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SOLUTION OF KDV AND COUPLED BURGER'S EQUATION VIA MAHGOUB HOMOTOPY PERTURBATION TRANSFORM SCHEME

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ABSTRACT. This article represents an elegant way to use Mahgoub Homotopy Perturbation Transform Scheme (MHPTS) to solve non-linear partial differential system, i.e. KdV system of third order, Coupled Hirota Satsuma KdV system and Coupled Burger's system of one and two Dimensions. The result shows that above method is very instinctive for solving system of Partial Differential Equation in facile manner.

1. INTRODUCTION

Day to day the manner of solving erratic or nonlinear partial differential system is becoming more and more elegant with the increasing rate of converting problems in physics, chemistry, engineering etc. to erratic partial form. Plenty of method had been developed to deal with this type of problems for finding pretty much results.

Accordingly, numerous strategies to bewilder out these issues are drawing in scientists as of late [1, 2]. The solution of the erratic term in the equation becomes much complex to reach any uninfluenced result. Different mode of solution had been suggested to perceive an accurate solution to the erratic systems of equations [3–7]. Then comes the era of Homotopy Perturbation Scheme

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(HPS)i.e. merging the Homotopy with topology and classical perturbation proficiency, which had been used to figure out many linear and erratic differential equations [8–15]. In this article, Homotopy Perturbation is merged with Mahgoub transformation and the variational iteration method to create extremely constructive manner to treat erratic terms.

The association of Homotopy Transform Scheme with Mahgoub Transform is used with He's Polynomial, which provides the nearest result to the erratic Partial Differential System namely a KdV system of third order. Coupled Hirota Satsuma KdV system and Coupled Burger's system of one and two Dimensions. Various methods have been applied to solve this system of equations [22–26]. Working with new Mahgoub Transform [17–21] which is restraint free in time domain made this consortium much easier to estimate the best outputs of the taken classification of PDE.

2. Mahgoub transform

The Mahgoub transform [17] indicated by $\mathfrak{m}(.)$ and Mahgoub transform of $\mathfrak{m}(T(t^\diamond))$ is settled by the given intact system:

$$\mathfrak{m}(T(t^\diamond)) = D(j) = j\int_0^\infty T(t^\diamond)e^{-jt^\diamond}dt^\diamond. \quad t^\diamond \geq 0, \tag{2.1}$$

and $\mathcal{C}_1 \leq j \leq \mathcal{C}_2$. Within the set θ, (2.1) is specified as,

$$\theta = \{T(t^\diamond) : \exists \mathbb{M}, \mathcal{C}_1, \mathcal{C}_2 > 0. |T(t^\diamond)| < Me^{|t^\diamond|/\mathcal{C}_q}\}.$$

Remark 2.1. *Hence, all further property about the Mahgoub transform can be studied in [17–21].*

3. Mahgoub Homotopy Perturbation transform scheme

For interpreting the procedure of aforementioned scheme, let us deal with a general erratic partial Differential system;

$$DT(X,t^\diamond) + AT(X,t^\diamond) + BT(X,t^\diamond) = 0. \tag{3.1}$$

Holds initial circumstances,

$$T(X,0) = F(X).$$

Here A and B are the linear and erratic differential arranger w.r.t.X and D is the linear differential arranger w.r.t. $t^\diamond$. Taking Mahgoub transform on both hands of above equation (3.1), we reached at

$$\mathfrak{m}[DT(X,t^\diamond)+AT(X,t^\diamond)+BT(X,t^\diamond)]=0.$$

Implementing derivative means of Mahgoub transform, our equation reduced to,

$$T(X,j)=F(X)-1/j\mathfrak{m}[AT(X,t^\diamond)-BT(X,t^\diamond)].$$

Again using Mahgoub Inverse transform both side

$$T(X,t^\diamond)=F(X,t^\diamond)-\mathfrak{m}^{-1}\Big[\frac{1}{j}\mathfrak{m}[AT(X,t^\diamond)-BT(X,t^\diamond)]\Big]. \tag{3.2}$$

$F(X,t^\diamond)$ arises from provided source and given initial condition. So, it's the time to utilize the Homotopy Perturbation scheme:

$$T(X,t^\diamond)=\sum_{n=0}^{\infty}p^nT_n(X,t^\diamond), \tag{3.3}$$

and for decomposing erratic (non-linear) terms

$$BT(X,t^\diamond)=\sum_{n=0}^{\infty}p^nT_n(T). \tag{3.4}$$

Consuming He's Polynomials T_n, which is given by

$$H_n(T_0,T_1,\dots,T_n)=\frac{1}{n!}\frac{\partial^n}{\partial p^n}\Big[B\Big(\sum_{i=o}^{\infty}p^iT_i\Big)\Big]_{p=o},$$

for $n=0,1,2,3,\dots$ Now arrangements of equation (3.3) and (3.4) with (3.2)

$$\sum_{n=0}^{\infty}p^nT_n(X,t^\diamond)=F(X,t^\diamond)-p\mathfrak{m}^{-1}\left[\frac{1}{j}\mathfrak{m}\left[\left\{\begin{matrix}\Big(A\sum_{n=0}^{\infty}p^nT_n(X,t^\diamond)\Big)\\ +\sum_{n=0}^{\infty}p^nH_n(T)\end{matrix}\right.\right]\right].$$

The above association of equations represents the unify of Mahgoub transform and the Homotopy Perturbation Scheme employing He's polynomials. Associating exponent of p, under mentioned approximation holds,

$$p^0:T_0(X,t^\diamond)=F(X,t^\diamond),$$

$$p^1:T_1(X,t^\diamond)=-\mathfrak{m}^{-1}\Big[\frac{1}{j}\mathfrak{m}[AT_0(X,t^\diamond)-H_0(T)]\Big],$$

placing $p=1$ gives nearby solution of (3.1),

$$T(X,t^{\diamond}) = T_0(X,t^{\diamond}) + T_1(X,t^{\diamond}) + T_2(X,t^{\diamond}) + \dots .$$

4. Solution of the Mentioned partial differential systems

To express methodology of the Mahgoub Homotopy Perturbation transformation scheme, will share under mentioned four ideal arrangement of erratic partial differential Equation.

Case 1. Deal with the arrangement of two homogeneous KdV equation of grade three.

$$\begin{aligned} T_{t^{\diamond}} &= T_{XXX} + TT_X + YY_X, \\ Y_{t^{\diamond}} &= -2Y_{XXX} + TY_X \end{aligned}$$

holds initial circumstances

$$\begin{aligned} T(X,0) &= \Big(3 - 6\tanh^2\frac{X}{2}\Big), \\ Y(X,0) &= -\Big(3t^{\diamond}\sqrt{2}\tanh^2\frac{X}{2}\Big). \end{aligned}$$

Utilizing aforementioned method and applying initial condition, the equation above reduced to

$$T(X,j) = \Big(3 - 6\tanh^2\frac{X}{2}\Big) + \frac{1}{j}\mathfrak{m}\Big[T_{XXX} + TT_X + YY_X\Big],$$

$$Y(X,j) = \Big(-3t\sqrt{2}\tanh^2\frac{X}{2}\Big) + \frac{1}{j}\mathfrak{m}\Big[-2Y_{XXX} + TY_X\Big].$$

Again, utilizing inverse Mahgoub transformation, it reduces to

$$T(X,t^{\diamond}) = \Big(3 - 6\tanh^2\frac{X}{2}\Big) + \mathfrak{m}\Big[\frac{1}{j}K[T_{XXX} + TT_X + YY_X]\Big],$$

$$Y(X,t^{\diamond}) = \Big(-3t\sqrt{2}\tanh^2\frac{X}{2}\Big) + \mathfrak{m}\Big[\frac{1}{j}K[-2Y_{XXX} + TY_X]\Big].$$

Implementing the Homotopy Perturbation scheme i.e.

$$T(X,t^{\diamond}) = T_0 + T_1 p + T_2 p^2 + \dots, \quad Y(X,t^{\diamond}) = Y_0 + Y_1 p + Y_2 p^2 + \dots$$

$$\sum_{n=0}^{\infty} p^n T_X(X,t^\diamond) = \Big(3 - 6\tanh^2\frac{X}{2}\Big)$$
$$-p\mathfrak{m}^{-1}\left[\frac{1}{j}\mathfrak{m}\begin{cases}\Big(\sum_{n=0}^{\infty} p^n T_n(X,t^\diamond)_{XXX}\Big)\\ +\sum_{n=0}^{\infty} p^n H_n^1(T,Y)\end{cases}\right]$$

and

$$\sum_{n=0}^{\infty} p^n T_X(X,t^\diamond) = \Big(-3t\sqrt{2}\tanh^2\frac{X}{2}\Big)$$
$$-p\mathfrak{m}^{-1}\left[\frac{1}{j}\mathfrak{m}\begin{cases}\Big(\sum_{n=0}^{\infty} p^n T_n(X,t^\diamond)_{XXX}\Big)\\ +\sum_{n=0}^{\infty} p^n H_n^2(T,Y)\end{cases}\right].$$

In above equations $H_n^k(T)$ (for$k = 1, 2$) are He's polynomials representing erratic terms. Some elements of He's polynomials mentioned below,

$$H_0^1(T) = T_0 T_{0X} + Y_0 Y_{0X},$$
$$H_1^1(T) = (T_1 T + T_0 T_{1X}) + (Y_1 Y_{0X} + Y_0 Y_{1X}).$$

Likewise,

$$H_0^2(T) = T_0 Y_{0X}, \quad H_1^2(T) = (T_1 Y_{0X} + T_0 Y_{1X}).$$

Associating same power of p, following approximation holds

$$p^0 : T_0(X,t^\diamond) = \Big(3 - 6\tan h^2\frac{X}{2}\Big),$$
$$Y_0(X,t^\diamond) = -\Big(3t\sqrt{2}\tan h^2\frac{X}{2}\Big);$$
$$p^1 : T_1(X,t^\diamond) = -6\sec h^2\frac{X}{2}\tan h\frac{X}{2},$$
$$Y_1(X,t^\diamond) = -3t\sqrt{2}\sec h^2\frac{X}{2}\tan h\frac{X}{2};$$
$$p^2 : T_2(X,t^\diamond) = \frac{3}{2}t^{\diamond 2}\Big(2\sec h^2\frac{X}{2} + 7\sec h^4\frac{X}{2} - 15\sec h^6\frac{X}{2}\Big),$$
$$Y_2(X,t^\diamond) = \frac{3t\sqrt{2}}{4}t^{\diamond 2}\Big(2\sec h^2\frac{X}{2} + 21\sec h^4\frac{X}{2} - 24\sec h^6\frac{X}{2}\Big).$$

Placing $p = 1$ results the nearby solution mentioned below:

$$\begin{aligned} T(X,t^{\diamond}) &= \Big(3 - 6\tanh^2\frac{X}{2}\Big) \pm 6\sec h^2\frac{X}{2}\tanh\frac{X}{2} \\ &\quad + \frac{3}{2}t^{\diamond^2}\Big(2\sec h^2\frac{X}{2} + 7\sec h^4\frac{X}{2} - 15\sec h^6\frac{X}{2}\Big)\ldots \end{aligned}$$

$$\begin{aligned} Y(X,t^{\diamond}) &= -\Big(3t\sqrt{2}\tanh^2\frac{X}{2}\Big) \pm 3t^{\diamond}\sqrt{2}\sec h^2\frac{X}{2}\tanh\frac{X}{2} \\ &\quad + \frac{3t^{\diamond}\sqrt{2}}{4}t^{\diamond^2}\Big(2\sec h^2\frac{X}{2} + 21\sec h^4\frac{X}{2} - 24\sec h^6\frac{X}{2}\Big)\ldots \end{aligned}$$

Remark 4.1. *The end result is just like that acquired with Homotopy Perturbation scheme [16] and [23].*

Case 2. Recognize the popularized coupled Hirota Satsuma KdV system.

$$\begin{aligned} T_{t^{\diamond}} &= \frac{1}{2}T_{XXX} - 3TT_X + 3(YR)_X, \\ Y_{t^{\diamond}} &= 3TY_X - Y_{XXX}, \\ R_{t^{\diamond}} &= 3TR_X - R_{XXX}. \end{aligned}$$

Subject to opening circumstances,

$$\begin{aligned} T(X,0) &= -\frac{1}{3} + 2\tanh^3 X, \\ Y(X,0) &= \tanh X, \\ R(X,0) &= \frac{8}{3}\tanh X. \end{aligned}$$

Now utilizing the aforementioned method subject to opening circumstances, the Equation reduces to

$$\begin{aligned} T(X,j) &= [T(X,0)] + \frac{1}{j}\mathfrak{m}\Big[\frac{1}{2}T_{XXX} - 3TT_X + 3(YR)_X\Big], \\ Y(X,j) &= [Y(X,0)] + \frac{1}{j}\mathfrak{m}\Big[3TY_X - Y_{XXX}\Big], \\ R(X,j) &= [R(X,0)] + \frac{1}{j}\mathfrak{m}\Big[3TR_X - R_{XXX}\Big]. \end{aligned}$$

Implement Inverse Mahgoub transform both side and initial condition

$$T(X,t^{\diamond}) = \Big(-\frac{1}{3} + 2\tanh^2 X\Big) + \mathfrak{m}^{-1}\Big[\frac{1}{j}\mathfrak{m}\Big[\frac{1}{2}T_{XXX} - 3TT_X + 3(YR)_X\Big]\Big],$$

$$Y(X,t^{\diamond}) = \tanh X + \mathfrak{m}^{-1}\Big[\frac{1}{j}\mathfrak{m}\Big[3TY_X - Y_{XXX}\Big]\Big],$$

$$R(X,t^{\diamond}) = \frac{8}{3}\tanh x + \mathfrak{m}^{-1}\Big[\frac{1}{j}\mathfrak{m}\Big[3TR_X - R_{XXX}\Big]\Big].$$

Now implementing Homotopy Perturbation scheme

$$\begin{aligned}\sum_{n=0}^{\infty} p^n T(X,t^{\diamond}) &= \Big(-\frac{1}{3} + 2\tanh^3 X\Big) \\ &\quad -p\mathfrak{m}^{-1}\Big[\frac{1}{j}\mathfrak{m}\Big[\sum_{n=0}^{\infty} p^n H_n^1(T) + \Big(\sum_{n=0}^{\infty} p^n T(X,t^{\diamond})_{XXX}\Big)\Big]\Big],\end{aligned}$$

$$\sum_{n=0}^{\infty} p^n Y_X(X,t^{\diamond}) = (\tanh X) + p\mathfrak{m}^{-1}\Big[\frac{1}{j}\mathfrak{m}\Big[\sum_{n=0}^{\infty} p^n H_n^2(T) - \Big(\sum_{n=0}^{\infty} p^n Y_n(X,t^{\diamond})_{XXX}\Big)\Big]\Big],$$

$$\begin{aligned}\sum_{n=0}^{\infty} p^n R_X(X,t^{\diamond}) &= \Big(\frac{8}{3}\tanh X\Big) \\ &\quad + p\mathfrak{m}^{-1}[\frac{1}{j}\mathfrak{m}\Big[\sum_{n=0}^{\infty} p^n H_n^3(T) - \Big(\sum_{n=0}^{\infty} p^n R_n(X,t)_{XXX}\Big)\Big]\Big]\end{aligned}$$

In above equations H_n^k (for $k = 1,2,3$) represents He's polynomials corresponds the erratic expressions. Only some elements for He's polynomials are mentioned below

$$\begin{aligned}H_0^1(T) &= -3T_0T_{0X} + 3Y_0R_{0X} + 3R_0Y_{0X} \\ H_1^1(T) &= -3(T_1T_{0X} + T_{1X}T_0) + 3(Y_0R_{1X} + Y_1R_{0X}) + 3(R_1Y_{0X} + R_0Y_1X).\end{aligned}$$

likewise $H_0^2(T) = 3T_0Y_{0X}$

$$H_1^2(T) = 3(T_1Y_{0X} + T_0Y_{1X}).$$

Associating same power of p, following are received

$$\begin{aligned}&p^0 : T_0(X,t^{\diamond}) = -\frac{1}{3} + 2\tanh^3 X, \quad Y_0(X,t^{\diamond}) = \tanh X, \\ &R_0(X,t^{\diamond}) = \frac{8}{3}\tanh X, \quad p^1 : T_1(X,t^{\diamond}) = 4t^{\diamond}\sec h^2 X \tanh X, \\ &Y_1(X,t^{\diamond}) = t^{\diamond}\sec h^2 X, \quad R_1(X,t^{\diamond}) = \frac{8}{3}t^{\diamond}\sec h^2 X.\end{aligned}$$

Placing $p = 1$ the following 3 approximation are received

$$T(X,t^\diamond) = -\frac{1}{3} + 4t\sec h^2 X\tanh X + 4t\sec h^2 X(1-3\tanh^2 X)\dots$$
$$Y(X,t^\diamond) = \tanh X + t\sec h^2 X + (-t^{\diamond 2})\sec h^2 X\tanh X\dots$$
$$R(X,t^\diamond) = \frac{8}{3}\tanh X + \frac{8}{3}t^\diamond\sec h^2 X - \frac{8}{3}t^{\diamond 2}\sec h^2 X\tanh X\dots.$$

Remarks: The result here is as same as with HPTM with He's polynomial [16] and [23].

Case 3. Acknowledge the one dimensional coupled burger's equation

$$T_t = T_{XX} + 2TT_X - (TY)_X, \quad Y_{t^\diamond} = Y_{XX} + 2YY_X - (TY)_X.$$

Holding to initial circumstances,

$$T(X,0) = \cos X, \quad Y(X,0) = \cos X.$$

Implementing Mahgoub transform for given equations,

$$T(X,j) = \cos X + \Big[\frac{1}{j}\Big[\mathfrak{m}\Big(T_{XX} + 2TT_X - (TY)_X\Big)\Big]\Big],$$
$$Y(X,j) = \cos X + \Big[\frac{1}{j}\Big[\mathfrak{m}\Big(Y_{XX} + 2YY_X - (TY)_X\Big)\Big]\Big].$$

Utilizing inverse of Mahgoub Transform both side,

$$T(X,t^\diamond) = \cos X + \mathfrak{m}^{-1}\Big[\frac{1}{j}\Big[\mathfrak{m}\Big(T_{XX} + 2TT_X - (TY)_X\Big)\Big]\Big],$$
$$Y(X,t^\diamond) = \cos X + \mathfrak{m}^{-1}\Big[\frac{1}{j}\Big[\mathfrak{m}\Big(Y_{XX} + 2YY - (TY)_X\Big)\Big]\Big].$$

Now we Apply HPTS,

$$\sum_{n=0}^{\infty} p^n T_X(X,t^\diamond) = \cos X - p\mathfrak{m}^{-1}\Big[\frac{1}{j}\Big[\Big[\sum_{n=0}^{\infty} p^n H_n^1(T) + (\sum_{n=0}^{\infty} p^n T_n(X,t^\diamond)_{XX}\Big)\Big]\Big],$$

$$\sum_{n=0}^{\infty} p^n Y_X(X,t^\diamond) = \cos X - p\mathfrak{m}^{-1}\Big[\frac{1}{j}\Big[\sum_{n=0}^{\infty} p^n H_n^2(T) + (\sum_{n=0}^{\infty} p^n Y_n(X,t^\diamond)_{XX}\Big)\Big]\Big].$$

In above equations $H_n^k(T)$ (for $k = 1, 2$) corresponds to He's polynomials representing erratic terms. Some elements for He's polynomials were undermentioned

$$\begin{aligned}
H_0^1(T) &= 2T_0T_{0X} - (T_0Y_{0X} + T_{0X}Y_0)\\
H_1^1(T) &= 2(T_1T_{0X} + T_0T_{1X}) - (T_1Y_{0X} + T_0Y_{1X} + T_{0X}Y_1 + T_{1X}Y_0)\\
H_2^1(T) &= 2(T_2T_{0X} + T_1T_{1X} + T_0T_{2X})\\
&\qquad - (T_2u_{0X} + T_1Y_{1X} + T_0Y_{2X} + Y_2T_{0X} + Y_1T_{1X} + Y_0T_{2X})\dots\\
H_0^2(T) &= 2Y_0Y_{0X} - (T_0Y_{0X} + T_{0X}Y_0)\\
H_1^2(T) &= 2(Y_1Y_{0X} + T_0Y_{1X}) - (T_1Y_{0X} + T_0Y_{1X} + T_{0X}Y_1 + T_{1X}Y_0)\dots
\end{aligned}$$

Associating comparable power of p,

$$p^0 : T_0(X,t^\diamond) = \cos X, Y_0(X,t^\diamond) = \cos X,$$
$$p^1 : T_1(X,t^\diamond) = -t^\diamond \cos X, Y_1(X,t^\diamond) = -t^\diamond \cos X.$$

placing $p = 1$ then

$$T(X,t^\diamond) = T_0 + T_1p + T_2p^2 + \dots,$$

$$T(X,t^\diamond) = \cos X\Big(1 - t^\diamond + \frac{t^{\diamond^2}}{2} - \frac{t^{\diamond^3}}{2}\dots\Big) = \cos X e^{-t^\diamond}, \tag{4.1}$$

and $Y(X,t^\diamond) = Y_0 + Y_1p + Y_2p^2 \dots,$

$$Y(X,t^\diamond) = \cos X\Big(1 - t^\diamond + \frac{t^{\diamond^2}}{2} - \frac{t^{\diamond^3}}{2}\dots\Big) = \cos X e^{-t^\diamond}. \tag{4.2}$$

Remark 4.2. *Equations* (4.1) *and* (4.2) *are very close to that received by Homotopy Perturbation scheme with He's Polynomial [16], [22] and [24].*

Case 4. Look at two dimensional connected burger's equation

$$T_{t^\diamond} - \Cap^2 T - 2T \Cap T + (TY)_X + (TY)_Z = 0,$$
$$Y_{t^\diamond} - \Cap^2 Y - 2Y \Cap Y + (TY)_X + (TY)_Z = 0.$$

Subjected to the settings

$$T(X,Z,0) = \cos(X+Z), \quad Y(X,Z,0) = \cos(X+Z).$$

Implementing Mahgoub transform both side, we obtain

$$T(X,Z,j) = \cos(X+Z) + \frac{1}{j}[\mathfrak{m}(\Cap^2 T + 2T \Cap T - (TY)_X - (TY)_Z)],$$

$$Y(X,Z,j) = \cos(X+Z) + \frac{1}{j}[\mathfrak{m}(⋒^2 Y + 2Y ⋒ Y - (TY)_X - (TY)_Z)].$$

Utilizing Mahgoub inverse both side,

$$T(X,Z,t^\diamond) = \cos(X+Z) + \mathfrak{m}^{-1}[\frac{1}{j}[\mathfrak{m}(⋒^2 T + 2T ⋒ T - (TY)_X - (TY)_Z)]],$$

$$Y(X,Z,t^\diamond) = \cos(X+Z) + \mathfrak{m}^{-1}[\frac{1}{j}[\mathfrak{m}(⋒^2 Y + 2Y ⋒ Y - (TY)_X - (TY)_Z)]].$$

Here we implement $t\ \hat{H}$omotopy Perturbation scheme

$$\sum_{n=0}^{\infty} p^n T_X(X,t^\diamond) = \cos(X+Z) + p\mathfrak{m}^{-1}\Big[\frac{1}{j}\mathfrak{m}\Big[\sum_{n=0}^{\infty} p^n H_n^1(T) + \Big(\sum_{n=0}^{\infty} p^n T_n(X,t^\diamond)_{XX}\Big) + \Big(\sum_{n=0}^{\infty} p^n T_n(X,t^\diamond)_{ZZ}\Big)\Big]\Big],$$

$$\sum_{n=0}^{\infty} p^n Y_X(X,t^\diamond) = \cos(X+Z) - p\mathfrak{m}^{-1}\Big[\frac{1}{j}\mathfrak{m}\Big[\sum_{n=0}^{\infty} p^n H_n^2(T) + \Big(\sum_{n=0}^{\infty} p^n Y_n(X,t^\diamond)_{XX}\Big) + \Big(\sum_{n=0}^{\infty} p^n Y_n(X,t^\diamond)_{ZZ}\Big)\Big]\Big],$$

In above equations $H_n^k(T)$ (for $k = 1,2$) corresponds to He's polynomials representing erratic terms. Some elements for He's Polynomialwere undermentioned,

$$H_0^1(T) = 2T_0 ⋒ T_0 - T_0 ⋒ Y_0 - Y_0 ⋒ T_0 \ldots$$

and

$$H_0^2(T) = 2Y_0 ⋒ Y_0 - T_0 ⋒ Y_0 - Y_0 ⋒ T_0 \ldots.$$

Association with comparable power of p, we get

$$\begin{aligned}
&p^0 : T_0(X,Z,t^\diamond) = \cos(X+Z), Y_0(X,Z,t^\diamond) = \cos(X+Z),\\
&p^1 : T_1(X,Z,t^\diamond) = -2t^\diamond \cos(X+Z),\\
&Y_1(X,Z,t^\diamond) = -2t^\diamond \cos(X+Z),\\
&p^2 : T_2(X,Z,t^\diamond) = 2t^{\diamond^2} \cos(X+Z),\\
&Y_2(X,Z,t^\diamond) = 2t^{\diamond^2} \cos(X+Z),
\end{aligned}$$

placing $p = 1$ holds nearby solution as:

$$T(X,Z,t^\diamond) = T_0 + T_1 p + T_2 p^2 + \ldots,$$

Similarly, $Y(X,Z,t^{\diamond}) = Y_0 + Y_1 p + Y_2 p^2 \ldots,$

$$T(X,Z,t^{\diamond}) = \cos(X+Z)\Big(1 - 2t + \frac{4t^2}{2!} - \frac{8t^3}{3!}\ldots\Big),$$

$$\begin{aligned} Y(X,Z,t^{\diamond}) &= \cos(X+Z)\Big(1 - 2t + \frac{4t^2}{2!} - \frac{8t^3}{3!}\ldots\Big) \\ &= \cos X e^{-2t^{\diamond}}. \end{aligned}$$

Remark 4.3. *The results here are as similar as the when computed with Laplace Transformation with HPTM [16] and [24].*

5. Conclusions and comparative results

This article concludes that Mahgoub Transform can be successfully used with Homotopy Perturbation scheme to find the approximate result. The most important benefit of this scheme is to get over the absence of fulfilled given initial circumstances and to develop Homotopy, which is an unmanageable assignment in case of HPM. As well as this new transform method needs very less computing task, hence shows fast convergence to get approximate explanation of erratic system of PDE's. The correctness and reliability of this modern technique are guaranteed. The end result that states the Mahgoub transform scheme is an elementary and significant implement. The conclusion is identical to He's Polynomial [16] and [22–24]. All of them are coincides. The conclusions by all techniques are identical.

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