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A FUZZY INVENTORY MODEL WITH ENVIRONMENTALLY RESPONSIBLE WASTE DISPOSAL

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ABSTRACT. In this paper, our objective is to dispose the wastages that occur in manufacturing processes so as to maintain a clean and green environment. We have been introduced the trapezoidal fuzzy numbers. Our aim is to calculate optimum EOQ and minimum total cost of these models by using the Beta distribution method for defuzzifying the fuzzy total production, inventory cost and by using the Extension of Lagrangean Method for solving the problem. The proposed model is verified using an illustration.

1. INTRODUCTION

Fuzzy is a concept which has various boundaries of application according to the necessity and the circumstances it is used. The concept seems to be vague sometimes and has no specific meaning. Inventory refers to goods that are in various stages of being made ready for sale. In general, it is the stock that is kept ready for the sale of the next year. In many occasions, the cost of waste disposal is not taken into consideration while formulating an EOQ model to get the minimum total cost. Methods like landfill, decomposing, incineration and few more are used in dealing the municipal waste. Among all these methods, the incineration method is comparatively both economical and also good for the environment.

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Dobos and Richter [2] derived a production or recycling model and from that, for further development, Jaber et al. [4, 5] proposed EOQ of repair and waste disposal which has entropy cost. A. Roy and K. Maity developed the concept and suggested an EOQ with remanufacturing of defective items. A.M.A. El Saandany and M.Y. Jaber [3] went for an EOQ model in which returns of subassemblies has been dealt in a new way. Sawakhande S.M. and R.T. Ajdhav came up with a study of Renewable Energy from Bio- Waste Municipal Corporation. W. Ritha and Nivetha Martin suggested another EOQ model that explains the benefits of incineration as the method for disposing the wastes. W. Ritha and I. Antonitte Vinolin [7] derived another EOQ in which waste disposal is done with environmental concern. Chang et al. [1] derived an EOQ for the uncertain demand. Recently, authors suggested an EOQ in fuzzy to calculate the minimum total cost when the cost includes the cost of waste disposal also [6].

In this paper, we are going to propose an inventory model to dispose the wastages that occur in manufacturing processes so as to maintain a clean and green environment. The solution is discussed in both crisp and fuzzy senses. We use trapezoidal fuzzy numbers to discuss the same under fuzzy sense. An illustration is shown to verify the proposed model numerically and the derivation of the model ends up with a conclusion and suggestions for further studies.

2. Definitions and Methodologies

Definition 2.1. *Fuzzy Set: A fuzzy set $\tilde{A}$ in X is defined as the set of ordered pairs $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X\}$, where $\mu_{\tilde{A}} : X \to [0, 1]$ is a the the membership function of $x \in X$ in $\tilde{A}$.*

Definition 2.2. *Trapezoidal Fuzzy Number: $\tilde{A} = (a, b, c, d)$ is called a trapezoidal fuzzy number in which the membership function is*

$$\mu_{\tilde{A}}(x) \begin{cases} L(x) = \frac{x-a}{b-a}, & a \le x \le b \\ 1, & b \le x \le c \\ R(x) = \frac{d-x}{d-c}, & c \le x \le d \\ 0, & \textit{otherwise} \end{cases} .$$

Definition 2.3. *Beta Distribution Method: The Beta distribution function is a function that maps the set of trapezoidal numbers to the real line R. The real number* *that corresponds to the trapezoidal fuzzy number* $\tilde{A} = (a_1, a_2, a_3, a_4)$ *is given by*

$$\beta(\tilde{A}) = \frac{2a_1 + 7a_2 + 7a_3 + 2a_4}{18}.$$

Lagrange Method:

This method is used to find the solution of non-linear LPP when the constraints are taken as equations. Later, it is developed so that it is used for solving constraints with inequalities. Assume that the problem is to minimize $y = f(x)$, subject to the constraints $g_i(x) \geq 0, i = 1, 2, ...m.x \geq 0$ is also included. Then the method has the following steps.

Step 1: Solve the LPP Min $y = f(x)$. If the result satisfies all the constraints, stop. Otherwise, put $k = 1$ and go to the next step.

Step 2: Convert any k constraints into equalities and minimize $f(x)$ subject to these k constraints using Lagrangean method. If the result is feasible according to the remaining constraints, then repeat. If all constraints are considered at a stretch without getting a feasible solution, go to the next step.

Step 3: If $K = m$, stop. It means that there is no feasible solution. If not, put $k = k + 1$ and proceed from step 2.

Fundamental Operations:

Let $\tilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$ be two trapezoidal fuzzy numbers. then the fundamental operations are defined as follows.

- $\tilde{A} \oplus \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$
- $\tilde{A} \otimes \tilde{B} = (a_1 b_1, a_2 b_2, a_3 b_3, a_4 b_4)$
- $\tilde{A} \ominus \tilde{B} = (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1)$
- $\tilde{A} \oslash \tilde{B} = (\frac{a_1}{b_4}, \frac{a_2}{b_3}, \frac{a_3}{b_2}, \frac{a_4}{b_1})$
- $\alpha\tilde{A} = \begin{cases} \alpha a_1, \alpha a_2, \alpha a_3, \alpha a_4, \alpha \geq 0 \\ \alpha a_4, \alpha a_3, \alpha a_2, \alpha a_1, \alpha < 0 \end{cases}$

Notations:

A - Ordering Cost
S - Setup Cost
H - Holding Cost
T_C - Total Cost
m - Proportion of waste produced
D - Demand Rate

P - Production Rate
Q - Economic Order Quantity
$\tilde{A}$ - Fuzzy ordering Cost
$\tilde{S}$ - Fuzzy setup Cost
$\tilde{H}$ - Fuzzy holding Cost
$\tilde{W}$ - Fuzzy Waste disposing Cost
$\tilde{m}$ - Fuzzy Proportion of waste produced
$\tilde{Q}^*$ - Fuzzy EOQ
$\tilde{T}_c$ - Fuzzy total cost

2.1. **Assumptions.**

- The demand rate is a known fixed quantity
- Shortages are not entertained.
- The time limit has no restriction.

2.2. **Mathematical model in Crisp Sense.** The total cost of this mathematical inventory model is the total of the Ordering cost, Holding cost, Setup Cost and the Waste disposal cost

$$T_c = \frac{Q}{2}\left[HD + \left(1 - \frac{P}{D}\right)A\right] + \frac{1}{Q}\left[AD + S + Wm\right].$$

By Differentiating w.r.t Q and equating to zero, we have

$$Q^* = \sqrt{\frac{2\left(AD + S + Wm\right)}{HD + \left(1 - \frac{P}{D}\right)A}}.$$

2.3. **Mathematical model in Fuzzy Sense:** Let us consider the Ordering cost, Setup cost, Holding cost, Waste disposal costand the proportion of waste produced as trapezoidal fuzzy numbers. Let

$$\tilde{A} = (a_1, a_2, a_3, a_4), \quad \tilde{S} = (s_1, s_2, s_3, s_4), \quad \tilde{H} = (h_1, h_2, h_3, h_4)$$

$$\tilde{W} = (w_1, w_2, w_3, w_4), \quad \tilde{m} = (m_1, m_2, m_3, m_4) \text{ and } \tilde{Q} = (Q_1, Q_2, Q_3, Q_4)$$

be Trapezoidal fuzzy numbers. Now, the fuzzy total cost is given by

$$\tilde{T}_c = \frac{\tilde{Q}}{2}\left[\tilde{H}D \oplus \left(1 \ominus \frac{P}{D}\right)\tilde{A}\right] \oplus \frac{1}{\tilde{Q}}\left[\tilde{A}D + \tilde{S} + \tilde{W}\tilde{m}\right],$$

$$\tilde{T}_c = \frac{(Q_1,Q_2,Q_3,Q_4)}{2}\left[(h_1,h_2,h_3,h_4)D \oplus \left(1-\frac{P}{D}\right)(a_1,a_2,a_3,a_4)\right]$$
$$\oplus \frac{1}{(Q_1,Q_2,Q_3,Q_4)}\Big[(a_1,a_2,a_3,a_4)D \oplus (s_1,s_2,s_3,s_4)$$
$$\oplus (w_1,w_2,w_3,w_4)(m_1,m_2,m_3,m_4)\Big]$$

$$\tilde{T}_c = \frac{Q_1}{2}\left[h_1D + \left(1-\frac{P}{D}\right)a_1\right] + \frac{1}{Q_1}\left[a_1D + +s_1 + w_1m_1\right]$$
$$\frac{Q_2}{2}\left[h_2D + \left(1-\frac{P}{D}\right)a_2\right] + \frac{1}{Q_2}\left[a_2D + +s_2 + w_2m_2\right],$$
$$\frac{Q_3}{2}\left[h_3D + \left(1-\frac{P}{D}\right)a_3\right] + \frac{1}{Q_3}\left[a_3D + +s_3 + w_3m_3\right], \tag{2.1}$$
$$\frac{Q_4}{2}\left[h_4D + \left(1-\frac{P}{D}\right)a_4\right] + \frac{1}{Q_4}\left[a_4D + +s_4 + w_4m_4\right].$$

We use Beta distribution function for defuzzification. By defuzzifying equation (2.1) using beta distribution function, we get

$$\tilde{T}_c = \frac{1}{18}\Big(2\Big[\frac{Q_1}{2}\left[h_1D + \left(1-\frac{P}{D}\right)a_1\right] + \frac{1}{Q_1}\left[a_1D + +s_1 + w_1m_1\right]\Big]$$
$$+ 7\Big[\frac{Q_2}{2}\left[h_2D + \left(1-\frac{P}{D}\right)a_2\right] + \frac{1}{Q_2}\left[a_2D + +s_2 + w_2m_2\right]\Big]$$
$$+ 7\Big[\frac{Q_3}{2}\left[h_3D + \left(1-\frac{P}{D}\right)a_3\right] + \frac{1}{Q_3}\left[a_3D + +s_3 + w_3m_3\right]\Big]$$
$$+ 2\Big[\frac{Q_4}{2}\left[h_4D + \left(1-\frac{P}{D}\right)a_4\right] + \frac{1}{Q_4}\left[a_4D + +s_4 + w_4m_4\right]\Big]\Big).$$

The extension of the Lagrangean method is used to solve Q_1, Q_2, Q_3 and $Q_4$to get minimum $\tilde{T}_c(Q)$.

Step1: Minimize

$$\begin{aligned}\tilde{T}_c = \frac{1}{18}\Big(2\Big[\frac{Q_1}{2}\left[h_1 D + \left(1-\frac{P}{D}\right)a_1\right] + \frac{1}{Q_1}\left[a_1 D + +s_1 + w_1 m_1\right]\Big] \\ + 7\Big[\frac{Q_2}{2}\left[h_2 D + \left(1-\frac{P}{D}\right)a_2\right] + \frac{1}{Q_2}\left[a_2 D + +s_2 + w_2 m_2\right]\Big] \\ + 7\Big[\frac{Q_3}{2}\left[h_3 D + \left(1-\frac{P}{D}\right)a_3\right] + \frac{1}{Q_3}\left[a_3 D + +s_3 + w_3 m_3\right]\Big] \\ + 2\Big[\frac{Q_4}{2}\left[h_4 D + \left(1-\frac{P}{D}\right)a_4\right] + \frac{1}{Q_4}\left[a_4 D + +s_4 + w_4 m_4\right]\Big]\Big).\end{aligned}$$

with $0 < Q_1 \leq Q_2 \leq Q_3 \leq Q_4$, $Q_2 - Q_1 \geq 0, Q_3 - Q_2 \geq 0,$ and $Q_4 - Q_3 \geq 0,$ where $Q_1 \geq 0.$

Now equating all the partial derivatives to zero and solving $Q_1, Q_2, Q_3,$ and Q_4, we get

$$\begin{aligned}\frac{\partial \tilde{T}_c}{\partial Q_1} = 0 \quad \text{then} \quad Q_1 = \sqrt{\frac{2(2a_4 D + 2s_4 + 2w_4 m_4)}{2h_1 D + 2\left(1-\frac{P}{D}\right)a_1}}, \\ \frac{\partial \tilde{T}_c}{\partial Q_2} = 0 \quad \text{then} \quad Q_2 = \sqrt{\frac{2(7a_3 D + 7s_3 + 7w_3 m_3)}{7h_2 D + 7\left(1-\frac{P}{D}\right)a_2}}, \\ \frac{\partial \tilde{T}_c}{\partial Q_3} = 0 \quad \text{then} \quad Q_3 = \sqrt{\frac{2(7a_2 D + 7s_2 + 7w_2 m_2)}{7h_3 D + 7\left(1-\frac{P}{D}\right)a_3}}, \\ \frac{\partial \tilde{T}_c}{\partial Q_4} = 0 \quad \text{then} \quad Q_4 = \sqrt{\frac{2(2a_1 D + 2s_1 + 2w_1 m_1)}{2h_4 D + 2\left(1-\frac{P}{D}\right)a_4}}.\end{aligned}$$

It shows that $Q_1 \geq Q_2$ and $Q_3 \geq Q_4$, which does not arrive the local optimum.

Step 2: The inequality constraint $Q_2 - Q_1 \geq 0$ is converted into equality as $Q_2 - Q_1 = 0$ and minimize $\tilde{T}_c(Q)$ subject to $Q_2 - Q_1 = 0$.

Fix the constraint as 1. (i.e) $\lambda(Q_2 - Q_1)$

$$\begin{aligned}\tilde{T}_c = \frac{1}{18}\Big(2\Big[\frac{Q_1}{2}\left[h_1 D + \left(1-\frac{P}{D}\right)a_1\right] + \frac{1}{Q_4}\left[a_1 D + +s_1 + w_1 m_1\right]\Big] \\ + 7\Big[\frac{Q_2}{2}\left[h_2 D + \left(1-\frac{P}{D}\right)a_2\right] + \frac{1}{Q_3}\left[a_2 D + +s_2 + w_2 m_2\right]\Big]\end{aligned}$$

$$+7\Big[\frac{Q_3}{2}\left[h_3D+\left(1-\frac{P}{D}\right)a_3\right]+\frac{1}{Q_2}\left[a_3D++s_3+w_3m_3\right]\Big]$$
$$+2\Big[\frac{Q_4}{2}\left[h_4D+\left(1-\frac{P}{D}\right)a_4\right]+\frac{1}{Q_1}\left[a_4D++s_4+w_4m_4\right]\Big]\Big)+\lambda(Q_2-Q_1).$$

Differentiating partially w.r.t. $Q_1, Q_2, Q_3,$ and Q_4, and minimizing $\tilde{T}_(Q)$, we get five kinds of results. Equating all partial derivatives to zero and solving $Q_1, Q_2, Q_3,$ and Q_4, we get

$$Q_1=Q_2=\sqrt{\frac{2(2a_4D+2s_4+2w_4m_4)+2(7a_3D+7s_3+7w_3m_3)}{2h_1D+2\left(1-\frac{P}{D}\right)a_1+7h_2D+7\left(1-\frac{P}{D}\right)a_2}},$$

$$Q_3=\sqrt{\frac{2(7a_2D+7s_2+7w_2m_2)}{7h_3D+7\left(1-\frac{P}{D}\right)a_3}},\quad Q_4=\sqrt{\frac{2(2a_1D+2s_1+2w_1m_1)}{2h_4D+2\left(1-\frac{P}{D}\right)a_4}}.$$

Because the above result shows that $Q_3 \geq Q_4$, it doesn't satisfy
$0 < Q_1 \leq Q_2 \leq Q_3 \leq Q_4$.
This means that the local optimum is not yet obtained.

Step 3: (Fix the constraint as 2)

$$\lambda_1(Q_2-Q_1)+\lambda_2(Q_3-Q_2)$$

$$\tilde{T}_c=\frac{1}{18}\Bigg(2\Big[\frac{Q_1}{2}\left[h_1D+\left(1-\frac{P}{D}\right)a_1\right]+\frac{1}{Q_4}\left[a_1D++s_1+w_1m_1\right]\Big]$$
$$+7\Big[\frac{Q_2}{2}\left[h_2D+\left(1-\frac{P}{D}\right)a_2\right]+\frac{1}{Q_3}\left[a_2D++s_2+w_2m_2\right]\Big]$$
$$+7\Big[\frac{Q_3}{2}\left[h_3D+\left(1-\frac{P}{D}\right)a_3\right]+\frac{1}{Q_2}\left[a_3D++s_3+w_3m_3\right]\Big]$$
$$+2\Big[\frac{Q_4}{2}\left[h_4D+\left(1-\frac{P}{D}\right)a_4\right]+\frac{1}{Q_1}\left[a_4D++s_4+w_4m_4\right]\Big]\Bigg)$$
$$+\lambda_1(Q_2-Q_1)+\lambda_2(Q_3-Q_2).$$

Differentiating partially w.r.t. $Q_1, Q_2, Q_3,$ and Q_4, and λ to minimizing $\tilde{T}_(Q)$, we get five kinds of results.

Equating all partial derivatives to '0' and solving Q_1, Q_2, Q_3, and Q_4, we get

$$Q_1 = Q_2 = Q_3 = \sqrt{\frac{\begin{matrix} 2(2a_4D + 2s_4 + 2w_4m_4) + 2(7a_3D + 7s_3 + 7w_3m_3) \\ +2(7a_2D + 7s_2 + 7w_2m_2) \end{matrix}}{\begin{matrix} 2h_1D + 2\left(1 - \frac{P}{D}\right)a_1 + 7h_2D + 7\left(1 - \frac{P}{D}\right)a_2 \\ +7h_3D + 7\left(1 - \frac{P}{D}\right)a_3 \end{matrix}}},$$

$$Q_4 = \sqrt{\frac{2(2a_1D + 2s_1 + 2w_1m_1)}{2h_4D + 2\left(1 - \frac{P}{D}\right)a_4}}.$$

It doesn't satisfy local optimum.

Step 4: Fix Constraints as 3. That is

$$\lambda_1(Q_2 - Q_1) + \lambda_2(Q_3 - Q_2) + \lambda_3(Q_4 - Q_3)$$

$$\begin{aligned} \tilde{T}_c = \frac{1}{18}\Bigg(& 2\Big[\frac{Q_1}{2}\left[h_1D + \left(1 - \frac{P}{D}\right)a_1\right] + \frac{1}{Q_4}\left[a_1D + +s_1 + w_1m_1\right]\Big] \\ & + 7\Big[\frac{Q_2}{2}\left[h_2D + \left(1 - \frac{P}{D}\right)a_2\right] + \frac{1}{Q_3}\left[a_2D + +s_2 + w_2m_2\right]\Big] \\ & + 7\Big[\frac{Q_3}{2}\left[h_3D + \left(1 - \frac{P}{D}\right)a_3\right] + \frac{1}{Q_2}\left[a_3D + +s_3 + w_3m_3\right]\Big] \\ & + 2\Big[\frac{Q_4}{2}\left[h_4D + \left(1 - \frac{P}{D}\right)a_4\right] + \frac{1}{Q_1}\left[a_4D + +s_4 + w_4m_4\right]\Big]\Bigg) \\ & + \lambda_1(Q_2 - Q_1) + \lambda_2(Q_3 - Q_2) + \lambda_3(Q_4 - Q_3) \end{aligned}$$

Then we simplify it all and we have

$$\tilde{Q}^* = \sqrt{\frac{2\Big[\begin{matrix}(2a_1 + 7a_2 + 7a_3 + 2a_4)D + (2s_1 + 7s_2 + 7s_2 + 2s_4) \\ +(2w_1m_1 + 7w_2m_2 + 7w_3m_3 + 2w_4m_4)\end{matrix}\Big]}{\Big[\begin{matrix}(2h_1 + 7h_2 + 7h_3 + 2h_4)D \\ +(2a_1 + 7a_2 + 7a_2 + 2a_4)\left(1 - \frac{P}{D}\right)\end{matrix}\Big]}} \tag{2.2}$$

Hence, equation (2.2) gives the EOQ in fuzzy sense and equation (**??**) gives the total cost in fuzzy sense.

2.4. **Numerical Example.**

2.4.1. *Crisp Sense:* Let $D = 1500$ per year, $P = 1000$ per year, $A = Rs.500$ per unit per year, $H = Rs.2$ per unit, $S = Rs.50000$, $W = Rs.50$ per unit, $m = 500$ units per year, then $Q^* = 22.8265$ and $T_c^* = Rs.72,284.16/-$

2.4.2. *Fuzzy Sense:* Let $D = 1500$ per year, $P = 1000$ per year,

$\tilde{A} = (300, 400, 600, 700)$,

$\tilde{S} = (30000, 40000, 60000, 70000)$, $\tilde{H} = (0.5, 1, 2.5, 3)$,

$\tilde{W} = (30, 40, 60, 70)$, $\tilde{m} = (300, 400, 600, 700)$,

then $\tilde{Q}^* = 24.37$ and $\tilde{T}_c^* = Rs.68040.5309/-$.

3. Conclusion

This paper proposed a fuzzy inventory model to dispose the wastages that occur in the manufacturing processes. The problem is established in both crisp and fuzzy sense. The beta distribution method is used for defuzzification. Also, the parameters like ordering cost, setup cost, holding cost, waste disposal cost and proportion of waste produced is taken as trapezoidal fuzzy numbers. The derived solutions are verified using numerical examples. This paper can be developed in the future for further research studies.

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