfts$ (X, ρ, ρ^*) is said to be an generalized double fuzzy $\mathcal{Z}$ extremely disconnected (briefly, $gDF\mathcal{Z}ed$) space if $G\mathcal{Z}C_{\rho,\rho^*}(\gamma, r, \kappa)$ is an (r, κ)-$gf\mathcal{Z}o$ set $\forall$ (r, κ)-$gf\mathcal{Z}o$ set $\gamma \in I^X$.*

Example 1. *The $dfts$ (X, ρ, ρ^*) is $gDF\mathcal{Z}ed$ space, where $X = \{l, m, n\}$ with the topologies*

$$\rho(\gamma) = \begin{cases} 1, & \text{if } \gamma \in \{\underline{0}, \underline{1}\}, \\ \frac{3}{4} & \gamma = \underline{0.5}, \\ \frac{1}{2} & \gamma \in \{\underline{0.3}, \underline{0.7}\}, \\ \frac{1}{4} & \gamma \in \{\underline{0.4}, \underline{0.6}\}, \\ 0 & \text{o.w.} \end{cases} \qquad \rho^*(\gamma) = \begin{cases} 0, & \text{if } \gamma \in \{\underline{0}, \underline{1}\}, \\ \frac{1}{4} & \gamma = \underline{0.5}, \\ \frac{1}{2} & \gamma \in \{\underline{0.3}, \underline{0.7}\}, \\ \frac{3}{4} & \gamma \in \{\underline{0.4}, \underline{0.6}\}, \\ 1 & \text{o.w.} \end{cases}$$

Proposition 2.2. *Let (X, ρ_1, ρ_1^*) and (Y, ρ_2, ρ_2^*) be $dfts$'s. If a function $f : (X, \rho_1, \rho_1^*) \to (Y, \rho_2, \rho_2^*)$ is a $gDF\mathcal{Z}Irr$, $gDF\mathcal{Z}O$ surjective function such that (X, ρ_1, ρ_1^*) is an $gDF\mathcal{Z}ed$ then, (Y, ρ_2, ρ_2^*) is $gDF\mathcal{Z}ed$.*

Theorem 2.1. *Let (X,ρ,ρ^*) be a $dfts$. Then:*

(i) (X,ρ,ρ^*) *is an $gDF\mathcal{Z}ed$ space.*

(ii) $\forall$ (r,κ)*-*$gf\mathcal{Z}c$ *set* γ, $G\mathcal{Z}I_{\rho,\rho^*}(\gamma,r,\kappa)$ *is an* (r,κ)*-*$gf\mathcal{Z}c$ *set.*

(iii) $\forall$ (r,κ)*-*$gf\mathcal{Z}o$ *set* γ, $G\mathcal{Z}C_{\rho,\rho^*}(\gamma,r,\kappa)\bigvee G\mathcal{Z}C_{\rho,\rho^*}(\underline{1}-G\mathcal{Z}C_{\rho,\rho^*}(\gamma,r,\kappa)r,\kappa)=\underline{1}$.

(iv) $\forall$ (r,κ)*-*$gf\mathcal{Z}o$ *sets* γ $\&$ μ $\ni$ $G\mathcal{Z}C_{\rho,\rho^*}(\gamma,r,\kappa)\vee\mu=\underline{1}$, *then* $G\mathcal{Z}C_{\rho,\rho^*}(\gamma,r,\kappa)\vee G\mathcal{Z}C_{\rho,\rho^*}(\mu,r,\kappa)=\underline{1}$

are equivalent.

Theorem 2.2. *A* $dfts$ (X,ρ,ρ^*) *is* $gDF\mathcal{Z}ed$ *space iff* $\forall$ *a* (r,κ)*-*$gf\mathcal{Z}o$ *set* γ $\&$ *a* (r,κ)*-*$gf\mathcal{Z}c$ *set* μ *set* $\ni\gamma\leq\mu$, $G\mathcal{Z}C_{\rho,\rho^*}(\gamma,r,\kappa)\leq G\mathcal{Z}I_{\rho,\rho^*}(\mu,r,\kappa)$.

3. Generalized Double Fuzzy $\mathcal{Z}$-Basically Disconnected Spaces

Definition 3.1. *Let* (X,ρ,ρ^*) *be a* $dfts$. *A fs* $\gamma\in I^X$ *is said to be an* (r,κ)*-generalized fuzzy*

(i) $\mathcal{Z}$*open-*F_σ *(briefly,* (r,κ)*-*$gf\mathcal{Z}o$*-*F_σ*) if* γ *is an* (r,κ)*-*$gf\mathcal{Z}o$ *and* (r,κ)*-fuzzy* F_σ *set.*

(ii) $\mathcal{Z}$*closed-*G_δ *(briefly,* (r,κ)*-*$gf\mathcal{Z}c$*-*G_δ*) if* γ *is an* (r,κ)*-*$gf\mathcal{Z}c$ *and* (r,κ)*-fuzzy* G_δ *set.*

(iii) $\mathcal{Z}$*clopen-*GF *(briefly,* (r,κ)*-*$gf\mathcal{Z}co$*-*GF*) if* γ *is an* (r,κ)*-*$gf\mathcal{Z}o$*-*F_σ $\&$ (r,κ)*-*$gf\mathcal{Z}c$*-*G_δ *set.*

(iv) $\mathcal{Z}G_\delta$*-closure of* γ *is defined by* $G\mathcal{Z}G_\delta C_{\rho,\rho^*}(\gamma,r,\kappa)=\bigwedge\{\mu\in I^X\backslash\gamma\leq\mu$ *and* μ *is* (r,κ)*-*$gf\mathcal{Z}c$*-*$G_\delta\}$.

(v) $\mathcal{Z}F_\sigma$*-interior of* γ *is defined by* $G\mathcal{Z}F_\sigma I_{\rho,\rho^*}(\gamma,r,\kappa)=\bigvee\{\mu\in I^X\backslash\mu\leq\gamma$ $\&$ μ *is* (r,κ)*-*$gf\mathcal{Z}o$*-*$F_\sigma\}$.

Remark 3.1. *Let* (X,ρ,ρ^*) *be a* $dfts$. $\forall$ $\gamma\in I^X$, $r\in I_0$ $\&$ $\kappa\in I_1$, *the statements hold:*

(i) $G\mathcal{Z}F_\sigma I_{\rho,\rho^*}(\gamma,r,\kappa)=G\mathcal{Z}G_\delta C_{\rho,\rho^*}(\underline{1}-\gamma,r,\kappa)$.

(ii) $G\mathcal{Z}G_\delta C_{\rho,\rho^*}(\gamma,r,\kappa)=G\mathcal{Z}F_\sigma I_{\rho,\rho^*}(\underline{1}-\gamma,r,\kappa)$.

Definition 3.2. *A* $dfts$ (X,ρ,ρ^*) *is called generalized double fuzzy* $\mathcal{Z}$*-basically (briefly* $gDF\mathcal{Z}$*-*b*) disconnected space if the* $G\mathcal{Z}G_\delta C_{\rho,\rho^*}(\gamma,r,\kappa)$ *is an* (r,κ)*-*$gf\mathcal{Z}o$*-*F_σ, $\forall$ (r,κ)*-*$gf\mathcal{Z}o$*-*F_σ γ *in* I^X.

Definition 3.3. *Let* (X,ρ_1,ρ_1^*) $\&$ (Y,ρ_2,ρ_2^*) *be* $dfts$*'s. A function* $f:(X,\rho_1,\rho_1^*)\to(Y,\rho_2,\rho_2^*)$ *is called* *(i)* *generalized double fuzzy $\mathcal{Z}F_\sigma$-open (briefly, $gDF\mathcal{Z}F_\sigma O$) if $f(\gamma)$ is an (r,κ)-$gf\mathcal{Z}o$-F_σ set in I^Y, $\forall$ (r,κ)-$gf\mathcal{Z}o$-F_σ in I^X.*

(ii) *generalized double fuzzy $\mathcal{Z}G_\delta$-continuous (briefly, $gDF\mathcal{Z}G_\delta Cts$) if $f^{-1}(\gamma)$ is an (r,κ)-$gf\mathcal{Z}c$-G_δ set in I^X, $\forall$ (r,κ)-fc and (r,κ)fuzzy G_δ set γ in I^Y.*

(iii) *generalized double fuzzy $\mathcal{Z}F_\sigma$-irresolute (briefly, $gDF\mathcal{Z}F_\sigma Irr$) if $f^{-1}(\gamma)$ is an (r,κ)-$gf\mathcal{Z}o$-F_σ set in I^X, $\forall$ (r,κ)-$gf\mathcal{Z}o$-F_σ in I^Y.*

Proposition 3.1. *Let (X,ρ_1,ρ_1^*) & (Y,ρ_2,ρ_2^*) be $dfts$'s. If a function $f:(X,\rho_1,\rho_1^*) \to (Y,\rho_2,\rho_2^*)$ is (i) $gDF\mathcal{Z}F_\sigma O$ surjective function, then $\forall$ fs γ in I^Y, $f^{-1}(G\mathcal{Z}G_\delta\ C_{\rho_2,\rho_2^*}(\gamma,\ r,\kappa)) \leq G\mathcal{Z}G_\delta C_{\rho_1,\rho_1^*}(f^{-1}\gamma,r,\kappa)$. (ii) $gDF\mathcal{Z}F_\sigma Irr$ function iff $f(G\mathcal{Z}G_\delta\ C_{\rho_1,\rho_1^*}(\gamma,r,\kappa)) \leq G\mathcal{Z}G_\delta C_{\rho_2,\rho_2^*}(f(\gamma),r,\kappa)$, $\forall$ fuzzy set γ in I^X.*

Theorem 3.1. *Let (X,ρ_1,ρ_1^*) & (Y,ρ_2,ρ_2^*) be $dfts$'s. A function $f:(X,\rho_1,\rho_1^*) \to (Y,\rho_2,\rho_2^*)$ is $gDF\mathcal{Z}F_\sigma Irr$, $gDF\mathcal{Z}F_\sigma O$ onto function $\ni$ (X,ρ_1,ρ_1^*) is a $gDF\mathcal{Z}$-b disconnected space, then (Y,ρ_2,ρ_2^*) is $gDF\mathcal{Z}$-b disconnected space.*

Theorem 3.2. *For a $dfts$ (X,ρ,ρ^*), the statements:*

(i) *(X,ρ,ρ^*) is a $gDF\mathcal{Z}$-b disconnected space.*

(ii) *$\forall$ an (r,κ)-$gf\mathcal{Z}c$-G_δ set γ, $G\mathcal{Z}F_\sigma I_{\rho,\rho^*}(\gamma,r,\kappa)$ is an (r,κ)-$gf\mathcal{Z}c$-G_δ set.*

(iii) *$\forall$ an (r,κ)-$gf\mathcal{Z}o$-F_σ set γ, $G\mathcal{Z}G_\delta C_{\rho,\rho^*}(\gamma,r,\kappa) \vee G\mathcal{Z}G_\delta C_{\rho,\rho^*}(\underline{1} - G\mathcal{Z}G_\delta C_{\rho,\rho^*}(\gamma, r,\kappa) r,\kappa) = \underline{1}$.*

(iv) *$\forall$ an (r,κ)-gfo-F_σ set γ & $\mu \ni G\mathcal{Z}G_\delta C_{\rho,\rho^*}(\gamma,r,\kappa) \vee \mu = \underline{1}$, $G\mathcal{Z}G_\delta C_{\rho,\rho^*}(\gamma,r,\kappa) \vee G\mathcal{Z}G_\delta C_{\rho,\rho^*}(\mu,r,\kappa) = \underline{1}$*

are equivalent.

Theorem 3.3. *A $dfts$ (X,ρ,ρ^*) is a $gDF\mathcal{Z}$-b disconnected iff $\forall$ (r,κ)-$gf\mathcal{Z}o$-F_σ set γ and (r,κ)-$gf\mathcal{Z}c$-G_δ set $\mu \ni \gamma \leq \mu$, $G\mathcal{Z}G_\delta C_{\rho,\rho^*}(\gamma,r,\kappa) \leq G\mathcal{Z}F_\sigma I_{\rho,\rho^*}(\mu,r,\kappa)$.*

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