ADV MATH SCI JOURNAL
Advances in Mathematics: Scientific Journal **9** (2020), no.4, 1905–1912
ISSN: 1857-8365 (printed); 1857-8438 (electronic)
https://doi.org/10.37418/amsj.9.4.49 Spec. Issue on NCFCTA-2020

AN EDGE PRIME OF SOME GRAPHS

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ABSTRACT. Let $G=(V,E)$ be a (l,m) graph. A bijection $g:E\to\{1,2,...,m\}$ is said to be an edge prime labeling if for each edge $ab\in E$, we have

$$gcd(g^{+}(a),g^{+}(b))=1,$$

where $g^{+}(a)=\sum_{ac\in E}g(ac)$. Moreover, a bijection $g:E\to\{1,2,...,m\}$ is semiedge prime labeling if for each $ab\in E$, either $gcd(g^{+}(a),g^{+}(b))=1$ or $g^{+}(a)=g^{+}(b)$. A graph that admits an edge prime (or semiedge prime) labeling is said to be an edge prime (or semiedge prime) graph. In this paper, we prove that if G has an edge prime, then $G\cup P_n$ is an edge prime graph. Also, we obtain $\theta(3^{[m]})\odot\theta(3^{[n]})$, $n\not\equiv 0(mod\ 5)$ and some graphs superimposing of path are an edge (or semiedge) prime graph.

1. INTRODUCTION

Let $G=(V,E)$ be a simple, finite, undirected graph with vertex set V and edge set E of order $|V|=l$ and $|E|=m$. For all other notations and terminology in graph, we refer Balakrishnan. R and Renganathan. K [1]. A graph labeling is an assignment of integers to the vertices or edges or both subject to the certain conditions. An excellent survey on graph labeling is maintained by Gallian [2]. A graph with vertex set V is said to have a prime labeling if its vertices are labeled with distinct integers $1,2,3,...,|V|$ such that for each edge xy the lables assigned to x and y are relatively prime. A graph which admits

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2010 *Mathematics Subject Classification.* 05C78.
Key words and phrases. Prime labeling, Edge prime labeling, Semiedge prime labeling.

prime labeling is called a prime graph. The notion of a prime labeling was originated by Roger Entinger and it was discussed in a paper by Tout et.al., [5]. The following is taken from [4] “Shiu et.al., [4] was introduced the concept an edge (or semiedge) prime graph.

Let $G = (V, E)$ be a graph with l vertices and m edges. A bijecction function $g : E \rightarrow \{1, 2, 3, ..., m\}$ is called an edge prime labeling if for each edge $ab \in E(G)$, we have $gcd(g^+(a), g^+(b)) = 1$, where $g^+(a) = \sum_{ac\in E} g(ac)$. Moreover, a bijection $g : E \rightarrow \{1, 2, 3, ..., m\}$ is a semiedge prime labeling if for each edge $ab \in E(G)$, we have either $gcd(g^+(a), g^+(b)) = 1$ or $g^+(a) = g^+(b)$. A graph which admits an edge (or semiedge) prime labeling is called an edge (or semiedge) prime graph. They [3] obtained a necessary and sufficient condition for disjoint union of path to be edge prime.

We use (x, y) instead of $gcd(x, y)$ if there is no ambiguous. They [3] also determined that all 2- regular graphs, bipartite and tripartite graphs are edge prime. Also, they [3] investigated that bipartite and tripartite graphs are a semiedge prime. The generalized theta graph $\theta(S_1, S_2, ..., S_k)$ consists of a pair of end vertices joined by $k \geq 3$ internally disjoint paths of lengths $S_1, S_2, ..., S_k$, $k \geq 1$.

Let $DS(m, n)$ be the double star with $V(DS(m, n)) = \{a, b, r_i, s_j : 1 \leq i \leq m, 1 \leq j \leq n\}$ and $E(DS(m, n)) = \{ab, ar_i, bs_j : 1 \leq i \leq m, 1 \leq j \leq n\}$.” The following definition taken from [3] “If $G_1(p_1, q_1)$ and $G_2(p_2, q_2)$ are two connected graphs, then the graph obtained by superimposing any selected vertex of G_2 on any selected vertex of G_1 is denoted by $G_1 \odot G_2$. The resultant graph $G = G_1 \odot G_2$ contains $p_1 + p_2 - 1$ vertices $q_1 + q_2$ edges.”

In the present work C_n denoted by the cycle with n vertices and P_n denotes the path of n vertices. In the wheel $W_n = C_n + K_1$, the vertex corresponding to K_1 is called the open vertex and the vertices, corresponding to C_n are called the rim vertices.

In this paper, we investigate if G is an edge prime graph, then $G \cup P_n$ is edge prime graph, $\theta(3^{[m]}) \odot \theta(3^{[n]})$, $n \not\equiv 0 (mod\ 5)$, bipartite and tripartite graphs superimposing of path are edge (a semiedge) prime graphs.

2. MAIN RESULTS

Theorem 2.1. *If G has an edge prime, then $G \cup P_n$ is an edge prime.*

Proof. Let $G(d^*, e^*)$ be an edge prime graph. Define $g : E(G) \to \{1, 2, ..., e^*\}$ with the property that for each edge $rs \in E(G)$, the numbers $g^+(r), g^+(s)$ are relatively prime. Consider the path P_n with vertex set $\{c_h : 1 \leq h \leq n\}$ edge set $\{c_h c_{h+1} : 1 \leq h \leq n-1\}$. Define a new graph $G^1 = G \cup P_n$ with vertex set $V^1 = V \cup \{c_h : 1 \leq h \leq n\}$ and edge set $E^1 = E \cup \{c_h c_{h+1} :\leq h \leq n-1\}$. Define the bijective function $g : E^1(G^1) \to \{1, 2, ..., d^* + e^*, d^* + e^* + 1, ..., d^* + e^* + n - 1\}$ by $g(ab) = f(ab)$ for all $ab \in E(G)$, $g(c_h c_{h+1}) = g + h, 1 \leq h \leq n-1$. To prove that G^1 is an edge prime graph. In earlier, G is an edge prime graph, it is enough to prove that for any $ab \in E^1$ which is not in G, the numbers $g^+(a), g^+(b)$ are relatively prime, For any edge $c_h c_{h+1} \in E^1$, $2 \leq h \leq n-1$, $(g^+(c_h), g^+(c_{h+1})) = (2e^* + 2h - 1, 2e^* + 2h + 1) = 1$, $(g^+(c_1), g^+(c_2)) = (e^* + 1, 2e^* + 2h - 1) = 1$, $(g^+(c_{n-1}), g^+(c_n)) = (2e^* + 2n - 3, e^* + n - 1) = 1$. It is easily verified that, for any edge $rs \in E(G)$, $g^+(r), g^+(s)$ are relatively prime. Hence G is an edge prime. □

Theorem 2.2. *The graph $G = K_{(2,m)} \odot P_n$ is an edge prime.*

Proof. Let $G = K_{2,m} \odot P_n$ be a graph. Then $V(G) = \{a_\alpha, b_\beta, c_\gamma : 1 \leq \alpha \leq 2, 1 \leq \beta \leq m, 1 \leq \gamma \leq n\}$ and $E(G) = \{a_\alpha b_\beta : 1 \leq \alpha \leq 2, 1 \leq \beta \leq m\} \cup \{a_1 c_1\} \cup \{c_\gamma c_{\gamma+1} : 1 \leq \gamma \leq n-1\}$. G has $2m + n$ edges. Let $g : E(G) \to \{1, 2, ..., 2m + n\}$ be defined by, for each $1 \leq \beta \leq m$, $g(a_1 b_\beta) = 2\beta - 1, g(a_2 b_\beta) = 2m + 2 - 2\beta, g(a_1 c_1) = 2m + 1$, for each $1 \leq \gamma \leq n-1, g(c_\gamma c_{\gamma+1}) = 2m + 1 + \gamma$. Clearly, $g^+(a_2) = m^2 + m, g^+(a_1) = m^2 + 2m + 1, g^+(c_\gamma) = 4m + 2\gamma + 1, 1 \leq \gamma \leq n-1, g^+(c_n) = 2m + n$. It can be easily verified that $(g^+(a_1), g^+(b_\beta)) = (g^+(a_2), g^+(b_\beta)) = (g^+(a_1), g^+(c_1)) = (g^+(c_\alpha), g^+(c_{\alpha+1})) = 1$. Hence, G is an edge prime. □

Theorem 2.3. *The graph $G = \theta(3^{[m]}) \odot \theta(3^{[n]}), n \not\equiv 1 (mod 5)$ is an edge prime.*

Proof. Let $G = \theta(3^{[m]}) \odot \theta(3^{[n]}), n \not\equiv 0 (mod\ 5)$ be a graph.

Then $V(G) = \{a, b, c, a_w, b_w, r_x, c_x : 1 \leq w \leq n, 1 \leq x \leq m\}$ and $E(G) = \{aa_w, a_w b_w, bb_w : 1 \leq w \leq n\} \cup \{br_x, r_x c_x, cc_x : 1 \leq x \leq m\}$. Note that $|E(G)| = 3(n + m)$. Define $g : E(G) \to \{1, 2, ..., 3n + 3m\}$ by for each $1 \leq w \leq n$, $g(aa_w) = w, g(a_w b_w) = 2n + 1 - w, g(bb_w) = 2n + w$, for each $1 \leq w \leq m, g(br_x) = 3n + m + 1 - x, g(r_x c_x) = 3n + m + x, g(cc_x) = 3n + 3m + 1 - x$. Clearly, $g^+(a) = \frac{n(n+1)}{2}, g^+(a_w) = 2n + 1, g^+(b_w) = 4n + 1, g^+(b) = \frac{n(5n+1)}{2} + \frac{m(m+6n+1)}{2}, g^+(r_x) = 6n + 2m + 1, g^+(c_x) = 6n + 4m + 1, g^+(c) = \frac{m(5m+6n+1)}{2}$. It can be easily verified that, $(g^+(a), g^+(a_w)) = (g^+(a_w), g^+(b_w)) = (g^+(b), g^+(b_w)) = (g^+(b), g^+(r_x)) = (g^+(r_x), g^+(c_x)) = (g^+(c_x), g^+(c)) = 1$. Hence, G is an edge prime labeling. □

Theorem 2.4. *The graph* $\theta(3^{[m]}) \odot P_n, m \not\equiv 2(mod 5)$ *is an edge prime.*

Proof. Let $G = \theta(3^{[m]}) \odot P_n, m \not\equiv 2(mod 5)$ be a graph. Then $V(G) = \{a, b, a_\alpha, b_\alpha,, c_\beta : 1 \leq \alpha \leq m, 1 \leq \beta \leq n\}$ and $E(G) = \{aa_\alpha, a_\alpha b_\alpha, b_\alpha b : 1 \leq \alpha \leq m\} \cup \{bc_1\} \cup \{c_\gamma c_{\gamma+1} : 1 \leq \gamma \leq n-1\}$. Hence, $|E(G)| = 3m + n$. Let the labeling $g : E(G) \to \{1, 2, ..., 3m + n\}$ by $g(aa_\alpha) = \alpha$, for each $1 \leq \alpha \leq m, g(a_\alpha b_\alpha) = 2m + 1 - \alpha, g(b_\alpha b) = 2m + 1 - \alpha, g(ac_\alpha) = 3m + 1, g(c_\gamma c_{\gamma+1}) = 3m + 1 + \gamma$ for $1 \leq \gamma \leq n - 1$. Clearly $g^+(a) = \frac{m^2+7m+2}{2}, g^+(a_\alpha) = 2m + 1, g^+(b_\alpha) = 4m + 1, g^+(b) = \frac{5m^2+m}{2}, g^+(c_\gamma) = 6m + 2\gamma + 1$ for $1 \leq \gamma \leq n - 1$ and $g^+(c_n) = 3m + n$. It's enough to prove that $(g^+(a), g^+(a_\alpha)) = (g^+(a_\alpha), g^+(b_\alpha)) = (g^+(b_\alpha), g^+(b)) = (g^+(a), g^+(c_1)) = (g^+(c_\alpha), g^+(c_{\alpha+1})) = 1$. Hence $\theta(3^{[m]}) \odot P_n, m \not\equiv 2(mod 5)$ is an edge prime. □

Theorem 2.5. *The graph* $\theta(4^{[m]}) \odot P_n, m \not\equiv 1(mod 3)$ *is an edge prime.*

Proof. Let $G = \theta(4^{[m]}) \odot P_n, m \not\equiv 1(mod 3)$ be a graph. Then $V(G) = \{r, b, s_d, t_d, a_d, c_e : 1 \leq d \leq m, 1 \leq e \leq n\}$ and $E(G) = \{rs_d, s_d t_d, t_d a_d, a_d b : 1 \leq d \leq m\} \cup \{rc_1, c_f c_{f+1} : 1 \leq f \leq n - 1\}$. Also,$|E(G)| = 4m + n$. Define a bijective function $g : E(G) \to \{1, 2, ..., 4m + n\}$ be as follows: $g(rs_d) = d$ for $1 \leq d \leq n, g(s_d t_d) = 2m + 1 - d$ for $1 \leq d \leq m, g(t_d a_d) = 2m + d, 1 \leq d \leq m, g(a_d b) = 4m + 1 - d$ for $1 \leq d \leq m, g(rc_1) = 4m + 1, g(c_f c_{f+1}) = 4m + f + 1$ for $1 \leq f \leq n - 1$. Clearly, $g^+(r) = \frac{m^2+9m+2}{2}, g^+(s_d) = 2n + 1, g^+(t_d) = 4m + 1, g^+(a_d) = 6m + 1, g^+(b) = \frac{7m^2+m}{2}, g^+(c_f) = 8m + 2f + 1$ for $1 \leq f \leq n - 1$ and $g^+(c_n) = 4m + n$. It can be easily verified that $(g^+(r), g^+(s_d)) = ((g^+(s_d), g^+(t_d)) = (g^+(t_d), g^+(a_d)) = (g^+(a_d), g^+(b) = (g^+(r)), g^+(c_1)) = (g^+(c_f), g^+(c_{f+1})) = 1$. Hence $\theta(4^{[m]}) \odot P_n, m \not\equiv 1(mod 3)$ is an edge prime. □

Theorem 2.6. *The graph* $\theta(m, m, m) \odot P_n$ *is edge prime for* $m \not\equiv 0(mod\ 7)$.

Proof. For $m = 3, 4$ the result follows from theorem 2.4 and 2.5. We may assume that $m \geq 5$. Let $G = \theta(m, m, m) \odot P_n, (m \not\equiv 0(mod\ 7))$ be a graph. Then $V(G) = \{a, b, r_i, s_i, t_i, c_j : 1 \leq i \leq n - 1, 1 \leq j \leq n\}$ and $E(G) = \{ar_1, as_1, at_1, r_{m-1}b, s_{m-1}b, t_{m-1}b, ac_1\} \cup \{r_i r_{i+1}, s_i s_{i+1}, t_i t_{i+1}, c_j c_{j+1} : 1 \leq i \leq m - 2, 1 \leq j \leq n - 1\}$. Clearly, $|E(G)| = 3m + n$. Let $g : E(G) \to \{1, 2, 3, ..., 3m + n\}$ be defined as follows: $g(ar_1) = 1, g(as_1) = 2, g(at_1) = 3, g(ac_1) = 3m + 1$.

$g(r_{i-1}r_i) = 3i, g(s_{i-1}s_i) = 3i-1, g(t_{i-1}t_i) = 3i-2$ for even $i \geq 2$.
$g(r_{i-1}r_i) = 3i-2, g(s_{i-1}s_i) = 3i-1, g(t_{i-1}t_i) = 3i$ for odd $i \geq 3$.
$g(r_{m-1}b) = 3m, g(s_{m-1}b) = 3m-1, g(t_{m-1}b) = 3m-2$ if m is even.
$g(r_{m-1}b) = 3m-2, g(s_{m-1}b) = 3m-1, g(t_{m-1}b) = 3m$ if m is odd.
$g(c_jc_{j+1}) = 3m+j+1$ for $1 \leq j \leq n-1$.

We note that $g^+(a) = 3m+7, g^+(r_i) = g^+(s_i) = g^+(t_i) = 6i+1$ for $1 \leq i \leq m-1, g^+(b) = 9m-3, g^+(c_j) = 6m+2j+1$ for $1 \leq j \leq n-1$. We know that, $(g^+(a), g^+(r_1)) = 1 = (g^+(a), g^+(c_1))$ for $1 \leq i \leq m-2, (g^+(r_i), g^+(r_{i+1})) = (6i+1, 6i+7) = 1$. Also, $(g^+(r_{m+1}), g^+(b)) = (6m-5, 9m-6) = 1, (g^+(c_j), g^+(c_{j+1})) = (6m+2j+1, 6m+2j+3) = 1$ for $1 \leq j \leq n-1$. Hence, G is an edge prime labeling. $\square$

Theorem 2.7. *For even $m = 2^l \geq 2$, $DS(m-1, m) \odot P_(n)$ is an edge prime if $m+1$ is prime.*

Proof. Let $G = DS(m-1, m) \odot P_n$ (for even $m = 2^l \geq 2, m+1$ is prime) be a graph. Then $V(G) = \{a, b, r_\alpha, s_\beta, t_\gamma : 1 \leq \alpha \leq m-1, 1 \leq \beta \leq m,$ $1 \leq \gamma \leq n\}$ and $E(G) = \{ab, at_1, ar_\alpha, bs_\beta, t_\gamma t_{\gamma+1} : 1 \leq \alpha \leq m-1, 1 \leq \beta \leq m,$ $1 \leq \gamma \leq n-1\}$. G has $2m+n$ edges. Define $g : E(G) \to \{1, 2, ..., 2m+n\}$ be as follows, $g(ab) = m+1, g(ar_\alpha) = \frac{(2\alpha-1)}{\{m+1\}}$ for $1 \leq \alpha \leq m-1, g(bs_\beta) = 2\beta$ for $1 \leq \beta \leq m, g(at_1) = 2m+1, g(t_\gamma t_{\gamma+1}) = 2m+\gamma+1$ for $1 \leq \gamma \leq n-1$. We have, $g^+(a) = m^2+2m+1, g^+(b) = (m+1)^2, g^+(t_\gamma) = 4m+2\gamma+1$ for $1 \leq \gamma \leq n-1, g^+(t_n) = 2m+n$. It can be easily verified that $(g^+(a), g^+(b)) = (g^+(a), g^+(r_\alpha)) = (g^+(b), g^+(s_\alpha)) = (g^+(a), g^+(t_1)) = (g^+(t_\alpha), g^+(t_{\alpha+1})) = 1$. Hence, G is an edge prime. $\square$

Theorem 2.8. *For odd $m = 2^l - 1 \geq 1$, $DS(m, m) \odot P_n$ is an edge prime if m^2+m+1 is prime.*

Proof. Let $G = DS(m, m) \odot P_n$, (for odd $m = 2^l - 1 \geq 1$, m^2+m+1 is prime) be a graph. Then $V(G) = \{a, b, r_\alpha, s_\beta, t_\gamma : 1 \leq \alpha, \beta \leq m, 1 \leq \gamma \leq n\}$ and $E(G) = \{ab, at_1, ar_\alpha, bs_\beta, t_\gamma t_{\gamma+1} : 1 \leq \alpha, \beta \leq m, 1 \leq \gamma \leq n-1\}$. Here, $|E(G)| = 2m+n+1$. Define a bijective function $g : E(G) \to \{1, 2, ..., 2m+n+1\}$ by $g(ab) = 1$, $g(ar_\alpha) = 2\alpha+1$ for $1 \leq \alpha \leq m$, $g(bs_\beta) = 2\beta$ for $1 \leq \beta \leq m$, $g(at_1) = 2m+2$, $g(t_\gamma t_{\gamma+1}) = 2m+\gamma+2$ for $1 \leq \gamma \leq n-1$. Clearly, $g^+(a) = m^2+4m+3$,

$g^+(b) = m^2+m+1$, $g^+(t_\gamma) = 4m+2\gamma+3$, $g^+(t_n) = 2m+n+1$. It can be easily verified that $(g^+(a), g^+(b)) = ((g^+(a), g^+(r_\alpha)) = (g^+(b), g^+(s_\alpha)) = (g^+(a), g^+(t_1)) = (g^+(t_\gamma), g^+(t_{\gamma+1})) = 1$. Hence, G is an edge prime. □

Theorem 2.9. *The complete tripartite graph $K_{1,1,n}$ is edge prime.*

Proof. Define $g : E(K_{2,n}) \to \{1, 2, ..., 2n\}$ by $g(s_1t_j) = 2j - 1$ and $g(s_2t_j) = 2n + 2 - 2j$, $1 \leq j \leq n$. Then $g^+(t_j) = 2n + 1$ for all $j, g^+(s_1) = n^2$ and $g^+(s_2) = n^2 + n$. The labeling g is called the basic labeling of $K_{2,n}$. If we add an edge s_1s_2 and get the graph $K_{1,1,n}$. Define $g_1 : E(K_{1,1,n}) \to \{1, 2, ..., 2n + 1\}$ by follow the same g and $g_1(s_1s_2) = 2n + 1$. Then thus we have $g_1^+(t_j) = 2n + 1$ for all j, $g_1^+(s_1) = (n + 1)^2$ and $g_1^+(s_2) = (n + 1)^2 + n$. Hence $K_{1,1,n}$ is an edge prime. □

Theorem 2.10. *$W_m \odot P_n$ is semiedge prime.*

Proof. Let $G = W_m \odot P_n$ be a graph. Then $V(G) = \{a, b_d, c_e : 1 \leq d \leq m, 1 \leq e \leq n\}$ and $E(G) = \{ab_d : 1 \leq d \leq n\} \cup \{b_db_{d+1} : 1 \leq d \leq n - 1\} \cup \{b_1b_n\} \cup \{ac_1\} \cup \{c_ec_{e+1} : 2 \leq e \leq n - 1\}$. Note that, $|E(G)| = 2m + n$. Let the labeling $g : E(G) \to \{1, 2, ..., 2m + n\}$ be defined as follows and consider the following cases.

Case 1: m is even.

$g(aa_d) = 2m - 2d + 1$, for $1 \leq d \leq m$, g$(a_da_{d+1}) = d + 1$ for odd d, $g(a_da_{d+1}) = m + d$ for even d, $g(ac_1) = 2m + 1, g(c_ec_{e+1}) = 2m + 1 + e$ for $1 \leq e \leq n - 1$. Observe that $g^+(a) = m^2 + 2m + 1$, $g^+(a_1) = 4m + 1$, $g^+(a_d) = 3m + 1$ for $2 \leq d \leq m$, $g^+(c_e) = 4m + 2e + 1$ for $1 \leq e \leq n - 1, g^+(c_n) = 2m + n$.

Case 2: m is odd.

$g(aa_d) = 2m - 2d + 1$, for $1 \leq d \leq m$, $g(a_da_{d+1}) = d + 1$ for odd d, $g(a_da_{d+1}) = m + d + 1$, for even d, $g(ac_1) = 2m + 1, g(c_ec_{e+1}) = 2m + e + 1$ for $1 \leq e \leq n - 1$. Observe that $g^+(a) = m^2 + 2m + 1$, $g(a_d) = 3m + 2$, for $1 \leq e \leq m$. $g^+(c_e) = 4m + 1 + 2e$ for $1 \leq e \leq n - 1$, $g^+(c_n) = 2m + n$, We know that $(g^+(a), g^+(a_d)) = (g^+(a_d), g^+(a_{d+1})) = (g^+(a), g^+(c_d)) = (g^+(c_d), g^+(c_{d+1})) = 1$. Hence, $W_m \odot P_n$ is an semiedge prime. □

Theorem 2.11. *The graph $P(2, m) \odot P_n$ is semiedge prime if $m \geq 6$.*

Proof. Let $G = P(2, m) \odot P_n$ be a graph $(m \geq 6)$. Then

$V(G) = \{a_d : 1 \leq d \leq m\} \cup \{b_e : 1 \leq e \leq n\}$ and

$E(G) = \{a_d a_{d+1} : 1 \leq d \leq m-1\} \cup \{a_d a_{d+2} : 1 \leq d \leq n-2\} \cup \{a_1 b_1\} \cup \{b_e b_{(e+1)} : 1 \leq e \leq n-1\}$. Also, $|E(G)| = 2m+n-3$. Define a bijective function $g : E(G) \to \{1, 2, 3, ..., 2m+n-3\}$ be as follows $g(a_d a_{d+1}) = d$ for $1 \leq d \leq m-1$. $g(a_d a_{d+2}) = 2m-d-2$ for $1 \leq d \leq m-2$. $g(a_1 b_1) = 2m-2.g(b_e b_{e+1}) = 2m+e-2$ for $1 \leq e \leq n-1$. Observe that, $g^+(a_1) = 4m-2, g^+(a_2) = 2m-1 = g^+(a_m)$, $g^+(a_{m-1}) = 3m-2$, $g^+(b_1) = 4d-3$ for $3 \leq d \leq m-2$,$g^+(b_e) = 4m+2(e-1)-3$ for $1 \leq e \leq n-1$ and $g^+(b_n) = 2m+n-3$. It is easily to verified that every two adjacent vertices labels that are relatively prime. □

Theorem 2.12. *The graph $K_{2,m} \cup W_n$ is semiedge prime.*

Proof. Let $G = K_{2,m} \cup W_n$ be a graph. Then

$V(G) = \{a_1, a_2, b_i : 1 \leq i \leq m\} \cup \{r, r_i : 1 \leq i \leq n\}$ and

$E(G) = \{a_1 b_i, a_2 b_i : 1 \leq i \leq m\} \cup \{rr_i : 1 \leq i \leq n\} \cup \{r_i r_{i+1} : 1 \leq i \leq n-1\}$. Also, $|E(G)| = 2(m+n)$. Define $g : E(G) \to \{1, 2, ..., 2m+2n\}$ by as follows for each $1 \leq i \leq m, g(a_1 b_i) = 2i-1, g(a_2 b_i) = 2m+2-2i$, Consider the following cases.

<u>Case 1: n is even.</u>

$g(r_i r_{i+1}) = 2m+i+1$ for odd $i, g(r_i r_{i+1}) = 2m+n+i$ for even i, $g(rr_i) = 2m+2n-2i+1$ for $1 \leq i \leq n$. Observe that $g^+(r) = 2mn+n^2$, $g^+(r_1) = 6m+4n+1, g^+(r_i) = 6m+3n+1$.

<u>Case 2: n is odd.</u>

$g(r_i r_{i+1}) = 2m+i+1$ for odd i, $g(r_i r_{i+1}) = 2m+n+i+1$ for even i, $g(rr_i) = 2m+2n-2i+1$ for $1 \leq i \leq n$. Clearly, $g^+(r) = 2mn+n^2$, $g^+(s_i) = 6m+3n+2$ for $1 \leq i \leq n$. It is easily to verified that every two adjacent vertices labels that are relatively prime. □

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