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INTERVAL VALUED PICTURE FUZZY SOFT SET IN PATTERN RECOGNITION

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ABSTRACT. This paper deals with interval valued picture fuzzy soft set (IVPFSS) as a generalization of picture fuzzy soft set. We introduce the concept of normalized Euclidean distance between IVPFSS and establish that it is a metric. An algorithm based on this distance of IVPFSS is developed and an illustration is provided.

1. INTRODUCTION

The concept of picture fuzzy set was introduced by Cuong [1,2] as extensions of fuzzy sets and intuitionistic fuzzy sets (Atanassov). They defined some Operations on picture fuzzy sets established some of their properties. Cuong et al. [3] constructed main operations for fuzzy inference processes in picture fuzzy systems. Yang et al. [4] defined picture fuzzy soft set and introduced an algorithm by using level soft set and picture fuzzy soft set to solve decision making problems. Motivated by these concepts we have developed interval valued picture fuzzy soft set. This paper deals with interval valued picture fuzzy soft set (IVPFSS) as a generalization of picture fuzzy soft set. We introduce the concept of normalized Euclidean distance between IVPFSS and establish that it is

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a metric. An algorithm based on this distance of IVPFSS is developed and an illustration is provided.

2. Interval Valued Picture Fuzzy Soft Set

In this section some operations on IVPFSS are defined and some of their properties.

Definition 2.1. *Let U be an universe and E be a set of parameters. Let $IVPFSS(U)$ denote the set of all interval valued picture fuzzy soft set over U and $A \subseteq E$. A pair (P_F, A) is an IVPFSS over U, where P_F is a mapping given by $P_F : A \to IVPFSS(U)$ and*

$$(P_F, A) = \{x, [\underline{\mu}_{P_{F(e)}}(x), \overline{\mu}_{P_{F(e)}}(x)], [\underline{\eta}_{P_{F(e)}}(x), \overline{\eta}_{P_{F(e)}}(x)], [\underline{\nu}_{P_{F(e)}}(x), \overline{\nu}_{P_{F(e)}}(x)]); x \in U, e \in A\}.$$

For any parameter $e \in A$, $P_{F(e)}$ is an IVPFSS.

Definition 2.2. *The necessity operator on an $IVPFSS(P_F, A)$ denoted by*

$$(P_F, A) = \{(x, [\underline{\mu}_{P_{F(e)}}(x), \overline{\mu}_{P_{F(e)}}(x)], [1 - \underline{\eta}_{P_{F(e)}}(x), 1 - \overline{\eta}_{P_{F(e)}}(x)], [1 - \underline{\nu}_{P_{F(e)}}(x), 1 - \overline{\nu}_{P_{F(e)}}(x)]; x \in U, e \in A\}.$$

Definition 2.3. *The possibility operator on an $IVPFSS(P_F, A)$ denoted by*

$$(P_F, A) = \{(x, [1 - \underline{\mu}_{P_{F(e)}}(x), 1 - \overline{\mu}_{P_{F(e)}}(x)], [\underline{\eta}_{P_{F(e)}}(x), \overline{\eta}_{P_{F(e)}}(x)], [\underline{\nu}_{P_{F(e)}}(x), \overline{\nu}_{P_{F(e)}}(x)]; x \in U, e \in A\}.$$

Definition 2.4. *The complement of an $IVPFSS(P_F, A)$ denoted by*

$$(P_F, A)^C = \{(x, [\underline{\nu}_{P_{F(e)}}(x), \overline{\nu}_{P_{F(e)}}(x)], [\underline{\eta}_{P_{F(e)}}(x), \overline{\eta}_{P_{F(e)}}(x)], [\underline{\mu}_{P_{F(e)}}(x), \overline{\mu}_{P_{F(e)}}(x)]; x \in U, e \in A\}.$$

Definition 2.5. *Let $A, B \subseteq E$, (P_F, A) is an interval valued picture fuzzy soft subset of (P_F, B) denoted by $(P_F, A) \Subset (P_F, B)$, if, and only if,*

(i) $A \subseteq B$.

(ii) $\forall e \in A, P_{F(e)}$ *is an interval valued picture fuzzy soft subset of $P_{G(e)}$, i.e, $\forall x \in U, \forall e \in A$, $\underline{\mu}_{P_{F(e)}}(x) \leq \underline{\mu}_{P_{G(e)}}(x)$, $\overline{\mu}_{P_{F(e)}}(x) \leq \overline{\mu}_{P_{G(e)}}(x)$, $\underline{\eta}_{P_{F(e)}}(x) \geq \underline{\eta}_{P_{G(e)}}(x)$, $\overline{\eta}_{P_{F(e)}}(x) \geq \overline{\eta}_{P_{G(e)}}(x)$, $\underline{\nu}_{P_{F(e)}}(x) \geq \underline{\nu}_{P_{G(e)}}(x)$, $\overline{\nu}_{P_{F(e)}}(x) \geq \overline{\nu}_{P_{G(e)}}(x)$.*

Further (P_G, B) is called an interval valued picture fuzzy superset of (P_F, A) and is denoted by $(P_G, B) \Supset (P_F, A)$.

Definition 2.6. *The degree of non-determinacy of an element $x \in U$, $e \in A$ to the $IVPFSS(P_F, A)$ is defined as*

$$\underline{\pi}_{P_{F(e)}}(x) = (1 - \underline{\mu}_{P_{F(e)}}(x) - \underline{\eta}_{P_{F(e)}}(x) - \underline{\nu}_{P_{F(e)}}(x))$$

and

$$\overline{\pi}_{P_{F(e)}}(x) = 1 - (\overline{\mu}_{P_{F(e)}}(x) - \overline{\eta}_{P_{F(e)}}(x) - \overline{\nu}_{P_{F(e)}}(x)).$$

Definition 2.7. *Let $\alpha \in [0,1]$ be a fixed number. Given an $IVPFSS(P_F, A)$, the operator*

$$\begin{aligned}\mathbb{D}_\alpha(P_F, A) = \{x, &[(\underline{\mu}_{P_{F(e)}}(x) + \alpha\underline{\pi}_{P_{F(e)}}(x)), (\overline{\mu}_{P_{F(e)}}(x) + \alpha\overline{\pi}_{P_{F(e)}}(x))],\\ &[(\underline{\eta}_{P_{F(e)}}(x) + (1-\alpha)(\underline{\pi}_{P_{F(e)}}(x)), (\overline{\eta}_{P_{F(e)}}(x) + (1-\alpha)(\overline{\pi}_{P_{F(e)}}(x))],\\ &[(\underline{\nu}_{P_{F(e)}}(x) + (1-\alpha)(\underline{\pi}_{P_{F(e)}}(x)), (\overline{\nu}_{P_{F(e)}}(x) + (1-\alpha)(\overline{\pi}_{P_{F(e)}}(x))];\\ &x \in U,\ e \in A\}.\end{aligned}$$

Theorem 2.1. *For every $IVPFSS(P_F, A)$ and $\alpha, \beta \in [0,1]$, if $\alpha \le \beta$ then $\mathbb{D}_\alpha(P_F, A) \Subset \mathbb{D}_\beta(P_F, B)$.*

Proof. We have

$$\begin{aligned}\mathbb{D}_\alpha(P_F, A) = \{x, &[(\underline{\mu}_{P_{F(e)}}(x) + \alpha\underline{\pi}_{P_{F(e)}}(x)), (\overline{\mu}_{P_{F(e)}}(x) + \alpha\overline{\pi}_{P_{F(e)}}(x))],\\ &[(\underline{\eta}_{P_{F(e)}}(x) + (1-\alpha)(\underline{\pi}_{P_{F(e)}}(x)), (\overline{\eta}_{P_{F(e)}}(x) + (1-\alpha)(\overline{\pi}_{P_{F(e)}}(x))],\\ &[(\underline{\nu}_{P_{F(e)}}(x) + (1-\alpha)(\underline{\pi}_{P_{F(e)}}(x)), (\overline{\nu}_{P_{F(e)}}(x) + (1-\alpha)(\overline{\pi}_{P_{F(e)}}(x))];\\ &x \in U,\ e \in A\}\end{aligned}$$

and

$$\begin{aligned}\mathbb{D}_\beta(P_F, A) = \{x, &[(\underline{\mu}_{P_{F(e)}}(x) + \beta\underline{\pi}_{P_{F(e)}}(x)), (\overline{\mu}_{P_{F(e)}}(x) + \beta\overline{\pi}_{P_{F(e)}}(x))],\\ &[(\underline{\eta}_{P_{F(e)}}(x) + (1-\beta)(\underline{\pi}_{P_{F(e)}}(x)), (\overline{\eta}_{P_{F(e)}}(x) + (1-\beta)(\overline{\pi}_{P_{F(e)}}(x))],\\ &[(\underline{\nu}_{P_{F(e)}}(x) + (1-\beta)(\underline{\pi}_{P_{F(e)}}(x)), (\overline{\nu}_{P_{F(e)}}(x) + (1-\beta)(\overline{\pi}_{P_{F(e)}}(x))];\\ &x \in U,\ e \in A\}.\end{aligned}$$

Similarly $\overline{\mu}_{P_{F(e)}}(x) + \alpha\overline{\pi}_{P_{F(e)}}(x) \le \overline{\mu}_{P_{F(e)}}(x) + \beta\overline{\pi}_{P_{F(e)}}(x)$, since $\alpha < \beta$, $(1-\alpha) > (1-\beta)$ and $(\underline{\eta}_{P_{F(e)}}(x) + (1-\beta)(\underline{\pi}_{P_{F(e)}}(x)) \le (\underline{\eta}_{P_{F(e)}}(x) + (1-\alpha)(\underline{\pi}_{P_{F(e)}}(x))$.

Similarly $(\overline{\eta}_{P_{F(e)}}(x) + (1-\beta)\overline{\pi}_{P_{F(e)}}(x)) \le (\overline{\eta}_{P_{F(e)}}(x) + (1-\alpha)(\overline{\pi}_{P_{F(e)}}(x))$ and $(\underline{\nu}_{P_{F(e)}}(x) + (1-\beta)(\underline{\pi}_{P_{F(e)}}(x)) \le (\underline{\nu}_{P_{F(e)}}(x) + (1-\alpha)(\underline{\pi}_{P_{F(e)}}(x))$.

Similarly, $(\overline{\nu}_{P_{F(e)}}(x) + (1-\beta)\overline{\pi}_{P_{F(e)}}(x)) \le (\overline{\nu}_{P_{F(e)}}(x) + (1-\alpha)(\overline{\pi}_{P_{F(e)}}(x))$.

Hence, it follows that $\mathbb{D}_\alpha(P_F, A) \Subset \mathbb{D}_\beta(P_F, B)$. □

Definition 2.8. *For $\alpha, \beta \in [0,1]$, $\alpha + 2\beta \le 1$ the operator $\mathbb{F}_{\alpha,\beta}$ for an $IVPFSS(P_F, A)$ is defined as*

$$\mathbb{F}_{\alpha,\beta}(P_F, A) = \{x, [(\underline{\mu}_{P_{F(e)}}(x) + \alpha\underline{\pi}_{P_{F(e)}}(x)), (\overline{\mu}_{P_{F(e)}}(x) + \alpha\overline{\pi}_{P_{F(e)}}(x))],$$

$$[(\underline{\eta}_{P_{F(e)}}(x) + \beta\underline{\pi}_{P_{F(e)}}(x)), (\overline{\eta}_{P_{F(e)}}(x) + \beta\overline{\pi}_{P_{F(e)}}(x))],$$
$$[(\underline{\nu}_{P_{F(e)}}(x) + \beta\underline{\pi}_{P_{F(e)}}(x)), (\overline{\nu}_{P_{F(e)}}(x) + \beta\overline{\pi}_{P_{F(e)}}(x))];$$
$$x \in U,\ e \in A\}.$$

Theorem 2.2. *For $IVPFSS(P_F, A)$ and $\forall\ \alpha, \beta, \gamma \in [0,1]$ such that $\alpha + 2\beta \leq 1$, the following hold:*

(i) $\mathbb{F}_{\alpha,\beta}(P_F, A)$ *is an IVPFSS,*
(ii) $If\ 0 \leq \gamma \leq \alpha$ *then* $\mathbb{F}_{\gamma,\beta}(P_F, A) \Subset \mathbb{F}_{\alpha,\beta}(P_F, A)$*,*
(iii) $If\ 0 \leq \gamma \leq \beta$ *then* $\mathbb{F}_{\alpha,\beta}(P_F, A) \Subset \mathbb{F}_{\alpha,\gamma}(P_F, A)$*,*
(iv) $(\mathbb{F}_{\alpha,\beta}(P_F, A)^c)^c = \mathbb{F}_{\alpha,\beta}(P_F, A)$.

Proof.
(i) Consider,

$$\frac{\overline{\mu}_{P_{F(e)}}(x) + \alpha(\overline{\pi}_{P_{F(e)}}(x))}{2} + \frac{\overline{\eta}_{P_{F(e)}}(x) + \beta(\overline{\pi}_{P_{F(e)}}(x))}{2} + \frac{\overline{\nu}_{P_{F(e)}}(x) + \beta(\overline{\pi}_{P_{F(e)}}(x))}{2}$$
$$= \frac{\overline{\mu}_{P_{F(e)}}(x)}{2} + \frac{\overline{\eta}_{P_{F(e)}}(x)}{2} + \frac{\overline{\nu}_{P_{F(e)}}(x)}{2} + (\alpha + 2\beta)(\frac{\overline{\pi}_{P_{F(e)}}(x)}{2})$$
$$\leq \frac{\overline{\mu}_{P_{F(e)}}(x)}{2} + \frac{\overline{\eta}_{P_{F(e)}}(x)}{2} + \frac{\overline{\nu}_{P_{F(e)}}(x)}{2} + \frac{1 - \overline{\mu}_{P_{F(e)}}(x) - \overline{\eta}_{P_{F(e)}}(x) - \overline{\nu}_{P_{F(e)}}(x)}{2}$$
$$\leq \frac{1}{2} < 1.$$

Hence $\mathbb{F}_{\alpha,\beta}(P_F, A)$ is an IVPFSS.

(ii) $$\mathbb{F}_{\gamma,\beta}(P_F, A) = \{x, [(\underline{\mu}_{P_{F(e)}}(x) + \gamma\underline{\pi}_{P_{F(e)}}(x)), (\overline{\mu}_{P_{F(e)}}(x) + \gamma\overline{\pi}_{P_{F(e)}}(x))],$$
$$[(\underline{\eta}_{P_{F(e)}}(x) + \beta\underline{\pi}_{P_{F(e)}}(x)), (\overline{\eta}_{P_{F(e)}}(x) + \beta\overline{\pi}_{P_{F(e)}}(x))],$$
$$[(\underline{\nu}_{P_{F(e)}}(x) + \beta\underline{\pi}_{P_{F(e)}}(x)), (\overline{\nu}_{P_{F(e)}}(x) + \beta\overline{\pi}_{P_{F(e)}}(x))],$$
$$x \in U,\ e \in A\}$$
$$\mathbb{F}_{\alpha,\beta}(P_F, A) = \{x, [(\underline{\mu}_{P_{F(e)}}(x) + \alpha\underline{\pi}_{P_{F(e)}}(x)), (\overline{\mu}_{P_{F(e)}}(x) + \alpha\overline{\pi}_{P_{F(e)}}(x))],$$
$$[(\underline{\eta}_{P_{F(e)}}(x) + \beta\underline{\pi}_{P_{F(e)}}(x)), (\overline{\eta}_{P_{F(e)}}(x) + \beta\overline{\pi}_{P_{F(e)}}(x))],$$
$$[(\underline{\nu}_{P_{F(e)}}(x) + \beta\underline{\pi}_{P_{F(e)}}(x)), (\overline{\nu}_{P_{F(e)}}(x) + \beta\overline{\pi}_{P_{F(e)}}(x))],$$
$$x \in U,\ e \in A\}$$

Now, $\underline{\mu}_{P_{F(e)}}(x) + \gamma\underline{\pi}_{P_{F(e)}}(x) \leq \underline{\mu}_{P_{F(e)}}(x) + \alpha\underline{\pi}_{P_{F(e)}}(x)$, since$\gamma \leq \alpha$.

Similarly, we have $\overline{\mu}_{P_{F(e)}}(x) + \gamma\overline{\pi}_{P_{F(e)}}(x) \leq \overline{\mu}_{P_{F(e)}}(x) + \alpha\overline{\pi}_{P_{F(e)}}(x)$. Hence it follows that $\mathbb{F}_{\gamma,\beta}(P_F, A) \Subset \mathbb{F}_{\alpha,\beta}(P_F, A)$.

(iii) $$\begin{aligned}\mathbb{F}_{\alpha,\beta}(P_F,A) = \{x, &[(\underline{\mu}_{P_{F(e)}}(x) + \alpha\underline{\pi}_{P_{F(e)}}(x)), (\overline{\mu}_{P_{F(e)}}(x) + \alpha\overline{\pi}_{P_{F(e)}}(x))],\\ &[(\underline{\eta}_{P_{F(e)}}(x)+\beta\underline{\pi}_{P_{F(e)}}(x)), (\overline{\eta}_{P_{F(e)}}(x)+\beta\overline{\pi}_{P_{F(e)}}(x))],\\ &[(\underline{\nu}_{P_{F(e)}}(x)+\beta\underline{\pi}_{P_{F(e)}}(x)), (\overline{\nu}_{P_{F(e)}}(x)+\beta\overline{\pi}_{P_{F(e)}}(x))],\\ &x \in U, e \in A\}\\ \mathbb{F}_{\alpha,\gamma}(P_F,A) = \{x, &[(\underline{\mu}_{P_{F(e)}}(x) + \alpha\underline{\pi}_{P_{F(e)}}(x)), (\overline{\mu}_{P_{F(e)}}(x) + \alpha\overline{\pi}_{P_{F(e)}}(x))],\\ &[(\underline{\eta}_{P_{F(e)}}(x)+\gamma\underline{\pi}_{P_{F(e)}}(x)), (\overline{\eta}_{P_{F(e)}}(x)+\gamma\overline{\pi}_{P_{F(e)}}(x))],\\ &[(\underline{\nu}_{P_{F(e)}}(x)+\gamma\underline{\pi}_{P_{F(e)}}(x)), (\overline{\nu}_{P_{F(e)}}(x)+\gamma\overline{\pi}_{P_{F(e)}}(x))],\\ &x \in U, e \in A\}\end{aligned}$$

Now, $\underline{\mu}_{P_{F(e)}}(x) + \gamma\underline{\pi}_{P_{F(e)}}(x) \leq \underline{\mu}_{P_{F(e)}}(x) + \beta\underline{\pi}_{P_{F(e)}}(x)$, since$\gamma \leq \beta$. Similarly, we have $\overline{\mu}_{P_{F(e)}}(x) + \gamma\overline{\pi}_{P_{F(e)}}(x) \leq \overline{\mu}_{P_{F(e)}}(x) + \beta\overline{\pi}_{P_{F(e)}}(x)$. Hence, it follows that $\mathbb{F}_{\alpha,\beta}(P_F,A) \Subset \mathbb{F}_{\alpha,\gamma}(P_F,A)$.

(iv) $$\begin{aligned}(\mathbb{F}_{\alpha,\beta}(P_F,A)^c = \{x, &[(\underline{\nu}_{P_{F(e)}}(x) + \alpha\underline{\pi}_{P_{F(e)}}(x)), (\overline{\nu}_{P_{F(e)}}(x) + \alpha\overline{\pi}_{P_{F(e)}}(x))],\\ &[(\underline{\eta}_{P_{F(e)}}(x) + \beta\underline{\pi}_{P_{F(e)}}(x)), (\overline{\eta}_{P_{F(e)}}(x) + \beta\overline{\pi}_{P_{F(e)}}(x))],\\ &[(\underline{\mu}_{FP(e)}(x) + \beta\underline{\pi}_{FP(e)}(x)), (\overline{\mu}_{FP(e)}(x) + \beta\overline{\pi}_{FP(e)}(x))],\\ &x \in U, e \in A\}.\end{aligned}$$

Then
$$\begin{aligned}(\mathbb{F}_{\alpha,\beta}(P_F,A)^c)^c = \{x, &[(\underline{\mu}_{P_{F(e)}}(x) + \alpha\underline{\pi}_{P_{F(e)}}(x)), (\overline{\mu}_{P_{F(e)}}(x) + \alpha\overline{\pi}_{P_{F(e)}}(x))],\\ &[(\underline{\eta}_{P_{F(e)}}(x) + \beta\underline{\pi}_{P_{F(e)}}(x)), (\overline{\eta}_{P_{F(e)}}(x) + \beta\overline{\pi}_{P_{F(e)}}(x))],\\ &[(\underline{\nu}_{P_{F(e)}}(x) + \beta\underline{\pi}_{P_{F(e)}}(x)), (\overline{\nu}_{P_{F(e)}}(x) + \beta\overline{\pi}_{P_{F(e)}}(x))],\\ &x \in U, e \in A\}.\\ = \mathbb{F}_{\alpha,\beta}(P_F,A).&\end{aligned}$$
□

Definition 2.9. *Let $\alpha, \beta \in [0,1]$. Given an $IVPFSS(P_F,A)$, the operator $\mathbb{G}_{\alpha,\beta}$ is defined as*
$$\begin{aligned}\mathbb{G}_{\alpha,\beta}(P_F,A) = \{(x, &[\alpha\underline{\mu}_{P_{F(e)}}(x), \alpha\overline{\mu}_{P_{F(e)}}(x)], [\beta\underline{\eta}_{P_{F(e)}}(x), \beta\overline{\eta}_{P_{F(e)}}(x)],\\ &[\beta\underline{\nu}_{P_{F(e)}}(x), \beta\overline{\nu}_{P_{F(e)}}(x)]); x \in U, e \in A\}.\end{aligned}$$
Obviously, $\mathbb{G}_{1,1}(P_F,A) = (P_F,A)$ and $\mathbb{G}_{0,0}(P_F,A) = \phi$.

Theorem 2.3. *For every $IVPFSS(P_F,A)$ and $\alpha,\beta,\gamma \in [0,1]$,*

(i) $\mathbb{G}_{\alpha,\beta}(P_F,A)$ *is an IVPFSS.*
(ii) *If $\alpha \leq \gamma$ then $\mathbb{G}_{\alpha,\beta}(P_F,A) \Subset \mathbb{G}_{\gamma,\beta}(P_F,A)$,*
(iii) *If $\beta \leq \gamma$ then $\mathbb{G}_{\alpha,\beta}(P_F,A) \Supset \mathbb{G}_{\alpha,\gamma}(P_F,A)$,*
(iv) *If $\delta \in [0,1]$ then $\mathbb{G}_{\alpha,\beta}(\mathbb{G}_{\gamma,\delta}(P_F,A)) = \mathbb{G}_{\alpha\gamma,\beta\delta}(P_F,A) = \mathbb{G}_{\gamma,\delta}(\mathbb{G}_{\alpha,\beta}(P_F,A))$,*
(v) $(\mathbb{G}_{\alpha,\beta}(P_F,A)^c)^c = \mathbb{G}_{\beta,\alpha}(P_F,A)$.

Proof.

(i) Clearly $\mathbb{G}_{\alpha,\beta}(P_F, A)$ is an IVPFSS.

(ii) $$\begin{aligned}\mathbb{G}_{\alpha,\beta}(P_F, A) &= \{(x, [\alpha\underline{\mu}_{P_{F(e)}}(x), \alpha\overline{\mu}_{P_{F(e)}}(x)], [\beta\underline{\eta}_{P_{F(e)}}(x), \beta\overline{\eta}_{P_{F(e)}}(x)],\\ &\qquad [\beta\underline{\nu}_{P_{F(e)}}(x), \beta\overline{\nu}_{P_{F(e)}}(x)]); x \in U, e \in A\},\\ \mathbb{G}_{\gamma,\beta}(P_F, A) &= \{(x, [\gamma\underline{\mu}_{P_{F(e)}}(x), \gamma\overline{\mu}_{P_{F(e)}}(x)], [\beta\underline{\eta}_{P_{F(e)}}(x), \beta\overline{\eta}_{P_{F(e)}}(x)],\\ &\qquad [\beta\underline{\nu}_{P_{F(e)}}(x), \beta\overline{\nu}_{P_{F(e)}}(x)]); x \in U, e \in A\}.\end{aligned}$$

Since $\alpha \le \gamma$, $\alpha\underline{\mu}_{P_{F(e)}}(x) \le \gamma\underline{\mu}_{P_{F(e)}}(x)$ and $\alpha\overline{\mu}_{P_{F(e)}}(x) \le \gamma\overline{\mu}_{P_{F(e)}}(x)$. Then $\mathbb{G}_{\alpha,\beta}(P_F, A) \Subset \mathbb{G}_{\gamma,\beta}(P_F, A)$.

(iii) $$\begin{aligned}\mathbb{G}_{\alpha,\beta}(P_F, A) &= \{(x, [\alpha\underline{\mu}_{P_{F(e)}}(x), \alpha\overline{\mu}_{P_{F(e)}}(x)], [\beta\underline{\eta}_{P_{F(e)}}(x), \beta\overline{\eta}_{P_{F(e)}}(x)],\\ &\qquad [\beta\underline{\nu}_{P_{F(e)}}(x), \beta\overline{\nu}_{P_{F(e)}}(x)]); x \in U, e \in A\},\\ \mathbb{G}_{\alpha,\gamma}(P_F, A) &= \{(x, [\alpha\underline{\mu}_{P_{F(e)}}(x), \alpha\overline{\mu}_{P_{F(e)}}(x)], [\gamma\underline{\eta}_{P_{F(e)}}(x), \gamma\overline{\eta}_{P_{F(e)}}(x)],\\ &\qquad [\gamma\underline{\nu}_{P_{F(e)}}(x), \gamma\overline{\nu}_{P_{F(e)}}(x)]); x \in U, e \in A\}.\end{aligned}$$

Since $\beta \le \gamma$, $\beta\underline{\eta}_{P_{F(e)}}(x) \le \gamma\underline{\eta}_{P_{F(e)}}(x)$, $\beta\overline{\eta}_{P_{F(e)}}(x) \le \gamma\overline{\eta}_{P_{F(e)}}(x)$ and $\beta\underline{\nu}_{P_{F(e)}}(x) \le \gamma\underline{\nu}_{P_{F(e)}}(x)$, $\beta\overline{\nu}_{P_{F(e)}}(x) \le \gamma\overline{\nu}_{P_{F(e)}}(x)$. Then $\mathbb{G}_{\alpha,\beta}(P_F, A) \Supset \mathbb{G}_{\alpha,\gamma}(P_F, A)$.

(iv) $$\begin{aligned}\mathbb{G}_{\alpha,\beta}(\mathbb{G}_{\gamma,\delta}(P_F, A)) &= \{(x, [\alpha\gamma\underline{\mu}_{P_{F(e)}}(x), \alpha\gamma\overline{\mu}_{P_{F(e)}}(x)],\\ &\qquad [\beta\delta\underline{\eta}_{P_{F(e)}}(x), \beta\delta\overline{\eta}_{P_{F(e)}}(x)],\\ &\qquad [\beta\delta\underline{\nu}_{P_{F(e)}}(x), \beta\delta\overline{\nu}_{P_{F(e)}}(x)]); x \in U, e \in A\}\\ &= \{(x, [(\alpha\gamma)\underline{\mu}_{P_{F(e)}}(x), (\alpha\gamma)\overline{\mu}_{P_{F(e)}}(x)],\\ &\qquad [(\beta\delta)\underline{\eta}_{P_{F(e)}}(x), (\beta\delta)\overline{\eta}_{P_{F(e)}}(x)],\\ &\qquad [(\beta\delta)\underline{\nu}_{P_{F(e)}}(x), (\beta\delta)\overline{\nu}_{P_{F(e)}}(x)]); x \in U, e \in A\}\\ &= \mathbb{G}_{\alpha\gamma,\beta\delta}(P_F, A),\\ \mathbb{G}_{\gamma,\delta}(\mathbb{G}_{\alpha,\beta}(P_F, A)) &= \{(x, [\gamma\alpha\underline{\mu}_{P_{F(e)}}(x), \gamma\alpha\overline{\mu}_{PF(e)}(x)],\\ &\qquad [\delta\beta\underline{\eta}_{P_{F(e)}}(x), \delta\beta\overline{\eta}_{P_{F(e)}}(x)],\\ &\qquad [\delta\beta\underline{\nu}_{P_{F(e)}}(x), \delta\beta\overline{\nu}_{P_{F(e)}}(x)]); x \in U, e \in A\}\\ &= \{(x, [(\gamma\alpha)\underline{\mu}_{P_{F(e)}}(x), (\gamma\alpha)\overline{\mu}_{P_{F(e)}}(x)],\\ &\qquad [(\delta\beta)\underline{\eta}_{P_{F(e)}}(x), (\delta\beta)\overline{\eta}_{P_{F(e)}}(x)],\\ &\qquad [(\delta\beta)\underline{\nu}_{P_{F(e)}}(x), (\delta\beta)\overline{\nu}_{P_{F(e)}}(x)]); x \in U, e \in A\}\\ &= \{(x, [(\alpha\gamma)\underline{\mu}_{P_{F(e)}}(x), (\alpha\gamma)\overline{\mu}_{P_{F(e)}}(x)],\\ &\qquad [(\beta\delta)\underline{\eta}_{P_{F(e)}}(x), (\beta\delta)\overline{\eta}_{P_{F(e)}}(x)],\\ &\qquad [(\beta\delta)\underline{\nu}_{P_{F(e)}}(x), (\beta\delta)\overline{\nu}_{P_{F(e)}}(x)]); x \in U, e \in A\}\\ &= \mathbb{G}_{\alpha\gamma,\beta\delta}(P_F, A).\end{aligned}$$

From the above, it follows that

$$\mathbb{G}_{\alpha,\beta}(\mathbb{G}_{\gamma,\delta}(P_F,A)) = \mathbb{G}_{\alpha\gamma,\beta\delta}(P_F,A) = \mathbb{G}_{\gamma,\delta}(\mathbb{G}_{\alpha,\beta}(P_F,A)).$$

(v) $(P_F,A)^c = \{(x,[\underline{\gamma}_{P_{F(e)}}(x),\overline{\gamma}_{P_{F(e)}}(x)],[\underline{\eta}_{P_{F(e)}}(x),\overline{\eta}_{P_{F(e)}}(x)],$
$[\underline{\mu}_{P_{F(e)}}(x),\overline{\mu}_{P_{F(e)}}(x)]);x\in U,e\in A\},$
$\mathbb{G}_{\alpha,\beta}(P_F,A)^c = \{(x,[\alpha\underline{\gamma}_{P_{F(e)}}(x),\alpha\overline{\gamma}_{P_{F(e)}}(x)],[\alpha\underline{\eta}_{P_{F(e)}}(x),\alpha\overline{\eta}_{P_{F(e)}}(x)],$
$[\beta\underline{\mu}_{P_{F(e)}}(x),\beta\overline{\mu}_{P_{F(e)}}(x)]);x\in U,e\in A\},$
$(\mathbb{G}_{\alpha,\beta}(P_F,A)^c)^c = \{(x,[\beta\underline{\mu}_{P_{F(e)}}(x),\beta\overline{\mu}_{P_{F(e)}}(x)],[\alpha\underline{\eta}_{P_{F(e)}}(x),\alpha\overline{\eta}_{P_{F(e)}}(x)],$
$[\alpha\underline{\gamma}_{P_{F(e)}}(x),\alpha\overline{\gamma}_{P_{F(e)}}(x)]);x\in U,e\in A\},$
$= \mathbb{G}_{\beta,\alpha}(P_F,A).$
Then $(\mathbb{G}_{\alpha,\beta}(P_F,A)^c)^c = \mathbb{G}_{\beta,\alpha}(P_F,A).$ $\square$

3. Normalized Euclidean Distance between IVPFSS

In this section we define normalized Euclidean distance between IVPFSS and establish that it is a metric.

Definition 3.1. *Let $X=\{x_1,x_2,\dots,x_i\}$be an universal set, $E=\{e_1,e_2,\dots,e_j\}$ be a set of parameters and $(P_F,A),(P_G,B)$ two IVPFSS on X. Then the normalized Euclidean distance between (P_F,A) and (P_G,B) is defined as*

$$\begin{aligned}&\mathcal{D}_\epsilon\langle(P_F,A),(P_G,B)\rangle = \\ &\Big\{\tfrac{1}{6mn}\sum_{m=1}^{i}\sum_{n=1}^{j}[|\underline{\mu}_{P_{F(e_i)}}(x_j)-\underline{\mu}_{P_{G(e_i)}}(x_j)|+|\overline{\mu}_{P_{F(e_i)}}(x_j)-\overline{\mu}_{P_{G(e_i)}}(x_j)|\\ &+|\underline{\eta}_{P_{F(e_i)}}(x_j)-\underline{\eta}_{P_{G(e_i)}}(x_j)|+|\overline{\eta}_{P_{F(e_i)}}(x_j)-\overline{\eta}_{P_{G(e_i)}}(x_j)|\\ &+|\underline{\nu}_{P_{F(e_i)}}(x_j)-\underline{\nu}_{P_{G(e_i)}}(x_j)|+|\overline{\nu}_{P_{F(e_i)}}(x_j)-\overline{\nu}_{P_{G(e_i)}}(x_j)|\\ &+|\underline{\pi}_{P_{F(e_i)}}(x_j)-\underline{\pi}_{P_{G(e_i)}}(x_j)|+|\overline{\pi}_{P_{F(e_i)}}(x_j)-\overline{\pi}_{P_{G(e_i)}}(x_j)|]\Big\}.\end{aligned}$$

Theorem 3.1. *Let $IVPFSS(U)$ be the set of all IVPFSS over U. Then the distance function $\mathcal{D}_\epsilon$ from $IVPFSS(U)$ to the set of non-negative real numbers is a metric.*

Proof. Let $(P_F,A),(P_G,B)$ and (P_H,C) be three $IVPFSS(U)$ over U.

(i) $\mathcal{D}_\epsilon\langle(P_F,A),(P_G,B)\rangle > 0$ follows from Definition 3.1.

(ii) $\mathcal{D}_\epsilon\langle(P_F,A),(P_G,B)\rangle = 0$

$$\begin{aligned}&\Leftrightarrow (\underline{\mu}_{P_{F(e_i)}}(x_j)-\underline{\mu}_{P_{G(e_i)}}(x_j)+\overline{\mu}_{P_{F(e_i)}}(x_j)-\overline{\mu}_{P_{G(e_i)}}(x_j))\\ &+(\underline{\eta}_{P_{F(e_i)}}(x_j)-\underline{\eta}_{P_{G(e_i)}}(x_j)+\overline{\eta}_{P_{F(e_i)}}(x_j)-\overline{\eta}_{P_{G(e_i)}}(x_j))\\ &+(\underline{\nu}_{P_{F(e_i)}}(x_j)-\underline{\nu}_{P_{G(e_i)}}(x_j)+\overline{\nu}_{P_{F(e_i)}}(x_j)-\overline{\nu}_{P_{G(e_i)}}(x_j))\end{aligned}$$

$$
\begin{aligned}
&+(\underline{\pi}_{P_{F(e_i)}}(x_j)-\underline{\pi}_{P_{G(e_i)}}(x_j)+\overline{\pi}_{P_{F(e_i)}}(x_j)-\overline{\pi}_{P_{G(e_i)}}(x_j))=0\\
\Leftrightarrow\ &\underline{\mu}_{P_{F(e_i)}}(x_j)=\underline{\mu}_{P_{G(e_i)}}(x_j),\overline{\mu}_{P_{F(e_i)}}(x_j)=\overline{\mu}_{P_{G(e_i)}}(x_j),\\
&\underline{\eta}_{P_{F(e_i)}}(x_j)=\underline{\eta}_{P_{G(e_i)}}(x_j),\overline{\eta}_{P_{F(e_i)}}(x_j)=\overline{\eta}_{P_{G(e_i)}}(x_j),\\
&\underline{\nu}_{P_{F(e_i)}}(x_j)=\underline{\nu}_{P_{G(e_i)}}(x_j),\overline{\nu}_{P_{F(e_i)}}(x_j)=\overline{\nu}_{P_{G(e_i)}}(x_j),\\
&\underline{\pi}_{P_{F(e_i)}}(x_j)=\underline{\pi}_{P_{G(e_i)}}(x_j)\text{ and }\overline{\pi}_{P_{F(e_i)}}(x_j)=\overline{\pi}_{P_{G(e_i)}}(x_j)\\
\Leftrightarrow\ &(P_F,A)=(P_G,B).
\end{aligned}
$$

(iii) Clearly, $\mathcal{D}_\epsilon\langle(P_F,A),(P_G,B)\rangle=\mathcal{D}_\epsilon\langle(P_G,B),(P_F,A)\rangle$.

(iv) Assume that $(P_F,A),(P_G,B)$ and (P_H,C) are IVPFSS over U. Then for all $i\in\{1,2,\cdots,m\},j\in\{1,2,\cdots,n\}$,

$$
\begin{aligned}
&(\underline{\mu}_{P_{F(e_i)}}(x_j)-\underline{\mu}_{P_{G(e_i)}}(x_j)+\overline{\mu}_{P_{F(e_i)}}(x_j)-\overline{\mu}_{P_{G(e_i)}}(x_j))\\
&+(\underline{\eta}_{P_{F(e_i)}}(x_j)-\underline{\eta}_{P_{G(e_i)}}(x_j)+\overline{\eta}_{P_{F(e_i)}}(x_j)-\overline{\eta}_{P_{G(e_i)}}(x_j))\\
&+(\underline{\nu}_{P_{F(e_i)}}(x_j)-\underline{\nu}_{P_{G(e_i)}}(x_j)+\overline{\nu}_{P_{F(e_i)}}(x_j)-\overline{\nu}_{P_{G(e_i)}}(x_j))\\
&+(\underline{\pi}_{P_{F(e_i)}}(x_j)-\underline{\pi}_{P_{G(e_i)}}(x_j)+\overline{\pi}_{P_{F(e_i)}}(x_j)-\overline{\pi}_{P_{G(e_i)}}(x_j))\\
&\quad=(\underline{\mu}_{P_{F(e_i)}}(x_j)-\underline{\mu}_{P_{H(e_i)}}(x_j)+\underline{\mu}_{P_{H(e_i)}}(x_j)-\underline{\mu}_{P_{G(e_i)}}(x_j))\\
&\qquad+(\overline{\mu}_{P_{F(e_i)}}(x_j)-\overline{\mu}_{P_{H(e_i)}}(x_j)+\overline{\mu}_{P_{H(e_i)}}(x_j)-\overline{\mu}_{P_{G(e_i)}}(x_j))\\
&\qquad+(\underline{\eta}_{P_{F(e_i)}}(x_j)-\underline{\eta}_{P_{H(e_i)}}(x_j)+\underline{\eta}_{P_{H(e_i)}}(x_j)-\underline{\eta}_{P_{G(e_i)}}(x_j))\\
&\qquad+(\overline{\eta}_{P_{F(e_i)}}(x_j)-\overline{\eta}_{P_{H(e_i)}}(x_j)+\overline{\eta}_{P_{H(e_i)}}(x_j)-\overline{\eta}_{P_{G(e_i)}}(x_j))\\
&\qquad+(\underline{\nu}_{P_{F(e_i)}}(x_j)-\underline{\nu}_{P_{H(e_i)}}(x_j)+\underline{\nu}_{P_{H(e_i)}}(x_j)-\underline{\nu}_{P_{G(e_i)}}(x_j))\\
&\qquad+(\overline{\nu}_{P_{F(e_i)}}(x_j)-\overline{\nu}_{P_{H(e_i)}}(x_j)+\overline{\nu}_{P_{H(e_i)}}(x_j)-\overline{\nu}_{P_{G(e_i)}}(x_j))\\
&\qquad+(\underline{\pi}_{P_{F(e_i)}}(x_j)-\underline{\pi}_{P_{H(e_i)}}(x_j)+\underline{\pi}_{P_{H(e_i)}}(x_j)-\underline{\pi}_{P_{G(e_i)}}(x_j))\\
&\qquad+(\overline{\pi}_{P_{F(e_i)}}(x_j)-\overline{\pi}_{P_{H(e_i)}}(x_j)+\overline{\pi}_{P_{H(e_i)}}(x_j)-\overline{\pi}_{P_{G(e_i)}}(x_j))\\
&\quad\le(\underline{\mu}_{P_{F(e_i)}}(x_j)-\underline{\mu}_{P_{H(e_i)}}(x_j))+(\underline{\mu}_{P_{H(e_i)}}(x_j)-\underline{\mu}_{P_{G(e_i)}}(x_j))\\
&\qquad+(\overline{\mu}_{P_{F(e_i)}}(x_j)-\overline{\mu}_{P_{H(e_i)}}(x_j))+(\overline{\mu}_{P_{H(e_i)}}(x_j)-\overline{\mu}_{P_{G(e_i)}}(x_j))\\
&\qquad+(\underline{\eta}_{P_{F(e_i)}}(x_j)-\underline{\eta}_{P_{H(e_i)}}(x_j))+(\underline{\eta}_{P_{H(e_i)}}(x_j)-\underline{\eta}_{P_{G(e_i)}}(x_j))\\
&\qquad+(\overline{\eta}_{P_{F(e_i)}}(x_j)-\overline{\eta}_{P_{H(e_i)}}(x_j))+(\overline{\eta}_{P_{H(e_i)}}(x_j)-\overline{\eta}_{P_{G(e_i)}}(x_j))\\
&\qquad+(\underline{\nu}_{P_{F(e_i)}}(x_j)-\underline{\nu}_{P_{H(e_i)}}(x_j))+(\underline{\nu}_{P_{H(e_i)}}(x_j)-\underline{\nu}_{P_{G(e_i)}}(x_j))\\
&\qquad+(\overline{\nu}_{P_{F(e_i)}}(x_j)-\overline{\nu}_{P_{H(e_i)}}(x_j))+(\overline{\nu}_{P_{H(e_i)}}(x_j)-\overline{\nu}_{P_{G(e_i)}}(x_j))\\
&\qquad+(\underline{\pi}_{P_{F(e_i)}}(x_j)-\underline{\pi}_{P_{H(e_i)}}(x_j))+(\underline{\pi}_{P_{H(e_i)}}(x_j)-\underline{\pi}_{P_{G(e_i)}}(x_j))\\
&\qquad+(\overline{\pi}_{P_{F(e_i)}}(x_j)-\overline{\pi}_{P_{H(e_i)}}(x_j))+(\overline{\pi}_{P_{H(e_i)}}(x_j)-\overline{\pi}_{P_{G(e_i)}}(x_j))\\
&\quad=\mathcal{D}_\epsilon\langle(P_F,A),(P_H,C)\rangle+\mathcal{D}_\epsilon\langle(P_H,C),(P_G,B)\rangle.
\end{aligned}
$$

Thus, $\mathcal{D}_\epsilon\langle(P_F,A),(P_G,B)\rangle\le\mathcal{D}_\epsilon\langle(P_F,A),(P_H,C)\rangle+\mathcal{D}_\epsilon\langle(P_H,C),(P_G,B)\rangle$. This shows that $\mathcal{D}_\epsilon$ satisfies the triangle inequality. So $\mathcal{D}_\epsilon$ is a metric. □

4. Pattern Recognition Problem

Step 1: Construct an $IVPFSS(P_S, E)$, over U based on expert evaluation and this can be considered as the known pattern.

Step 2: Construct an $IVPFSS(P_{F_i}, E), \forall i = 1, 2, 3, \ldots$ over U based on the data available for the unknown pattern.

Step 3: Calculate the normalized Euclidean distance between (P_S, E) and (P_{F_i}, E).

Step 4: The pattern with less normalized Euclidean distance between (P_S, E) and (P_{F_i}, E) the pattern in the best suitable pattern.

Example 1. *Let three picture fuzzy soft sets $(P_{F_1}, E), (P_{F_2}, E)$ and (P_{F_3}, E) denote microwave oven of three different companies depending on the parameters $E = \{e_1, e_2, e_3, e_4, e_5\}$ where e_1= range of price ,e_2= solo, grill or convection,e_3= ordinary or inverter,e_4= electromagnetic radiation and e_5= dielectric material which are rated by four customers C_1, C_2, C_3 and C_4 on the five parameters. Depending on the rating of the customers the best microwave oven is to be selected depending upon the above parameters.*

Step 1. *The construction of known pattern is as follows (Table 1).*

U	C_1	C_2
e_1	[0.21,0.33],[0.45,0.52],[0.08,0.13]	[0.09,0.23],[0.29,0.31],[0.18,0.26]
e_2	[0.15,0.26],[0.35,0.43],[0.25,0.31]	[0.17,0.30],[0.12,0.29],[0.26,0.35]
e_3	[0.30,0.41],[0.23,0.35],[0.17,0.22]	[0.28,0.42],[0.19,0.36],[0.08,0.13]
e_4	[0.11,0.28],[0.20,0.37],[0.22,0.34]	[0.06,0.15],[0.25,0.45],[0.22,0.38]
e_5	[0.03,0.16],[0.32,0.44],[0.27,0.38]	[0.14,0.21],[0.16,0.50],[0.10,0.27]
U	C_3	C_4
e_1	[0.18,0.25],[0.07,0.36],[0.29,0.39]	[0.12,0.24],[0.09,0.42],[0.25,0.31]
e_2	[0.02,0.11],[0.23,0.49],[0.14,0.28]	[0.10,0.32],[0.18,0.22],[0.11,0.45]
e_3	[0.20,0.31],[0.13,0.27],[0.35,0.41]	[0.21,0.41],[0.03,0.30],[0.07,0.28]
e_4	[0.08,0.25],[0.21,0.30],[0.22,0.45]	[0.06,0.16],[0.29,0.49],[0.14,0.33]
e_5	[0.26,0.34],[0.16,0.42],[0.10,0.19]	[0.13,0.20],[0.08,0.52],[0.17,0.23]

Table 1. $IVPFSS(P_S, E)$ over U is the data from the previous records of the customer for the best microwave oven.

Step 2. *The construction of unknown pattern are given in Tables 2,3 and 4.*

U	C_1	C_2
e_1	[0.05,0.26],[0.13,0.33],[0.22,0.40]	[0.23,0.35],[0.07,0.28],[0.16,0.31]
e_2	[0.18,0.21],[0.08,0.29],[0.31,0.48]	[0.11,0.27],[0.18,0.32],[0.05,0.22]
e_3	[0.37,0.43],[0.17,0.25],[0.03,0.32]	[0.09,0.20],[0.38,0.43],[0.10,0.30]
e_4	[0.15,0.23],[0.29,0.35],[0.19,0.40]	[0.14,0.33],[0.26,0.53],[0.02,0.13]
e_5	[0.07,0.34],[0.14,0.42],[0.06,0.20]	[0.08,0.25],[0.12,0.40],[0.29,0.34]
U	C_3	C_4
e_1	[0.08,0.21],[0.13,0.39],[0.23,0.40]	[0.15,0.32],[0.24,0.48],[0.07,0.19]
e_2	[0.17,0.30],[0.05,0.27],[0.30,0.43]	[0.20,0.28],[0.11,0.30],[0.38,0.42]
e_3	[0.24,0.38],[0.19,0.48],[0.03,0.14]	[0.08,0.14],[0.25,0.51],[0.21,0.34]
e_4	[0.06,0.26],[0.31,0.41],[0.28,0.32]	[0.35,0.45],[0.18,0.39],[0.02,0.10]
e_5	[0.15,0.46],[0.02,0.12],[0.35,0.42]	[0.12,0.53],[0.05,0.16],[0.23,0.31]

Table 2. $IVPFSS(P_{F_1}, E)$ *over U gives the data rated by the customers for the microwave oven-1.*

U	C_1	C_2
e_1	[0.11,0.36],[0.22,0.44],[0.03,0.17]	[0.27,0.38],[0.09,0.27],[0.16,0.33]
e_2	[0.25,0.37],[0.08,0.15],[0.20,0.38]	[0.12,0.35],[0.30,0.43],[0.05,0.21]
e_3	[0.05,0.40],[0.28,0.34],[0.12,0.25]	[0.08,0.17],[0.25,0.32],[0.47,0.50]
e_4	[0.18,0.32],[0.43,0.50],[0.06,0.14]	[0.24,0.40],[0.06,0.20],[0.13,0.36]
e_5	[0.09,0.21],[0.13,0.46],[0.26,0.33]	[0.19,0.29],[0.31,0.45],[0.03,0.18]
U	C_3	C_4
e_1	[0.22,0.35],[0.15,0.27],[0.08,0.32]	[0.07,0.22],[0.28,0.37],[0.12,0.41]
e_2	[0.15,0.29],[0.33,0.42],[0.19,0.26]	[0.15,0.30],[0.09,0.20],[0.27,0.35]
e_3	[0.09,0.36],[0.20,0.51],[0.05,0.12]	[0.23,0.36],[0.18,0.45],[0.05,0.16]
e_4	[0.11,0.45],[0.02,0.18],[0.25,0.30]	[0.12,0.29],[0.31,0.50],[0.11,0.21]
e_5	[0.24,0.31],[0.10,0.28],[0.04,0.41]	[0.08,0.17],[0.43,0.55],[0.03,0.26]

Table 3. $IVPFSS(P_{F_2}, E)$ *over U gives the data rated by the customers for the microwave oven-2.*

U	C_1	C_2
e_1	[0.09,0.28],[0.15,0.30],[0.22,0.41]	[0.15,0.21],[0.35,0.44],[0.07,0.25]
e_2	[0.11,0.32],[0.05,0.42],[0.12,0.25]	[0.09,0.19],[0.28,0.39],[0.13,0.20]
e_3	[0.21,0.34],[0.18,0.27],[0.08,0.36]	[0.26,0.30],[0.14,0.32],[0.03,0.34]
e_4	[0.07,0.29],[0.33,0.51],[0.10,0.20]	[0.17,0.33],[0.05,0.42],[0.18,0.22]
e_5	[0.14,0.21],[0.03,0.19],[0.37,0.48]	[0.06,0.24],[0.29,0.31],[0.16,0.43]

U	C_3	C_4
e_1	*[0.19,0.23],[0.32,0.41],[0.09,0.36]*	*[0.14,0.25],[0.08,0.22],[0.38,0.42]*
e_2	*[0.06,0.12],[0.21,0.39],[0.16,0.47]*	*[0.18,0.29],[0.16,0.30],[0.28,0.40]*
e_3	*[0.28,0.35],[0.03,0.18],[0.29,0.43]*	*[0.09,0.19],[0.23,0.44],[0.15,0.37]*
e_4	*[0.15,0.38],[0.24,0.31],[0.05,0.20]*	*[0.13,0.33],[0.05,0.17],[0.26,0.43]*
e_5	*[0.04,0.49],[0.17,0.29],[0.13,0.22]*	*[0.10,0.21],[0.48,0.54],[0.03,0.20]*

Table 4. $IVPFSS(P_{F_3}, E)$ over U gives the data rated by the customers for the microwave oven-3.

Step 3. *The normalized Euclidean distance between $\mathcal{D}_\epsilon\langle(P_S, E), (P_{F_i}, E)\rangle$ iS calculated using Definition 3.1. The values evaluated are as follows:*

$$\mathcal{D}_\epsilon\langle(P_S, E), (P_{F_1}, E)\rangle = 0.1523,$$
$$\mathcal{D}_\epsilon\langle(P_S, E), (P_{F_2}, E)\rangle = 0.1465,$$
$$\mathcal{D}_\epsilon\langle(P_S, E), (P_{F_3}, E)\rangle = 0.1375.$$

Step 4. *We observe that $\mathcal{D}_\epsilon\langle(P_S, E), (P_{F_3}, E)\rangle$ is the least distance. Hence microwave oven-3 is the best.*

References

[1] B. C. Cuong, V. Kreinovich: *Picture Fuzzy Sets - a new concept for computational intelligence problems*, Proceedings of the Third World Congress on Information and Communication Technologies WIICT, (2013), 1–6.

[2] B. C. Cuong: *Picture Fuzzy sets*, J. Comput. Sci. Cybern., **30** (2014) 409–420.

[3] B. C. Cuong, P. V. Hai: *Some fuzzy logic operators for picture fuzzy sets*, Seventh International Conference on Knowledge and Systems Engineering, (2015), 132–137.

[4] Y. Yang, C. Liang, S. Ji, T. Liu: *Adjustable soft discernibility matrix based on picture fuzzy soft sets and its application in decision making*, J. Int. Fuzzy Syst., **29** (2015), 1711–1722.

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