ADV MATH SCI JOURNAL

Advances in Mathematics: Scientific Journal **9** (2020), no.4, 1965–1971
ISSN: 1857-8365 (printed); 1857-8438 (electronic)
https://doi.org/10.37418/amsj.9.4.55 Spec. Issue on NCFCTA-2020

INTUITIONISTIC INTERVAL VALUED MULTI FUZZY SUBFIELDS OF A FIELD

M. VASU

ABSTRACT. In this paper, some theorems of intuitionistic interval valued multi fuzzy subfield of a field are defined and noted and also some definitions, results and properties are given.

1. INTRODUCTION

The fuzzy set was introduced by L.A.Zadeh [13], A lot of researchers explained on the generalization of the notation of fuzzy set ([3–10]). Some results of intuitionistic multi fuzzy subset was introduced by K.T. Atanassov [1] as a generalization of the notion of fuzzy set. Azriel Rosenfeld [2] was introduced a fuzzy group.

2. PRELIMINARIES

Definition 2.1. *Let X be a non-empty set. A fuzzy subset E of X is a function $E : X \to [0, 1]$.*

Definition 2.2. *A multi fuzzy subset E of a set X is defined as an object of the form $E = \{\langle x, E_1(x), E_2(x), E_3(x), ..., E_n(x)\rangle /x \in X\}$, where $E_i : X \to [0, 1]$ for every i. E is called multi fuzzy subset of X with dimension n. It is denoted as $E = \{E_1, E_2, E_3, ..., E_n\}$*

2010 *Mathematics Subject Classification.* 03E72.

Key words and phrases. Fuzzy set, multi fuzzy subset, intuitionistic multi fuzzy subset, Interval valued multi fuzzy subfield, and intuitionistic Interval valued multi fuzzy subfield.

Definition 2.3. *A intuitionistic fuzzy set (IFS) E in X is defined as an object having the form $E = \{(x, \mu_E(x), \nu_E(x))/x \in X\}$, where $\mu_E : X \to [0,1]$ and $\nu_E : X \to [0,1]$ define the degree of membership and the degree of non-membership of the element $x \in X$ respectively and all x in X satisfying $0 \leq \mu_E(x) + \nu_E(x) \leq 1$.*

Definition 2.4. *An intuitionistic multi fuzzy subset of A of a set X is defined as an object of the form $A = \{(x, \mu_{E_1}(x), \mu_{E_2}(x), ..., \mu_{E_n}(x), \nu_{E_1}(x), \nu_{E_2}(x), ..., \nu_{E_n}(x)/ x \in X\}$ where $\mu_{E_i}(x) : X \to [0,1]$ and $\nu_{E_i}(x) : X \to [0,1]$ for every i, define the degrees of membership and the degrees of non-membership of the element $x \in X$ respectively and for all $x \in X$ satisfying $0 \leq \mu_E(x) + \nu_E(x) \leq 1$, for every i. It is noted as $E = (\mu_E, \nu_E)$ where $\mu_E = \{(\mu_{E_1}(x), \mu_{E_2}(x), ..., \mu_{E_n}(x))\}$ and $\nu_E = \{(\nu_{E_1}(x), \nu_{E_2}(x), ..., \nu_{E_n}(x))\}$.*

Definition 2.5. *Let X be a non-empty set, An interval valued fuzzy subset E of X is a function $E : X \to D[0,1]$, where $D[0,1]$ is a collection of all subinterval of $[0,1]$.*

Definition 2.6. *An interval valued multi fuzzy subset of E of a set X is defined as an object of the form $E = \{\langle x, E_1(x), E_2(x), E_3(x), ..., E_n(x)\rangle /x \in X\}$ where $E_i(x) : X \to D[0,1]$ for every i, Here E is called an interval valued multi fuzzy subset of X with dimension n. This noted as $E = \langle E_1 E_2 ... E_n\rangle$.*

Definition 2.7. *An intuitionistic interval valued fuzzy set E in X is an object having the form $E = \{(x, \mu_E(x), \nu_E(x))/x \in X\}$, where $\mu_E : X \to D[0,1], \nu_E : X \to D[0,1]$ defined the degrees of membership, the degree of non-membership of the elements x in X respectively and for all x in X satisfying $0 \leq \mu_{E(x)} + \nu_{E(x)} \leq 1$.*

Definition 2.8. *Let E and F be any two intuitionistic interval valued multi fuzzy subset of X, we define the following relations and operations*

(i) *$E \leq F$ iff $\mu_{E_i}(x) \leq \mu_{F_i}(x)$ and $\nu_{E_i}(x) \geq \nu_{F_i}(x)$ for every x in X and for every i.*
(ii) *$E = F$ iff $\mu_{E_i}(x) = \mu_{F_i}(x)$ and $\nu_{E_i}(x) = \nu_{F_i}(x)$ for every x in X and for every i.*
(iii) *$E \cap F$ iff $E \cap F(x) = \{r\min\{\mu_{E_i}(x), \mu_{F_i}(x)\}, r\max\{\nu_{E_i}(x), \nu_{F_i}(x)\}\}$ for every x in X and for every i.*
(iv) *$E \cup F$ iff $E \cup F(x) = \{\{r\max \mu_{E_i}(x), \mu_{F_i}(x)\}, r\min\{\nu_{E_i}(x), \nu_{F_i}(x)\}\}$ for every x in X yand for every i.*

Definition 2.9. *Let $(F, +, \cdot)$ be a field an interval valued multi fuzzy subset E of F is said to be a interval valued multi fuzzy subfield of F if the following conditions are satisfied.*

(i) $E_i(x-y) \geq r\min\{E_i(x), E_i(y)\}$*, for every x, y in F, for every i.*

(ii) $E_i(xy^{-1}) \geq r\min\{E_i(x), E_i(y)\}$ *for every $x, y \neq 0$ in F for every i, where 0 is the additive identity element of F.*

Definition 2.10. *Let $(F, +, \cdot)$ be a field. An intuitionistic interval valued multi fuzzy subset E of F is said to be an intuitionistic interval valued multi fuzzy subfield of F, if it satisfies the following axioms*

(i) $\mu_{E_i}(x-y) \geq r\min\{\mu_{E_i}(x), \mu_{E_i}(y)\}$ *for every x, y in F for every i.*

(ii) $\mu_{E_i}(xy^{-1}) \geq r\min\{\mu_{E_i}(x), \mu_{E_i}(x)\}$ *for every $x, y \neq 0$ in F for every i.*

(iii) $\nu_{E_i}(x-y) \leq r\min\{\nu_{E_i}(x), \nu_{E_i}(y)\}$ *for every x, y in F for every i.*

(iv) $\nu_{E_i}(xy^{-1}) \leq r\min\{\nu_{E_i}(x), \nu_{E_i}(y)\}$ *for every $x, y \neq 0$ in F for every i, where 0 is the additive identify element in F.*

3. Some Properties

Note 1. $0 = [0, 0], \quad 1 = [1, 1]$.

Theorem 3.1. *If E is an intuitionistic interval valued multi fuzzy subfield $(F, +, \cdot)$, then $\mu_{E_i}(-x) = \mu_{E_i}(x)$ for all x in F and $\mu_{E_i}(x^{-1}) = \mu_{E_i}(x)$ for all $x \neq 0$ in F and $\nu_{E_i}(-x) = \nu_{E_i}(x)$ for every x in F and $\nu_{E_i}(x^{-1}) = \nu_{E_i}(x)$ for all $x \neq 0$ in F, $\mu_{E_i}(x) \leq \mu_{E_i}(0)$ for every x in F and $\mu_{E_i}(x) \leq \mu_{E_i}(1)$ for every $x \neq 0$ in F and $\nu_{E_i}(x) \geq \nu_{E_i}(0)$ for every x in F and $\nu_{E_i}(x) \geq \nu_{E_i}(1)$ for all $x \neq 0$ in F, for all i, where 0 and 1 are identify element in F.*

Proof. For x in F and $0, 1$ there are identify elements in F. Now $\mu_{E_i}(x) = \mu_{E_i}(-(-x) \geq \mu_{E_i}(-x) \geq \mu_{E_i}(x)$. Therefore,

$$\mu_{E_i}(-x) = \mu_{E_i}(x)$$

for every x in F and for every i;

$$\mu_{E_i}(x) = \mu_{E_i}((x^{-1})^{-1}) \geq \mu_{E_i}(x^{-1}) \geq \mu_{E_i}(x).\mu_{E_i}(x^{-1}) = \mu_{E_i}(x)$$

for every $x \neq 0$ in F and for every i. So,

$$\nu_{E_i}(x) = \nu_{E_i}(-(-x)) \leq \nu_{E_i}(-x) \leq \nu_{E_i}(x).$$

Therefore,

$$\nu_{E_i}(-x) = \nu_{E_i}(x)$$

for every x in F and for every i, and further,

$$\nu_{E_i}(x) = \nu_{E_i}(x^{-1})^{-1} \leq \nu_{E_i}(x^{-1}) \leq \nu_{E_i}(x)$$

for every x in F and for every i,

$$\mu_{E_i}(0) = \mu_{E_i}(x - x) \geq r\min\{\mu_{E_i}(x), \mu_{E_i}(-x)\} = \mu_{E_i}(x).$$

Therefore,

$$\mu_{E_i}(0) \geq \mu_{E_i}(x)$$

for all x in F and for every i.

Now

$$\mu_{E_i}(1) = \mu_{E_i}(xx^{-1}) \geq r\min\{\mu_{E_i}(x), \mu_{E_i}(x^{-1})\} = \mu_{E_i}(x).$$

Therefore,

$$\mu_{E_i}(1) \geq \mu_{E_i}(x)$$

for every $x \neq 0$ in F and for every i, and

$$\nu_{E_i}(0) = \nu_{E_i}(x - x) \leq r\max\{\nu_{E_i}(x), \nu_{E_i}(-x)\} = \nu_{E_i}(x).$$

Therefore,

$$\nu_{E_i}(0) \leq \nu_{E_i}(x)$$

for every x in F and for every i,

$$\nu_{E_i}(1) = \nu_{E_i}(xx^{-1}) \leq r\max\{\nu_{E_i}(x), \nu_{E_i}(x^{-1})\} = \nu_{E_i}(x).$$

Therefore,

$$\nu_{E_i}(1) \leq \nu_{E_i}(x)$$

for every $x \neq 0$ in F and for every i. □

Theorem 3.2. *If A is an intuitionistic yinterval valued multi fuzzy subfield of a field $(F, +, \cdot)$, then for each i,*

(i) $\mu_{E_i}(x - y) = \mu_{E_i}(0)$ *gives* $\mu_{E_i}(x) = \mu_{E_i}(y)$ *for each x and y in F.*
(ii) $\mu_{E_i}(xy^{-1}) = \mu_{E_i}(1)$ *gives* $\mu_{E_i}(x) = \mu_{E_i}(y)$ *for every x and $y = 0$ in F.*
(iii) $\nu_{E_i}(x - y) = \nu_{E_i}(0)$ *gives* $\nu_{E_i}(x) = \nu_{E_i}(y)$ *for every x and y in F.*
(iv) $\nu_{E_i}(xy^{-1}) = \nu_{E_i}(1)$ *gives* $\nu_{E_i}(x) = \nu_{E_i}(y)$ *for all x and $y \neq 0$ in F where 0 and 1 are identity elements in F.*

Theorem 3.3. *Let E be an intuitionistic interval valued multi fuzzy subset of a field $(F,+,\cdot)$. If for every i, $\mu_{E_i}(e)=\mu_{E_i}(e_1)=1$ and $\nu_{E_i}(e)=\nu_{E_i}(e_1)=0$ and*
$\mu_{E_i}(x-y)\geq r\min\{\mu_{E_i}(x),\mu_{E_i}(y)\}$ for every x and y in F,
$\mu_{E_i}(xy^{-1})\geq r\min\{\mu_{E_i}(x),\mu_{E_i}(y)\}$ for every x and $y\neq e$ in F and
$\nu_{E_i}(x-y)\leq r\max\{\nu_{E_i}(x),\nu_{E_i}(y)\}$ for every x and y in F,
$\nu_{E_i}(xy^{-1})\leq r\max\{\nu_{E_i}(x),\nu_{E_i}(y)\}$ for every x and $y\neq e$ in F,
then E is an intuitionistic multi fuzzy subfield of F, where e and e_1 are identity elements of F.

Proof. It is well defined. □

Theorem 3.4. *Let E be an intuitionistic multi fuzzy subfield of a field $(F,+,\cdot)$, then $H=\{x/x\in F:\mu_{E_i}(x)=1,\nu_{E_i}(x)=0$ for every $i\}$ is either empty or a subfield of F.*

Proof. If no element satisfies this conditions, then H is empty. If x and y in H, then

$$\begin{aligned}\mu_{E_i}(x-y)&\geq r\min\{\mu_{E_i}(x),\mu_{E_i}(-y)\}\\&\geq r\min\{\mu_{E_i}(x),\mu_{E_i}(y)\}\\&=r\min(1,1)\\&=1.\end{aligned}$$

Therefore, $\mu_{E_i}(x-y)=1$ for every x,y in H and for every i.

$$\begin{aligned}\mu_{E_i}(xy^{-1})&\geq r\min\{\mu_{E_i}(x),\mu_{E_i}(y^{-1})\}\\&\geq r\min\{\mu_{E_i}(x),\mu_{E_i}(y)\}\\&=r\min(1,1)\\&=1.\end{aligned}$$

Therefore, $\mu_{E_i}(xy^{-1})=1$ for every x and $y\neq 0$ in H and for every i.

$$\begin{aligned}\nu_{E_i}(x-y)&\leq r\max\{\nu_{E_i}(x),\nu_{E_i}(-y)\}\\&\leq r\max\{\nu_{E_i}(x),\nu_{E_i}(y)\}\\&=r\max(0,0)\\&=0\end{aligned}$$

Therefore, $\nu_{E_i}(x-y)=0$ for every x and y in H and for every i. $x-y,xy^{-1}\in H$.

Therefore, H is a subfield of F. Hence H is either empty or a subfield of F. $\square$

Theorem 3.5. *Let E be an intuitionistic inteval valued multi fuzzy subfield of a field $(F, +, \cdot)$, then for every i,*

(i) *If $\mu_{E_i}(x-y) = 1$ then $\mu_{E_i}(x) = \mu_{E_i}(y)$ for every x and $y \neq e$ in F.*
(ii) *If $\nu_{E_i}(x-y) = 0$ then $\nu_{E_i}(x) = \nu_{E_i}(y)$ for every x and y in F and if $\nu_{E_i}(xy^{-1}) = 0$, then $\nu_{E_i}(x) = \nu_{E_i}(y)$ for every x and $y \neq e_1$ in F.*

Here e and e_1 are identity elements of F.

Theorem 3.6. *If E be a Intuitionistic Interval valued multi fuzzy subfield of a field $(F, +, \cdot)$, then for every i:*

(i) *If $\mu_{E_i}(x-y) = 0$ then either $\mu_{E_i}(x) = 0$ or $\mu_{E_i}(y) = 0$ for every x, y in F and if $\mu_{E_i}(xy^{-1}) = 0$ then either $\mu_{E_i}(x) = 0$ or $\mu_{E_i}(y) = 0$ for every x and $y \neq e$ in F.*
(ii) *If $\nu_{E_i}(x-y) = 1$ then either $\nu_{E_i}(x) = 1$ or $\nu_{E_i}(y) = 1$ for every x, y in F and if $\nu_{E_i}(xy^{-1}) = 1$ then either $\nu_{E_i}(x) = 1$ or $\nu_{E_i}(y) = 1$ for every x and $y \neq e_1$ in F, where e and e_1 are identity elements of F.*

REFERENCES

[1] K. T. ATANASSOV: *Intuitionistic fuzzy sets theory and applications*, Physica-Verlag, A Springer-Verlag company, Bulgaria, 1999.
[2] A. ROSENFELD: *Fuzzy Groups*, Journal of mathematical analysis and applications, **35** (1971), 512–517.
[3] R. BISWAS: *Fuzzy fields and fuzzy linear spaces redefined, Fuzzy sets and systems*, North Holland, 1989.
[4] M. MUTHUSAMY, N. PALANIAPPAN, K. ARJUNAN: *Notes on intuitionistic fuzzy subfields*, International J. of Math.Sci. Engg. Appls., **4**(5) (2010), 345–354.
[5] P. BHATTACHARYA: *Fuzzy Subgroups: Some Characterizations*, Journal of Mathematical Analysis and Applications, **128** (1987), 241–252.
[6] R. KUMAR: *Fuzzy Algebra*, University of Delhi Publication Division, **1** (1993), 33–44.
[7] R. KUMAR: *Fuzzy irreducible ideals in rings*, Fuzzy Sets and Systems, **42**(1991), 369–379.
[8] S. ABOU-ZAID: *On generalized characteristic fuzzy subgroups of a finite group*, Fuzzy sets and systems, **43** (1991), 235–241.
[9] F. I. SIDKY, M. A. MISHREF: *Fuzzy cosets and cyclic and Abelian fuzzy subgroups*, Fuzzy sets and systems, **43** (1991), 243–250.

[10] P. SIVARAMAKRISHNA DAS: *Fuzzy groups and level subgroups*, Journal of Mathematical Analysis and Applications, **84** (1981), 264–269.

[11] W. B. VASANTHA KANDASAMY: *Smarandache fuzzy algebra*, American research press, Rehoboth, 2003.

[12] M. VASU, D. SIVAKUMAR, K. ARJUNAN: *A study on anti-Multi fuzzy subfield of a field*, Indian journal of research, **1**(1) (2012), 78–79.

[13] L. A. ZADEH: *Fuzzy sets*, Information and control, **8** (1965), 338–353.

DEPARTMENT OF MATHEMATICS
GOVERNMENT ARTS COLLEGE FOR WOMEN
SIVAGANGAI-630562, TAMIL NADU, INDIA