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SOME THEOREMS IN INTUITIONISTIC MULTI FUZZY SUBFIELDS OF A FIELD

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ABSTRACT. In this paper, some properties of intuitionistic multi fuzzy subfield of a field are defined and studied. Also some definitions, results and Theorems are given.

1. INTRODUCTION

After the introduction of fuzzy sets by L.A.Zadeh [13], several researchers explored on the generalization of the notion of fuzzy set ([3–10]). The concept of intuitionistic Multi fuzzy subset was introduced by K.T.Atanassov [1], as a generalization of the notion of fuzzy set. Azriel Rosenfeld [2] defined a fuzzy groups. Vasu.M, Sivakumar.D and Arjunan.K [12] defined an anti-Multi fuzzy subfield of a field. We introduce the concept of intuitionistic Multi fuzzy subfield of a field and established some results.

2. PRELIMINARIES

Definition 2.1. *Let X be a non-empty set. A fuzzy subset A of X is a function $A : X \to [0, 1]$.*

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Definition 2.2. *A multi fuzzy subset A of a set X is defined as an object of the form $A = \{\langle x, A_1(x), A_2(x), A_3(x), ..., A_n(x)\rangle / x \in X\}$, where $A_i : X \to [0,1]$ for all i. Here A is called multi fuzzy subset of X with dimension n. It is denoted as $A = \langle A_1, A_2, A_3, ..., A_n\rangle$.*

Definition 2.3. *An intuitionistic fuzzy set (IFS) A in X is defined as an object having the form $A = (x, \mu_{A_i}(x), \gamma_{A_i}(x))/x \in X$, where $\mu_{A_i} : X \to [0,1]$ and $\gamma_{A_i} : X \to [0,1]$ define the degree of membership and the degree of non-membership of the element $x \in X$ respectively and every x in X satisfying $0 \leq \mu_{A_i}(x)+\gamma_{A_i}(x) \leq 1$.*

Definition 2.4. *An intuitionistic multi fuzzy subset A of a set X is defined as an object of the form*

$$A = \Big(x, \mu_{A_1}(x),\ mu_{A_2}(x), ..., \mu_{A_n}(x), \gamma_{A_1}(x), \gamma_{A_2}(x), ..., \gamma_{A_n}(x)\Big)/x \in X,$$

where $\mu_{A_i} : X \to [0,1]$ and $\gamma_{A_i} : X \to [0,1]$ for all i, define the degrees of membership and the degrees of non-membership of the element $x \in X$ respectively and every x in X satisfying $0 \leq \mu_{A_i}(x) + \gamma_{A_i}(x) \leq 1$ for all i. It is denoted as $A_i = (\mu_{A_i}, \gamma_{A_i})$, where $\mu_{A_i} = (\mu_{A_1}, \mu_{A_2}, ..., \mu_{A_n})$ and $\gamma_{A_i} = (\gamma_{A_1}, \gamma_{A_2}, ..., \gamma_{A_n})$.

Definition 2.5. *Let A and B be any two intuitionistic multi fuzzy subset of X. We define the following relations and operations:*

(i) *$A \subseteq B$ if and only if $\mu A_i(x) \leq \mu B_i(x)$ and $\gamma A_i(x) \geq \gamma B_i(x)$ for all $x \in X$ and for all i.*
(ii) *$A = B$ if and only if $\mu A_i(x) = \mu B_i(x)$ and $\gamma A_i(x) = \gamma B_i(x)$ for all $x \in X$ and for all i.*
(iii) *$A \cap B$ if and only if $(A \cap B)(x) = \min \mu A_i(x), \mu B_i(x), \max \mu A_i(x), \gamma B_i(x)$ for all $x \in X$ and for all i.*
(iv) *$A \cup B$ if and only if $(A \cup B)(x) = \max \mu A_i(x), \mu B_i(x), \min \gamma A_i(x), \gamma B_i(x)$ for all $x \in X$ and for all i.*

Definition 2.6. *Let $(F, +, \cdot)$ be a field. A multi fuzzy subset A of F is said to be a multi fuzzy subfield (MFSF) of F if the following conditions are satisfied:*

(i) *$A_i(x - y) \geq \min A_i(x), A_i(y)$, for all $x, y \in F$, for all i,*
(ii) *$A_i(xy - 1) \geq \min A_i(x), A_i(y)$, for all $x, y \neq 0 \in F$, for all i, where 0 is the additive identity element of F.*

Definition 2.7. *Let $(F, +, \cdot)$ be a field. An intuitionistic multi fuzzy subset A of F is said to be an intuitionistic multi fuzzy subfield (ILFSF) of F if it satisfies the following axioms:*

(i) $\mu_{A_i}(x-y) \geq \min \mu_{A_i}(x), \mu_{A_i}(y)$, *for all $x, y \in F$, for all i.*
(ii) $\mu_{A_i}(xy-1) \geq \min \mu_{A_i}(x), \mu_{A_i}(y)$ *for all $x, y \neq 0$ in F, for all i.*
(iii) $\nu_{A_i}(x-y) \leq \min \nu_{A_i}(x), \nu_{A_i}(y)$ *for all x, y in F, for all i.*
(iv) $\nu_{A_i}(xy-1) \leq \min \nu_{A_i}(x), \nu_{A_i}(y)$ *for all $x, y \neq 0$ in F, for all i where 0 is the additive identity element of F.*

3. Some Properties

Theorem 3.1. *Let $(F, +, \cdot)$ be a field. If A is an intuitionistic Multi fuzzy subfield of F, then*

(i) $\mu_{A_i}(x+y) = \mu_{A_i}(x) \vee \mu_{A_i}(y)$ *with* $\mu_{A_i}(x) \neq \mu_{A_i}(y)$,
(ii) $\nu_{A_i}(x+y) = \nu_{A_i}(x) \wedge \nu_{A_i}(y)$ *with* $\nu_{A_i}(x) \neq \nu_{A_i}(y)$, *for each x and y in F.*
(iii) $\mu_{A_i}(xy) = \mu_{A_i}(x) \vee \mu_{A_i}(y)$ *with* $\mu_{A_i}(x) \neq \mu_{A_i}(y)$,
(iv) $\nu_{A_i}(xy) = \nu_{A_i}(x) \wedge \nu_{A_i}(y)$ *with* $\nu_{A_i}(x) \neq \nu_{A_i}(y)$, *for each x and $y \neq 0$ in F for all i.*

Theorem 3.2. *Let A be an intuitionistic Multi fuzzy subfield of a field $(F, +, \cdot)$.*

(i) *If $\mu_{A_i}(x) < \mu_{A_i}(y)$, for some x and y in F, then $\mu_{A_i}(x+y) = \mu_{A_i}(x) = \mu_{A_i}(y+x)$ for all i.*
(ii) *If $\nu_{A_i}(y) < \nu_{A_i}(x)$, for some x and y in F for all i, then $\nu_{A_i}(x+y) = \nu_{A_i}(x) = \nu_{A_i}(y+x)$.*
(iii) *If $\mu_{A_i}(x) < \mu_{A_i}(y)$, for some x and $y \neq 0$ in F for all i, then $\mu_{A_i}(xy) = \mu_{A_i}(x) = \mu_{A_i}(yx)$.*
(iv) *If $\nu_{A_i}(y) < \nu_{A_i}(x)$, for some x and $y \neq 0$ in F for all i, then $\nu_{A_i}(xy) = \nu_{A_i}(x) = \nu_{A_i}(yx)$.*

Proof. Let A be an intuitionistic Multi fuzzy subfield of a field F.

(i) Also we have $\mu_{A_i}(x) < \mu_{A_i}(y)$, for some x and y in F, Then, $\mu_{A_i}(x+y) \geq \mu_{A_i}(x) \vee \mu_{A_i}(y) = \mu_{A_i}(x)$; and $\mu_{A_i}(x) = \mu_{A_i}(x+y-y) \geq \mu_{A_i}(x+y) \vee \mu_{A_i}(-y) \geq \mu_{A_i}(x+y) \vee \mu_{A_i}(y) = \mu_{A_i}(x+y)$. Therefore, $\mu_{A_i}(x+y) = \mu_{A_i}(x)$. Hence $\mu_{A_i}(x+y) = \mu_{A_i}(x) = \mu_{A_i}(y+x)$.

(ii) Also we have $\nu_{A_i}(y) < \nu_{A_i}(x)$, for some x and y in F. Then, $\nu_{A_i}(x+y) \leq \nu_{A_i}(x) \wedge \nu_{A_i}(y) = \nu_{A_i}(x)$; and $\nu_{A_i}(x) = \nu_{A_i}(x+y-y) \leq \nu_{A_i}(x+y) \wedge \nu_{A_i}(-y) \leq \nu_{A_i}(x+y) \wedge \nu_{A_i}(y) = \nu_{A_i}(x+y)$. Therefore, $\nu_{A_i}(x+y) = \nu_{A_i}(x)$. Hence $\nu_{A_i}(x+y) = \nu_{A_i}(x) = \nu_{A_i}(y+x)$.

(iii) Also we have $\mu_{A_i}(x) < \mu_{A_i}(y)$, for some x and $y \neq 0$ in F for all i. Then, $\mu_{A_i}(xy) \geq \mu_{A_i}(x) \vee \mu_{A_i}(y) = \mu_{A_i}(x)$; and $\mu_{A_i}(x) = \mu_{A_i}(xyy-1) \geq \mu_{A_i}(xy) \vee \mu_{A_i}(y-1) \geq \mu_{A_i}(xy) \vee \mu_{A_i}(y) = \mu_{A_i}(xy)$. Therefore, $\mu_{A_i}(xy) = \mu_{A_i}(x)$. Hence $\mu_{A_i}(xy) = \mu_{A_i}(x) = \mu_{A_i}(yx)$.

(iv) Also we have $\nu_{A_i}(y) < \nu_{A_i}(x)$, for some x and $y \neq 0$ in F. Then, $\nu_{A_i}(xy) \leq \nu_{A_i}(x) \wedge \nu_{A_i}(y) = \nu_{A_i}(x)$; and $\nu_{A_i}(x) = \nu_{A_i}(xyy-1) \leq \nu_{A_i}(xy) \wedge \nu_{A_i}(y-1) \leq \nu_{A_i}(xy) \wedge \nu_{A_i}(y) = \nu_{A_i}(xy)$. Therefore $\nu_{A_i}(xy) = \nu_{A_i}(x)$. Hence $\nu_{A_i}(xy) = \nu_{A_i}(x) = \nu_{A_i}(yx)$.

□

Theorem 3.3. *Let A be an intuitionistic Multi fuzzy subfield of a field $(F, +, \cdot)$.*

(i) *If $\mu_{A_i}(x) > \mu_{A_i}(y)$, for some x and y in F, then $\mu_{A_i}(x+y) = \mu_{A_i}(y) = \mu_{A_i}(y+x)$.*

(ii) *If $\nu_{A_i}(y) > \nu_{A_i}(x)$, for some x and y in F for all i, then $\nu_{A_i}(x+y) = \nu_{A_i}(y) = \nu_{A_i}(y+x)$, for all x and y in F.*

(iii) *If $\mu_{A_i}(x) > \mu_{A_i}(y)$, for some x and $y \neq 0$ in F for all i, then $\mu_{A_i}(xy) = \mu_{A_i}(y) = \mu_{A_i}(yx)$.*

(iv) *If $\nu_{A_i}(y) > \nu_{A_i}(x)$, for some x and $y \neq 0$ in F for all i then $\nu_{A_i}(xy) = \nu_{A_i}(y) = \nu_{A_i}(yx)$.*

Proof. It is trivial. □

Theorem 3.4. *Let A be an intuitionistic Multi fuzzy subfield of a field $(F, +, \cdot)$.*

(i) *If $\mu_{A_i}(x) > \mu_{A_i}(y)$, for some x and y in F for all i, then $\mu_{A_i}(x+y) = \mu_{A_i}(y) = \mu_{A_i}(y+x)$.*

(ii) *If $\nu_{A_i}(y) < \nu_{A_i}(x)$, for some x and y in F for all i, then $\nu_{A_i}(x+y) = \nu_{A_i}(x) = \nu_{A_i}(y+x)$.*

(iii) *If $\mu_{A_i}(x) > \mu_{A_i}(y)$, for some x and $y \neq 0$ in F for all i, then $\mu_{A_i}(xy) = \mu_{A_i}(y) = \mu_{A_i}(yx)$.*

(iv) *If $\nu_{A_i}(y) < \nu_{A_i}(x)$, for some x and $y \neq 0$ in F for all i, then $\nu_{A_i}(xy) = \nu_{A_i}(x) = \nu_{A_i}(yx)$.*

Proof. It is trivial. □

Theorem 3.5. *Let A be an intuitionistic Multi fuzzy subfield of a field $(F, +, \cdot)$.*

(i) *If $\mu_{A_i}(x) < \mu_{A_i}(y)$, for some x and y in F for all i, then $\mu_{A_i}(x+y) = \mu_{A_i}(x) = \mu_{A_i}(y+x)$.*

(ii) *) If $\nu_{A_i}(y) > \nu_{A_i}(x)$, for some x and y in F for all i, then $\nu_{A_i}(x+y) = \nu_{A_i}(y) = \nu_{A_i}(y+x)$.*

(iii) *If $\mu_{A_i}(x) < \mu_{A_i}(y)$, for some x and $y \neq 0$ in F for all i, then $\mu_{A_i}(xy) = \mu_{A_i}(x) = \mu_{A_i}(yx)$.*

(iv) *If $\nu_{A_i}(y) > \nu_{A_i}(x)$, for some x and $y \neq 0$ in F for all i, then $\nu_{A_i}(xy) = \nu_{A_i}(y) = \nu_{A_i}(yx)$.*

Proof. It is trivial. □

Theorem 3.6. *Let A be an intuitionistic Multi fuzzy subfield of a field $(F, +, \cdot)$ such that $Im\ \mu_A = \alpha$ and $Im\ \nu_A = \beta$, where α and β in L. If $A = B \cup C$, where B and C are intuitionistic Multi fuzzy subfields of F, then either $B \subseteq C$ or $C \subseteq B$.*

Proof. Let $A = B \cup C = \langle (x), \mu_A(x), \nu_A(x) \rangle / x$ in F, $B = \langle (x), \mu_B(x), \nu_B(x) \rangle / x$ in F and $C = \langle (x), \mu_C(x), \nu_C(x) \rangle / x$ in F.

Case (i): Assume that $\mu_B(x) > \mu_C(x) and \mu_B(y) < \mu_C(y)$, for some x and y in R. Then,

$$\alpha = \mu_A(x) = \mu_B \cup \mu_C(x) = \mu_B(x) \vee \mu_C(x) = \mu_B(x) > \mu_C(x).$$

Therefore, $\alpha > \mu_C(x)$, and

$$\alpha = \mu_A(y) = \mu_B \cup \mu_C(y) = \mu_B(y) \vee \mu_C(y) = \mu_C(y) > \mu_B(y).$$

Therefore, $\alpha > \mu_B(y)$. So that, $\mu_C(y) > \mu_C(x)$ and $\mu_B(x) > \mu_B(y)$. Hence $\mu_B(x+y) = \mu_B(y)$, for all x and y in F and $\mu_C(x+y) = \mu_C(x)$, for all x and y in F. But then,

$$\alpha = \mu_A(x+y) = \mu_B \cup C(x+y) = \mu_B(x+y) \vee \mu_C(x+y) = \mu_B(y) \vee \mu_C(x) < \alpha. \tag{3.1}$$

Case (ii): Assume that $\nu_B(x) < \mu_C(x)$ and $\nu_B(y) > \nu_C(y)$, for some x and y in F. Then,

$$\beta = \nu_A(x) = \nu_B \cup C(x) = \nu_B(x) \wedge \nu_C(x) = \nu_B(x) < \nu_C(x).$$

Therefore, $\beta < \nu_C(x)$. And

$$\beta = \nu_A(y) = \nu_B \cup C(y) = \nu_B(y) \wedge \nu_C(y) = \nu_C(y) < \nu_B(y).$$

Therefore, $\beta < \nu_B(y)$. So that, $\nu_C(y) < \nu_C(x)$ and $\nu_B(x) < \nu_B(y)$. Hence $\nu_B(x+y) = \nu_B(y)$ and $\nu_C(x+y) = \nu_C(x)$, for x and y in F. But then,

(3.2) $\beta = \nu_A(x+y) = \nu_B \cup C(x+y) = \nu_B(x+y) \wedge \nu_C(x+y) = \nu_B(y) \wedge \nu_C(x) > \beta$

It is a contradiction by (3.1) and (3.2). Therefore, either $B \subseteq C$ or $C \subseteq B$ is true. □

Theorem 3.7. *If A_i and B_i are any two intuitionistic L-fuzzy subfields of a field $(F, +, \cdot)$, then their intersection $A_i \cap B_i$ is an intuitionistic L-fuzzy subfield of F.*

Theorem 3.8. *The intersection of a family of intuitionistic L-fuzzy subfields of a field $(F, +, \cdot)$ is an intuitionistic L-fuzzy subfield of F.*

Theorem 3.9. *If A is an intuitionistic Multi fuzzy subfield of a field $(F, +, \cdot)$, then A is an intuitionistic Multi fuzzy subfield of F.*

Proof. Let A be an intuitionistic Multi fuzzy subfield of a field F. Consider $A = \langle (x), \mu_A(x), \nu_A(x) \rangle$, for all x in F, we take

$$A = B = \langle (x), \mu_B(x), \nu_B(x) \rangle \,,$$

where $\mu_B(x) = \mu_A(x), \nu_B(x) = 1 - \mu_A(x)$. Clearly,

$$\mu_B(x-y) \geq \mu_B(x) \vee \mu_B(y),$$

for all x and y in F and $\mu_B(xy{-}1) \geq \mu_B(x) \vee \mu_B(y)$, for all x and $y \neq 0$ in F. Since A is an intuitionistic Multi fuzzy subfield of F, we have $\mu_A(x{-}y) \geq \mu_A(x) \vee \mu_A(y)$, for all x and y in F, which implies that $1 - \nu_B(x-y) \geq (1 - \nu_B(x)) \vee (1 - \nu_B(y))$, which implies that

$$\nu_B(x-y) \leq 1 - (1 - \nu_B(x)) \vee (1 - \nu_B(y)) = \nu_B(x) \wedge \nu_B(y).$$

Therefore,

$$\nu_B(x-y) \leq \nu_B(x) \wedge \nu_B(y),$$

for all x and y in F. And,$\mu_A(xy-1) \geq \mu_A(x) \vee \mu_A(y)$, for all x and $y \neq 0$ in F, which implies that $1 - \nu_B(xy-1) \geq (1 - \nu_B(x)) \vee (1 - \nu_B(y))$ which implies that $\nu_B(xy-1) \leq 1 - (1 - \nu_B(x)) \vee (1 - \nu_B(y)) = \nu_B(x) \vee \nu_B(y)$.

Therefore,

$$\nu_B(xy-1) \leq \nu_B(x) \wedge \nu_B(y),$$

for all x and $y \neq 0$ in F.

Hence $B = A$ is an intuitionistic Multi fuzzy subfield of a field R. □

Remark 3.1. *The converse of the above theorem is not true. It is shown by the following example.*

Example 1. *Consider the field $Z_5 = 0, 1, 2, 3, 4$ with addition modulo 5 and multiplication modulo 5 operations. Then*

$$A = \langle 0, 0.7, 0.2\rangle, \langle 1, 0.5, 0.1\rangle, \langle 2, 0.5, 0.4\rangle, \langle 3, 0.5, 0.1\rangle, \langle 4, 0.5, 0.4\rangle$$

is not an intuitionistic Multi fuzzy subfield of Z_5, but

$$A = \langle 0, 0.7, 0.3\rangle, \langle 1, 0.5, 0.5\rangle, \langle 2, 0.5, 0.5\rangle, \langle 3, 0.5, 0.5\rangle, \langle 4, 0.5, 0.5\rangle$$

is an intuitionistic Multi fuzzy subfield of Z_5.

Theorem 3.10. *If A is an intuitionistic Multi fuzzy subfield of a field $(F, +, \cdot)$, then A is an intuitionistic Multi fuzzy subfield of F.*

Proof. Let A be an intuitionistic Multi fuzzy subfield of a field F. That is $A = \langle (x), \mu_A(x), \nu_A(x)\rangle$, for all x in F. Let $A = B = \langle (x), \mu_B(x), \nu_B(x)\rangle$, where $\mu_B(x) = 1 - \nu_A(x), \nu_B(x) = \nu_A(x)$. Clearly,

$$\nu_B(x - y) \leq \nu_B(x) \wedge \nu_B(y),$$

for all x and y in F and $\nu_B(xy-1) \leq \nu_B(x) \wedge \nu_B(y)$, for all x and $y = 0$ in F. Since A is an intuitionistic Multi fuzzy subfield of F, we have $\nu_A(x-y) \leq \nu_A(x) \wedge \nu_A(y)$, for all x and y in F, which implies that

$$1 - \mu_B(x - y) \leq (1 - \mu_B(x)) \wedge (1 - \mu_B(y)),$$

which implies that

$$\mu_B(x - y) \geq 1 - (1 - \mu_B(x)) \wedge (1 - \mu_B(y)) = \mu_B(x) \vee \mu_B(y).$$

Therefore,

$$\mu_B(x - y) \geq \mu_B(x) \vee \mu_B(y),$$

for all x and y in F. And $\nu_A(xy - 1) \leq \nu_A(x) \wedge \nu A(y)$, for all x and y in F, which implies that

$$1 - \mu_B(xy - 1) \leq (1 - \mu_B(x)) \wedge (1 - \mu_B(y)),$$

which implies that

$$\mu_B(xy - 1) \geq 1 - (1 - \mu_B(x)) \wedge (1 - \mu_B(y)) = \mu_B(x) \vee \mu_B(y).$$

Therefore,

$$\mu_B(xy - 1)\mu_B(x) \vee \mu_B(y),$$

for all x and y in F. Hence $B = A$ is an intuitionistic Multi fuzzy subfield of a field F. $\square$

Remark 3.2. *The converse of the above theorem is not true. It is shown by the following example.*

Example 2. *Consider the field* $Z_5 = 0, 1, 2, 3, 4$ *with addition modulo 5 and multiplication modulo 5 operations. Then*

$$A = \langle 0, (0.5, 0.4, 0.3), (0.1, 0.2, 0.3) \rangle, \langle 1, 0.6, 0.4 \rangle, \langle 2, 0.5, 0.4 \rangle,$$

$$\langle 3, 0.6, 0.4 \rangle, \langle 4, 0.5, 0.4 \rangle$$

is not an intuitionistic Multi fuzzy subfield of Z_5*, but*

$$A = \langle 0, 0.9, 0.1 \rangle, \langle 1, 0.6, 0.4 \rangle, \langle 2, 0.6, 0.4 \rangle, \langle 3, 0.6, 0.4 \rangle, \langle 4, 0.6, 0.4 \rangle$$

is an intuitionistic Multi fuzzy subfield of Z_5*.*

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