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GENERALIZED FUZZY \mathcal{Z} -CLOSURE IRRESOLUTE MAPPINGS IN DOUBLE FUZZY TOPOLOGICAL SPACES

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ABSTRACT. We introduce and investigate some new class of mappings called double fuzzy generalized Z-open irresolute map, double fuzzy generalized Zclosed irresolute map, double fuzzy generalized Z-continuous map, double fuzzy generalized Z-irresolute map and double fuzzy generalized Z-closure irresolute map in double fuzzy topological spaces. Also, some of their fundamental properties are studied.

1. INTRODUCTION AND PRELIMINARIES

In 1986, Atanassov [2] started 'Intuitionistic fuzzy sets' and Coker [3] in 1997, initiated Intuitionistic fuzzy topological space". The term "double" instead of "intuitionistic" coined by Garcia and Rodabaugh [5] in 2005. In the previous two decades many analysts [8, 9] accomplishing more applications on double fuzzy topological spaces. From 2011, \mathcal{Z} -open sets and maps were introduced in topological spaces by El-Maghrabi and Mubarki [7].

X denotes a non-empty set, $I_1 = [0, 1)$, $I_0 = (0, 1]$, I = [0, 1], $0 = \underline{0}(X)$, $1 = \underline{1}(X)$, $r \in I_0$ & $\kappa \in I_1$ and always $1 \ge r + \kappa$. I^X is a family of all fuzzy sets on X. In 2002, double fuzzy topological spaces (briefly, dfts), (X, η, η^*) , (r, κ) -fuzzy open (resp. (r, κ) -fuzzy closed) (briefly (r, κ) -fo (resp. (r, κ) -fc)) were defined.

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All other undefined notions are from [1,4,6,7,9–11] and cited there in.

2. Generalized double fuzzy \mathcal{Z} -closure irresolute functions

In this section, some characterisations of generalized double fuzzy \mathcal{Z} -continuous (resp. (open, closed, closure) irresolute) functions are studied.

Definition 2.1. A map $f : (X, \rho_1, \rho_1^*) \to (Y, \rho_2, \rho_2^*)$ is called as ageneralized double fuzzy

- (i) Z-open irresolute (briefly, gDFZoIrr) if for every (r, κ) - $gfZoset \gamma \in I^X, \kappa \in I_1$ and $r \in I_0$, $f(\gamma)$ is an (r, κ) -gfZo in I^Y .
- (ii) \mathbb{Z} -closed irresolute (briefly, $gDF\mathbb{Z}cIrr$) if for every (r, κ) - $gf\mathbb{Z}c$ set $\gamma \in I^X$, $\kappa \in I_1$ and $r \in I_0$, $f(\gamma)$ is an (r, κ) - $gf\mathbb{Z}c$ in I^Y .
- (iii) Z-continuous (briefly, gDFZCts) if for every (r, κ) -fo set $\gamma \in I^Y, r \in I_0$ and $\kappa \in I_1, f^{-1}(\gamma)$ is an (r, κ) -gfZo in I_X .
- (iv) Z-irresolute (briefly, gDFZIrr) if $f^{-1}(\mu)$ is (r, κ) -gfZo set $\forall (r, \kappa)$ -gfZo set $\mu \in I^Y$.
- (v) \mathbb{Z} -closure irresolute (briefly, $gDF\mathbb{Z}Cl\ Irr$) if $f^{-1}(G\mathbb{Z}C_{\rho}, \rho^{*}(f(\gamma), r, \kappa))$ is an (r, κ) - $gf\mathbb{Z}c$ set for every (r, κ) - $gf\mathbb{Z}c$ set $\gamma \in I^{Y}, r \in I_{0}$ and $\kappa \in I_{1}$.

Theorem 2.1. For any bijective map $f : (X, \rho_1, \rho_1^*) \to (Y, \rho_2, \rho_2^*)$, the below stated statements are equivalent.

(i) f is gDFZCl Irr function.

(ii) For every fuzzy set $\gamma \in I^X$, $f(G\mathcal{Z}C_{\rho_1,\rho_1^*}(\gamma, r, \kappa)) \leq G\mathcal{Z}C_{\rho_2,\rho_2^*}(f(\gamma), r, \kappa)$.

(iii) For every fuzzy set
$$\gamma$$
 in I^{γ} , $GZC_{\rho_1,\rho_1^*}(f^{-1}(\gamma), r, \kappa) \leq f^{-1}(GZC_{\rho_2,\rho_2^*}(\gamma, r, \kappa))$.

Proof.

 $\begin{array}{ll} (i) \ \Rightarrow \ (ii) \ : \ \text{Suppose} \ \gamma \ \in \ I^X \ \text{and} \ G\mathcal{Z}C_{\rho_1,\rho_1^*}(f(\gamma),r,\kappa) \ \in \ I^Y \ \text{is an} \ (r,\kappa)-gf\mathcal{Z}c, \\ gf\mathcal{Z}c, \ \text{then by (i), we have} \ f^{-1}(G\mathcal{Z}C_{\rho_2,\rho_2^*} \ (f(\gamma),r,\kappa) \ \in \ I^X \ \text{is an} \ (r,\kappa)-gf\mathcal{Z}c, \\ \text{set,} \ r \ \in \ I_0 \ \text{and} \ \kappa \ \in \ I_1. \ \text{Therefore,} \ G\mathcal{Z}C_{\rho_2,\rho_2^*}(f^{-1}(G\mathcal{Z}C_{\rho_2,\rho_2^*}(f(\gamma),r,\kappa)),r,\kappa) = \\ f^{-1}(G\mathcal{Z}C_{\rho_2,\rho_2^*}(f(\gamma),r,\kappa)). \ \text{Since} \ \gamma \ \leq \ f^{-1}(f(\gamma)) \ \text{and} \ G\mathcal{Z}C_{\rho_1,\rho_1^*}(\gamma,r,\kappa) \ \leq \ G\mathcal{Z}C_{\rho_2,\rho_2^*}(f^{-1}(G\mathcal{Z}C_{\rho_2,\rho_2^*}(f(\gamma),r,\kappa)),r,\kappa) = \\ (f^{-1}(f(\gamma),r,\kappa)). \ \text{Also,} \ f(\gamma) \ \leq \ G\mathcal{Z}C_{\rho_2,\rho_2^*} \ (f(\gamma),r,\kappa). \ \text{Then} \ G\mathcal{Z}C_{\rho_1,\rho_1^*}(\gamma,r,\kappa) \ \leq \\ G\mathcal{Z}C_{\rho_2,\rho_2^*}(f^{-1}(G\mathcal{Z}C_{\rho_2,\rho_2^*}(f(\gamma),r,\kappa),r,\kappa)) = f^{-1}(G\mathcal{Z}C_{\rho_2,\rho_2^*}(f(\gamma),r,\kappa)). \end{array}$

 $(ii) \Rightarrow (iii)$: Suppose $\gamma \in I^Y$, by (ii), $f(GZC_{\rho_2,\rho_2^*}(f^{-1}(\gamma), r, \kappa)) \leq GZC_{\rho_2,\rho_2^*}(f^{-1}(\gamma), r, \kappa)) \leq GZC_{\rho_2,\rho_2^*}(\gamma, r, \kappa)$. That is, $f(GZC_{\rho_1,\rho_1^*}(f^{-1}(\gamma), r, \kappa)) \leq GZC_{\rho_2,\rho_2^*}(\gamma, r, \kappa)$.

 (γ, r, κ) . Therefore, $f^{-1}(f(GZC_{\rho_1, \rho_1^*}(f^{-1}(\gamma), r, \kappa))) \leq f^{-1}(GZC_{\rho_2, \rho_2^*}(\gamma, r, \kappa))$. Hence, $GZC_{\rho_1, \rho_1^*}(f^{-1}(\gamma), r, \kappa) \leq f^{-1}(GZC_{\rho_2, \rho_2^*}(\gamma, r, \kappa))$.

 $\begin{array}{l} (iii) \Rightarrow (i): \text{Suppose } \gamma \in I^Y \text{ is an } (r, \kappa) \text{-} gf \mathcal{Z}c \text{ set. Then, } G\mathcal{Z}C_{\rho_2,\rho_2^*}(\gamma, r, \kappa) = \gamma.\\ \text{By (iii), } G\mathcal{Z}C_{\rho_1,\rho_1^*}(f^{-1}(\gamma), r, \kappa) \leq f^{-1}(G\mathcal{Z}C_{\rho_2,\rho_2^*}(\gamma, r, \kappa)) = f^{-1}(\gamma). \text{ But } f^{-1}(\gamma)\\ \leq G\mathcal{Z}C_{\rho_1,\rho_1^*}(f^{-1}(\gamma), r, \kappa). \text{ Therefore, } f^{-1}(\gamma) = G\mathcal{Z}C_{\rho_2,\rho_2^*}(\gamma, r, \kappa) \text{ i.e., } f^{-1}(\gamma) \in I^X\\ \text{ is } (r, \kappa) \text{-} gf \mathcal{Z}c. \text{ Thus } f \text{ is } gDF\mathcal{Z}Cl Irr \text{ map.} \end{array}$

Proposition 2.1. The map $f : (X, \rho_1, \rho_1^*) \to (Y, \rho_2, \rho_2^*)$ is a $gDF\mathcal{Z}cIrr$ iff $\forall \gamma \in I^X$, $G\mathcal{Z}C_{\rho_1,\rho_1^*}(f(\gamma), r, \kappa) \leq f(G\mathcal{Z}C_{\rho_1,\rho_1^*}(\gamma, r, \kappa)).$

Proposition 2.2. For any gDFZCl Irr function $f : (X, \rho_1, \rho_1^*) \to (Y, \rho_2, \rho_2^*)$, GZ $\mathcal{B}_{\rho_1,\rho_1^*}(f^{-1}(\gamma), r, \kappa)$ is zero for every (r, κ) -gfZo set $\gamma \in I^Y$.

Definition 2.2. A dfts (X, ρ, ρ^*) is said to be a double fuzzy \mathcal{Z} - $(\rho, \rho^*)_{1/2}$ space (briefly, $DF\mathcal{Z}$ - $(\rho, \rho^*)_{1/2}$), if each (r, κ) - $f\mathcal{Z}c$ set is (r, κ) -fc set in X.

Proposition 2.3. For any dfts (X, ρ_1, ρ_1^*) and $DF\mathcal{Z}$ - $(\rho, \rho^*)_{1/2}$ space (Y, ρ_2, ρ_2^*) , if the map $f : (X, \rho_1, \rho_1^*) \to (Y, \rho_2, \rho_2^*)$ is a bijective, then

- (i) f and f^{-1} are gDFZCl Irr
- (ii) f is gDFZCts and gDFZoIrr.
- (iii) f is gDFZCts and gDFZcIrr.
- (iv) $f(GZC_{\rho_1,\rho_1^*}(\gamma, r, \kappa)) = GZC_{\rho_2,\rho_2^*}(f(\gamma), r, \kappa)$, for every $\gamma \in I^X$.

are equivalent.

Proof.

 $(i) \Rightarrow (ii)$: Suppose γ is an (r, κ) -gfZo set in X. Since f^{-1} is gDFZCl Irr, $(f^{-1})^{-1}(\gamma) \in I^Y$ is (r, κ) -gfZo set, so f is gDFZoIrr. Now, let $\nu \in I^Y$ be an (r, κ) -fZo set, then it is an (r, κ) -gfZo. But by hypothesis, f^{-1} are gDFZCl Irr, then $f^{-1}(\nu) \in I^X$ is an (r, κ) -gfZo, i.e, f is gDFZCts.

 $(ii) \Rightarrow (iii)$: Obvious.

 $\begin{array}{ll} (iii) \Rightarrow (iv) : \text{If } \gamma \in I^X \text{, then } \gamma \leq f^{-1}(f(\gamma)) \text{ and } f(\gamma) \leq G\mathcal{Z}C_{\rho_2,\rho_2^*}(f(\gamma),r,\kappa) \\ \Rightarrow \gamma \leq f^{-1}(G\mathcal{Z}C_{\rho_2,\rho_2^*}(f(\gamma),r,\kappa). \text{ Now, } G\mathcal{Z}C_{\rho_2,\rho_2^*}(f(\gamma),r,\kappa) \in I^Y \text{ is an } (r,\kappa) \\ gf\mathcal{Z}c \text{ set. But } (Y,\rho_2,\rho_2^*) \text{ is a } DF\mathcal{Z} \\ (\rho,\rho^*)_{1/2} \text{ space, and } G\mathcal{Z}C_{\rho_2,\rho_2^*}(f(\gamma),r,\kappa) \\ \text{ is an } (r,\kappa) \\ \text{-}fc \text{ set, then } G\mathcal{Z}C_{\rho_2,\rho_2^*}(f(\gamma),r,\kappa) \in I^Y \text{ is an } (r,\kappa) \\ \text{-}f\mathcal{Z}c \text{ set. Since } f \text{ is } gDF\mathcal{Z}Cts, \ f^{-1}(G\mathcal{Z}C_{\rho_2,\rho_2^*}(f(\gamma),r,\kappa)) \text{ is an } (r,\kappa) \\ \text{-}gf\mathcal{Z}c \text{ set, which implies, } G\mathcal{Z}C_{\rho_2,\rho_2^*}(f^{-1}(G\mathcal{Z}C_{\rho_2,\rho_2^*}(f(\gamma),r,\kappa)),r,\kappa) = f^{-1}(G\mathcal{Z}C_{\rho_2,\rho_2^*}(f(\gamma),r,\kappa)). \text{ But } G\mathcal{Z}C_{\rho_2,\rho_2^*} \quad (f(\gamma), r,\kappa) \leq G\mathcal{Z}C_{\rho_2,\rho_2^*}(f^{-1}(G\mathcal{Z}C_{\rho_2,\rho_2^*}(f(\gamma),r,\kappa)),r,\kappa) \text{ and } \end{array}$

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 $\begin{aligned} G\mathcal{Z}C_{\rho_2,\rho_2^*}(f(\gamma),r,\kappa) &\leq f^{-1}(G\mathcal{Z}C_{\rho_2,\rho_2^*}(f(\gamma),r,\kappa)). \text{ Therefore, } f(G\mathcal{Z}C_{\rho_1,\rho_1^*}(\gamma,r,\kappa)) &\leq \\ G\mathcal{Z}C_{\rho_2,\rho_2^*}(f(\gamma),r,\kappa). \text{ Also,} \end{aligned}$

 $G\mathcal{Z}C_{\rho_2,\rho_2^*}(f(\gamma),r,\kappa) \leq f(G\mathcal{Z}C_{\rho_1,\rho_1^*}(\gamma,r,\kappa)) \Rightarrow f(G\mathcal{Z}C_{\rho_1,\rho_1^*}(\gamma,r,\kappa)) = G\mathcal{Z}C_{\rho_2,\rho_2^*}(f(\gamma),r,\kappa).$

 $(iv) \Rightarrow (i)$: Let $\gamma \in I^X$, by hypothesis of (iv), $f(GZC_{\rho_1,\rho_1^*}(\gamma), r, \kappa) = GZC_{\rho_2,\rho_2^*}(f(\gamma), r, \kappa)$. Therefore, $f(GZC_{\rho_1,\rho_1^*}(f(\gamma), r, \kappa) \leq GZC_{\rho_2,\rho_2^*}(f(\gamma), r, \kappa)$. Then, f is a gDFZCl Irr function. Now, suppose $\nu \in I^Y$ is (r, κ) -gfZc. Then $GZC_{\rho_2,\rho_2^*}(\nu, r, \kappa) = \nu \Rightarrow f(GZC_{\rho_2,\rho_2^*}(\nu, r, \kappa)) = f(\nu)$.

But, by (iv), $GZC_{\rho_1,\rho_1^*}(f(\nu), r, \kappa) = f(GZC_{\rho_2,\rho_2^*}(\nu, r, \kappa))$. Therefore, $GZC_{\rho_1,\rho_1^*}(f(\nu), r, \kappa) = f(\nu)$, then $f(\nu) \in I^Y$ is an (r, κ) -gfZc set. Therefore, f^{-1} is a gDFZCl Irr.

Proposition 2.4. For any two dfts's (X, ρ_1, ρ_1^*) and (Y, ρ_2, ρ_2^*) and any map $f : (X, \rho_1, \rho_1^*) \to (Y, \rho_2, \rho_2^*)$.

- (i) If f is any function, then $GZE_{\rho_1,\rho_1^*}(f^{-1}(\delta), r, \kappa) \leq GZC_{\rho_1,\rho_1^*}(1-f^{-1}(\delta), r, \kappa, \forall fuzzy \text{ set } \delta \in I^Y.$
- (ii) If f is a gDFZCts function, then for every (r, κ) - $fZc \ \delta \in I^Y$, we have $GZ\mathcal{B}_{\rho_1,\rho_1^*}(f^{-1}(\delta), r, \kappa) = GZ\mathcal{F}_{\rho_1,\rho_1^*}(f^{-1}(\delta), r, \kappa).$

Proposition 2.5. Let (X, ρ_1, ρ_1^*) & (Y, ρ_2, ρ_2^*) be dfts's and let $f : (X, \rho_1, \rho_1^*) \rightarrow (Y, \rho_2, \rho_2^*)$ be gDFZCl Irr function $\ni (X, \rho_1, \rho_1^*)$ is $g^*DFZ \cdot (\rho, \rho^*)_{1/2}$ space. Then, for every (r, κ) -gfZc set $\delta \in I^Y$, the following statements hold:

(i) $G\mathcal{B}_{\rho_1,\rho_1^*}(f^{-1}(\delta), r, \kappa) = G\mathcal{F}_{\rho_1,\rho_1^*}(f^{-1}(\delta), r, \kappa).$ (ii) $G\mathcal{E}_{\rho_1,\rho_1^*}(f^{-1}(\delta), r, \kappa) = 1 - f^{-1}(\delta).$

Proposition 2.6. Let $f : (X, \rho_1, \rho_1^*) \to (Y, \rho_2, \rho_2^*)$ be gDFZ-closed map such that (Y, ρ_2, ρ_2^*) is g^*DFZ - $(\rho, \rho^*)_{1/2}$ space. Then, $\forall (r, \kappa)$ -gfZc set δ in I^X , the following statements hold:

(i) $G\mathcal{B}_{\rho_2,\rho_2^*}(f(\delta), r, \kappa) = G\mathcal{F}_{\rho_2,\rho_2^*}(f(\delta), r, \kappa).$ (ii) $G\mathcal{E}_{\rho_2,\rho_2^*}(f(\delta), r, \kappa) = 1 - f(\delta).$

Proposition 2.7. Let (X, ρ_1, ρ_1^*) , $(Y, \rho_2, \rho_2^*) \& (Z, \rho_3, \rho_3^*)$ be dfts's, $f : (X, \rho_1, \rho_1^*) \rightarrow (Y, \rho_2, \rho_2^*)$ and $g : (Y, \rho_2, \rho_2^*) \rightarrow (Z, \rho_3, \rho_3^*)$ be gDFZCl Irr functions $\ni (X, \rho_1, \rho_1^*)$ is g^*DFZ - $(\rho, \rho^*)_{1/2}$ space. Then, $\forall (r, \kappa)$ -gfZc set in Z, the following statements hold:

(i) $G\mathcal{B}_{\rho_1,\rho_1^*}((g \circ f)^{-1}(\delta), r, \kappa) = G\mathcal{F}_{\rho_1,\rho_1^*}((g \circ f)^{-1}(\delta), r, \kappa).$

(ii) $G\mathcal{E}_{\rho_1,\rho_1^*}((g \circ f)^{-1}(\delta), r, \kappa) = \underline{1} - (g \circ f)^{-1}(\delta).$

3. CONCLUSION

We have introduced and investigated some new class of mappings called double fuzzy generalized \mathcal{Z} -open irresolute map, double fuzzy generalized \mathcal{Z} -closed irresolute map, double fuzzy generalized \mathcal{Z} -continuous map, double fuzzy generalized \mathcal{Z} -closure irresolute map and double fuzzy generalized \mathcal{Z} -closure irresolute map in dfts. Also, some of their fundamental properties are studied.

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