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A FUZZY INVENTORY MODEL WITH PERMISSIBLE DELAY IN PAYMENTS

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ABSTRACT. In this paper, our aim is to frame a mathematical model where the payments by the retailer to the supplier can be delayed for a certain permissible period of time. We have to calculate the economic order quantity and the total cost under the circumstance where payments can be delayed. For fuzzy sense, we use trapezoidal fuzzy numbers. Defuzzification is done by using the Beta distribution method. The optimal solution is found using the Kuhn- Tucker conditions. A numerical example follows to justify the solution procedure.

1. INTRODUCTION

A fuzzy concept is understood by scientists as a concept which is to an extent applicable in a situation. That means the concept has gradations of significance or unsharp (variable) boundaries of application. A fuzzy statement is a statement which is true to some extent and that extent can often be represented by a scaled value. The best known example of a fuzzy concept around the world is an amber traffic light. Indeed, fuzzy concepts are widely used in traffic control systems.

Inventory is an accounting term that refers to goods that are in various stages of being made ready for sale, including

- Finished goods (that are available to be sold).

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- Work-in-progress (meaning in the process of being made).
- Raw materials (to be used to produce more finished goods).
- Inventory is generally the largest current asset - items expected to sell within the next year - a company has.

The term fuzzy logic was first used with 1965 by Lofty Zadeh a professor of UC Berkeley in a California. He observed that conventional computers Logic was not capable of manipulating data representing subjective or unclear human ideas. Fuzzy logic has been applied to various fields, from control theory to Al.According to this theory, we have a transfer function usually called the "membership function", which runs from the universe of discourse of real's, which is $[0, 1]$. The Fuzzy logic research continues to advance in the leading countries, if we talk in terms of economic & technical progress, even though with a slower walk in countries with more traditional scientific culture and static, as may well be the Spanish country. The fuzzy inventory models have also been explored by many other researchers. Gen et al solved an inventory control problem such that input data are described by triangular fuzzy numbers.

S.P. Agarwal and C.K. Jaggi [1] derived ordering polices of deteriorating items under permissible delay in payments. Manisha pal, Hare Krishna Marity discussed an inventory model for deteriorating items with permissible Delay in payments and inflation under price Dependent Demand. Goyal [5] first developed an EOQ model under the condition of permissible delay in payments. Shin et al [8] extended the model by considering quantity discount for freight cost, Shan and Shah developed probabilistic inventory model for deteriorating items when delay in payments is permitted. Ghosh investigated a stochastic inventory model with stock dependent demand under conditions of permissible delay in payments. For more details, refer [2–4, 6, 7, 9].

In this paper, we are going to propose a fuzzy inventory model with permissible delay in payments. The solution is discussed in both crisp and fuzzy numbers to discuss the same under permissible delay in payments. A numerical example is also illustrated to verify the proposed model and finally a conclusion is given for the discussed mathematical model.

2. Definitions and Methodologies

Definition 2.1. ***Fuzzy Set:*** *A fuzzy set $\tilde{A}$ in a universe of discourse X is defined as the following set of pairs $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X\}$. Here $\mu_{\tilde{A}} : X \to [0, 1]$ is a mapping called the membership value of $x \in X$ in a fuzzy set $\tilde{A}$.*

Definition 2.2. ***Trapezoidal Fuzzy Number:*** *A Trapezoidal Fuzzy Number $\tilde{A} = (a, b, c, d)$ is represented by membership function*

$$\mu_{\tilde{A}}(x) \begin{cases} L(x) = \frac{x-a}{b-a}, & a \leq x \leq b \\ 1, & b \leq x \leq c \\ R(x) = \frac{d-x}{d-c}, & c \leq x \leq d \\ 0, & \text{otherwise} \end{cases} .$$

Definition 2.3. ***Beta Distribution Method:*** *The Beta distribution function is a function from the set of all trapezoidal numbers to the real line R.*

If $\tilde{A} = (a_1, a_2, a_3, a_4)$ is a trapezoidal fuzzy number, then the corresponding real number obtained by the beta distribution function is given by

$$\beta(\tilde{A}) = \frac{2a_1 + 7a_2 + 7a_3 + 2a_4}{18}.$$

Definition 2.4. ***Kuhn- Tucker Conditions:*** *The development of the Kuhn-Tucker conditions is based on the Lagrangean method. Suppose that the problem is given by,*

Minimize $Y = f(x)$
Subject to $g_i(x) \leq 0$
$i = 1, 2, ..., m$.

The non negativity constraints may be converted into equations by using non negative surplus variables.

Let $\lambda = (\lambda_1, \lambda_2 \lambda_3 \lambda_4)$, $g(x) = (g_1(x), g_2(x), ..., g_n(x))$... and $s^2 = (s_1^2, s_2^2, ..., s_1^n)$.

The Kuhn-Tucker conditions need x and λ to be a stationary point of the minimization problem, which can be summarized as follows:

$$\begin{cases} \lambda_1 \leq 0 \\ \nabla f(x) - \lambda \nabla g(x) = 0, \\ \lambda_i g_i(x) = 0, i = 1, 2, ..., m \\ g_i(x) \geq 0, i = 1, 2, ..., m. \end{cases}$$

2.1. Arithmetic Operations under Function Principle.

Suppose $\tilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$ are two trapezoidal fuzzy numbers, then the arithmetic operations are defined as

- $\tilde{A} \oplus \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$
- $\tilde{A} \otimes \tilde{B} = (a_1 b_1, a_2 b_2, a_3 b_3, a_4 b_4)$
- $\tilde{A} \ominus \tilde{B} = (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1)$
- $\tilde{A} \oslash \tilde{B} = (\frac{a_1}{b_4}, \frac{a_2}{b_3}, \frac{a_3}{b_2}, \frac{a_4}{b_1})$
- $\alpha \tilde{A} = \begin{cases} \alpha a_1, \alpha a_2, \alpha a_3, \alpha a_4, \alpha \geq 0 \\ \alpha a_4, \alpha a_3, \alpha a_2, \alpha a_1, \alpha < 0 \end{cases}$

Notations:

D - Demand Rate
P - Production Rate
A - Ordering Cost
S - Setup Cost
H - Holding Cost
I - Interest earned per cycle
M - Permissible delay period
T - Cycle Time
Q - Economic Order Quantity
T_C - Total Cost
$\tilde{A}$ - Fuzzy ordering Cost
$\tilde{S}$ - Fuzzy setup Cost
$\tilde{H}$ - Fuzzy holding Cost
$\tilde{I}$ - Fuzzy Interest earned per cycle
$\tilde{Q}^*$ - Fuzzy economic order quantity
$\tilde{T}_c$ - Fuzzy total cost

2.2. Assumptions.

- The production rate is known and constant demand.
- The time period is infinite.
- Shortages are not allowed.
- When $T \leq M$, the account is settled at $T = M$ and we do not pay any interest change.

2.3. **Mathematical model in Crisp Sense.**

The total cost of the mathematical model in crisp sense is given by

$$T_c = \frac{Q}{2}\left[H + \frac{AD}{P}\right] + \frac{D}{Q}\left[A + SI\left(M - \frac{T}{2}\right)\right].$$

By Differentiating with respect to Q and equating to zero, we get

$$Q^* = \sqrt{\frac{2D\left[A + SI\left(M - \frac{T}{2}\right)\right]}{H + \frac{AD}{P}}}.$$

2.4. **Mathematical model in Fuzzy Sense.**

For the fuzzy mathematical model the ordering cost, setup cost, holding cost and the interest earned per year are considered as trapezoidal fuzzy numbers. Let

$$\tilde{A} = (a_1, a_2, a_3, a_4), \quad \tilde{S} = (s_1, s_2, s_3, s_4), \quad \tilde{H} = (h_1, h_2, h_3, h_4)$$

$$\tilde{I} = (i_1, i_2, i_3, i_4), \quad \tilde{Q} = (q_1, q_2, q_3, q_4)$$

be Trapezoidal fuzzy numbers. Now, the fuzzy total cost is given by

$$\tilde{T_c} = \frac{\tilde{Q}}{2}\left[\tilde{H} \oplus \frac{\tilde{A}D}{P}\right] \oplus \frac{D}{\tilde{Q}}\left[\tilde{A} \oplus \tilde{S}\tilde{I}\left(M - \frac{T}{2}\right)\right].$$

Substituting the trapezoidal fuzzy numbers, we get

$$\begin{aligned}
\tilde{T_c} = &\frac{(q_1, q_2, q_3, q_4)}{2}\left[(h_1, h_2, h_3, h_4) \oplus \frac{(a_1, a_2, a_3, a_4)D}{P}\right] \\
&\oplus \frac{D}{(q_4, q_3, q_2, q_1)}\left[(a_1, a_2, a_3, a_4) \oplus (s_1, s_2, s_3, s_4)(i_1, i_2, i_3, i_4)\left(M - \frac{T}{2}\right)\right] \\
\tilde{T_c} = &\frac{q_1}{2}\left[h_1 + \frac{a_1 D}{P}\right] + \frac{D}{q_4}\left[a_1 + \left(M - \frac{T}{2}\right)\right] i_1 s_1 \\
&\frac{q_2}{2}\left[h_2 + \frac{a_2 D}{P}\right] + \frac{D}{q_3}\left[a_2 + \left(M - \frac{T}{2}\right)\right] i_2 s_2, \\
&\frac{q_3}{2}\left[h_3 + \frac{a_3 D}{P}\right] + \frac{D}{q_2}\left[a_3 + \left(M - \frac{T}{2}\right)\right] i_3 s_3 \\
&\frac{q_4}{2}\left[h_4 + \frac{a_4 D}{P}\right] + \frac{D}{q_1}\left[a_4 + \left(M - \frac{T}{2}\right)\right] i_4 s_4.
\end{aligned} \tag{2.1}$$

By defuzzifying equation (2.1) using beta distribution function, we get

$$\begin{aligned}\tilde{T}_c = \frac{1}{18}\Big(2\Big[\frac{q_1}{2}\left[h_1 + \frac{a_1 D}{P}\right] + \frac{D}{q_4}\left[a_1 + \left(M - \frac{T}{2}\right)\right] i_1 s_1\Big] \\ + 7\Big[\frac{q_2}{2}\left[h_2 + \frac{a_2 D}{P}\right] + \frac{D}{q_3}\left[a_2 + \left(M - \frac{T}{2}\right)\right] i_2 s_2\Big] \\ + 7\Big[\frac{q_3}{2}\left[h_3 + \frac{a_3 D}{P}\right] + \frac{D}{q_2}\left[a_3 + \left(M - \frac{T}{2}\right)\right] i_3 s_3\Big] \\ + 2\Big[\frac{q_4}{2}\left[h_4 + \frac{a_4 D}{P}\right] + \frac{D}{q_1}\left[a_4 + \left(M - \frac{T}{2}\right)\right] i_4 s_4\Big]\Big).\end{aligned}$$

In the following steps, we use Kuhn- Tucker conditions to find the solutions of q_1, q_2, q_3 and q_4. Our aim is to minimize $\tilde{T}_c(Q)$, subject to $\lambda \leq 0$

$$\begin{aligned}&\nabla f(\tilde{T}_c) - \lambda \nabla g_i(Q) = 0,\\ &\lambda_i g_i(Q) = 0, i = 1, 2, ..., m\\ &g_i(Q) \geq 0, i = 1, 2, ..., m\\ &\text{with } 0 \leq q_1 \leq q_2 \leq q_3 \leq q_4\end{aligned}$$

It can be written as $q_2 - q_1 \geq 0, q_3 - q_2 \geq 0, q_4 - q_3 \geq 0, q_1 \geq 0$.

Condition 1: $\lambda_1, \lambda_2, \lambda_3, \lambda_4 \leq .$

Condition 2:

$$\frac{\partial}{\partial q_1}(T_c) - \lambda_1 \frac{\partial}{\partial q_1}(g_1(Q)) - \lambda_2 \frac{\partial}{\partial q_2}(g_2(Q)) - \lambda_3 \frac{\partial}{\partial q_3}(g_3(Q)) - \lambda_4 \frac{\partial}{\partial q_4}(g_4(Q)) = 0.$$

Differentiating the above equation w.r.t q_1, q_2, q_3, q_4 and equating to zero, we get

$$\frac{2}{18}\Big[\frac{q_1}{2}\left[h_1 + \frac{a_1 D}{P}\right] + \frac{D}{q_4}\left[a_1 + \left(M - \frac{T}{2}\right)\right] i_4 s_4 + \lambda_1 - \lambda_4\Big] = 0. \tag{2.2}$$

$$\frac{7}{18}\Big[\frac{q_2}{2}\left[h_2 + \frac{a_2 D}{P}\right] + \frac{D}{q_3}\left[a_2 + \left(M - \frac{T}{2}\right)\right] i_3 s_3 + \lambda_2 - \lambda_1\Big] = 0. \tag{2.3}$$

$$\frac{7}{18}\Big[\frac{q_3}{2}\left[h_3 + \frac{a_3 D}{P}\right] + \frac{D}{q_2}\left[a_3 + \left(M - \frac{T}{2}\right)\right] i_2 s_2 + \lambda_2 - \lambda_3\Big] = 0. \tag{2.4}$$

$$\frac{2}{18}\Big[\frac{q_4}{2}\left[h_4 + \frac{a_4 D}{P}\right] + \frac{D}{q_1}\left[a_4 + \left(M - \frac{T}{2}\right)\right] i_1 s_1 - \lambda_3\Big] = 0. \tag{2.5}$$

Condition 3: $\lambda_1(q_2 - q_1) = 0; \lambda_2(q_3 - q_2) = 0; \lambda_3(q_4 - q_3) = 0; \lambda_4 q_1 = 0.$

Condition 4: $q_2 - q_1 \geq 0, q_3 - q_2 \geq 0, q_4 - q_3 \geq 0, q_1 \geq 0.$

So we get $\lambda_4 = 0$. Then replace q_2 by q_1, q_3, by q_2, and q_4 by q_3 then $q_1 = q_2 = q_3 = q_4$. Adding equations (2.2), (2.3),(2.4) and (2.5), we get

$$\begin{aligned}
&= \frac{2}{18}\Big[\frac{Q_1}{2}\left[h_1 + \frac{a_1 D}{P}\right] + \frac{D}{Q_1}\left[a_4 + \left(M - \frac{T}{2}\right)\right] i_4 s_4\Big] \\
&+ \frac{7}{18}\Big[\frac{Q_2}{2}\left[h_2 + \frac{a_2 D}{P}\right] + \frac{D}{Q_2}\left[a_3 + \left(M - \frac{T}{2}\right)\right] i_3 s_3\Big] \\
&+ \frac{7}{18}\Big[\frac{Q_3}{2}\left[h_3 + \frac{a_3 D}{P}\right] + \frac{D}{Q_3}\left[a_2 + \left(M - \frac{T}{2}\right)\right] i_2 s_2\Big] \\
&+ \frac{2}{18}\Big[\frac{Q_4}{2}\left[h_4 + \frac{a_4 D}{P}\right] + \frac{D}{Q_4}\left[a_1 + \left(M - \frac{T}{2}\right)\right] i_1 s_1\Big].
\end{aligned}$$

By simplifying, we have

$$Q^* = \sqrt{\frac{2D\Big[2\left(a_1\left(M - \frac{T}{2}\right) s_1 i_1\right) + 7\left(a_2\left(M - \frac{T}{2}\right) s_2 i_2\right) + 7\left(a_3\left(M - \frac{T}{2}\right) s_3 i_3\right) + 2\left(a_4\left(M - \frac{T}{2}\right) s_4 i_4\right)\Big]}{(2h_1 + 7h_2 + 7h_3 + 2h_4) + \left[\frac{2a_1 D + 7a_2 D + 7a_3 D + 2a_4 D}{P}\right]}}.$$

This gives the fuzzy economic order quantity and the fuzzy total cost.

3. Numerical Example

Crisp sense:

$D = 1000$ units per cycle
$A = Rs.5$ per unit
$S = Rs.1500/-$
$I = Rs.0.12$
$M = 4$ months
$T = 6$ months
$H = Rs.25$ per unit
$P = 1200$ units per cycle, then
$Q^* = 112.63$
$Tc^* = Rs.3290/-$

Fuzzy Sense:

$D = 1000$ units per cycle
$A = (2, 3, 7, 8)$

$S = (1200, 1400, 1600, 1800)$
$I = (0.8, 0.10, 0.14, 0.16)$
$M = 4$ months
$T = 6$ months
$H = (22, 24, 26, 28)$
$P = 1200$ units per cycle, then
$Q^{*} = 139.85$
$Tc^{*} = Rs.4020/-$

4. CONCLUSION

In this paper, we have proposed a fuzzy inventory model in which payments can be delayed for a certain period of time. The model is established in both crisp and fuzzy sense. The Beta distribution method is used for defuzzification. Also the parameters like ordering cost, setup cost and holding cost are taken as trapezoidal fuzzy numbers. The derived solutions are verified using numerical examples.This paper can be developed in future for further research studies.

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