ADV MATH SCI JOURNAL
Advances in Mathematics: Scientific Journal **9** (2020), no.4, 2083–2092
ISSN: 1857-8365 (printed); 1857-8438 (electronic)
https://doi.org/10.37418/amsj.9.4.67 Spec. Issue on NCFCTA-2020

A STUDY ON SOFT (α_γ, β)-CONTINUOUS MAPPINGS IN SOFT TOPOLOGICAL SPACES

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ABSTRACT. In this paper the concept of soft (α_γ,β)-continuous mappings is introduced and some of their basic properties are studied. Further soft (γ,α-β)-open(closed) mappings are introduced and studied.

1. INTRODUCTION

The following notations are used throughout this paper: S-soft SS-soft set, STS-soft topological space, STSs-soft topological spaces, OS-open set, CS-closed set, TS-topological space, TSs-topological spaces, CMa-continuous mapping, Co Ma-continuous mapping, Ma-mapping, Op Ma-open mapping, Cl Ma-closed mapping, Bi Ma-bijective mapping.

Uncertain data are inherent in various fields such as economics, engineering and business mangement. Due to the importance of those applications and the increasing amount of uncertain data collected, accumulated, research on effective and efficient techniques that are dedicated to modeling uncertain data and handling uncertainties has attracted much interest in recent times.

Molodtsov [11] introduced SS theory as a general mathematical tool for dealing with uncertain fuzzy, not clearly defined objects. Maji et. al. [9,10], Chen et.

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2010 *Mathematics Subject Classification.* 54A05, 54A10, 54C08.

Key words and phrases. Soft γ-open set,soft β-open set,soft α_γ-open set, soft(α_γ,β)-continuous mappings,(γ,α_β)-open(closed) mappings.

al. [2,3], Kong et. al. [8], Xiao et. al. [15] and Pei and Miao [12] contributed many concepts to the soft set theory and applications.

Recently, Shabir and Naz [13,14] introduced the concept of STSs which are defined over an initial universe with a fixed set of parameters. They also described some of the important basic concepts of STSs. Aygunoglu et. al. [1], Zorlutuna et. al. [16]. Kalaivani and Sai Sundara Krishnan [4] introduced α-γ-OSs in TSs and studied their properties.

In this work in section 3 the concept of S (α_γ, β)- Co Mas have been introduced and investigated their properties. In section 4, we introduce S (γ,α_β)-Op Mas, (γ,α_β)-Cl Mas and studied many basic results.

2. Preliminaries

The soft set, null soft set, union, intersection, complement of soft sets, soft open sets, soft closed sets, soft interior, soft closure and soft topology are discussed in [3,5–7].

3. Soft (α_γ, β)-Continuous Mappings

Definition 3.1. *A Ma f_M :$(X_s, \tau_s, E_s) \to (Y_s, \sigma_s, K_s)$ is said to be a S(α_γ, β)-Co Ma if the inv image of every S β-OS in (Y_s, σ_s, K_s) is a S α_γ-OS in (X_s, τ_s, E_s).*

Definition 3.2. *A Ma f :$(X_s, \tau_s, E_s) \to (Y_s, \sigma_s, K_s)$ is said to be a S(γ, β)-Co Ma if the inv image of every S β-OS in (Y_s, σ_s, K_s) is a S γ-OS in (X_s, τ_s, E_s).*

Example 1. *Let $X_s = \{x_1, x_2, x_3\}$, $Y_s = \{y_1, y_2, y_3\}$, $\tau_s = \big\{\phi, X_s, (F_1, E_s), (F_2, E_s), (F_3, E_s), (F_4, E_s)\big\}$ and $\sigma_s = \{\phi, Y_s, (G_1, E_s), (G_2, E_s), (G_3, E_s), (G_4, E_s)\}$, where (F, E_s) and (G, K_s) are defined as $F_1(E_s) = (e_1, \{x_1\})$, $F_2(E_s) = (e_2, \{x_2\})$, $F_3(E_s) = (e_3, \{x_1, x_2\})$, $F_4(E_s) = (e_3, \{x_1, x_3\})$, $G_1(E_s) = (k_1, \{y_1\})$, $G_2(E_s) = (k_2, \{y_2\})$, $G_3(E_s) = (k_3, \{y_1, y_2\})$, $G_4(E_s) = (k_3, \{y_1, y_3\})$ and (X_s, τ_s, E_s) and (Y_s, σ_s, K_s) are STSs.*

Define Operations $\gamma_s : \tau_s \to P(X_s)$ and $\beta_s : \sigma_s \to P(Y_s)$ by

$$(H, E_S)^{\gamma_s} = \begin{cases} (H, E_S) & if x_2 \notin (H, E_S) \\ cl((H, E_S)) & if\ x_2 \in (H, E_S) \end{cases}$$

$$(G, E_S)^{\beta_s} = \begin{cases} cl((G, E_S)) & if x_2 \notin (G, E_S) \\ (G, E_S) \cup \{c\} & if x_2 \in (G, E_S) \end{cases} .$$

Then

$$\tau_{s(\alpha_\gamma)} = \{(F_1, E_s), (F_2, E_s), (F_3, E_s), (F_4, E_s)\}$$

and

$$\sigma_{s(\beta_s)} = \big\{(G_1, K_s), (G_2, K_s), (G_3, K_s), (G_4, K_s)\big\}.$$

Define $u_M : X_s \to Y_s$ *and* $p_M : E_s \to K_s$ *as u*(x_1) $= y_1$,*u*(x_2) $= y_2$ *and u*(x_3) $=$ y_3*; p*(e_1) $= k_1$,*p*(e_2) $= k_2$ *and p*(e_3) $= k_3$.*Let* $f_{up} : (X_s, \tau_s, E_s) \to (y_s, \sigma_s, K_s)$ *be a So Ma. Then* (G, K_s) *is a S* β*-OS in* Y_s *and* $f^{-1}_{pu}(G, K)$= (F, E_s) *is a S* α_γ*-OS in* X_s*. Hence* f_{pu} *is a S (*α_γ,β*)-Co Ma.*

Theorem 3.1. *For a Ma* $f_M : (X_s, \tau_s, E_s) \to (Y_s, \sigma_s, K_s)$*,the following statements are equivalent:*

(i) f_M *is a S(*α_γ,β*)-Co Ma;*

(ii) *For each point* x *in* X_s *and each S* β*-OS V in* Y_s *such that f(x)* $\in V$*, there exists a S* α_γ*-OS W in* X_s *such that* $x \in$ *W,*$f_M(W) \subseteq$ *V;*

(iii) *The inverse image of each S* β*-CS in* Y_s *is a S* α_γ*-CS in* X_s*.*

Proof.

$(i) \Rightarrow (ii)$ Let $x \in Y_s$ and V be any S β-OS of Y_s containing $f_M(x)$. Set W = $f^{-1}_M(V)$ then by Definition 3.1, W is a S α_γ-OS containing x and $f_M(W) = f_M(f^{-1}_M(V)) \subseteq V$.

$(ii) \Rightarrow (iii)$ Let F be a S β-CS in (Y_s, σ). Set V = $Y_s - F$, then V is a S β-OS in Y_s. Let $x \in f^{-1}_M(V)$, by(ii) there exists a S α_γ-OS W of X_s containing x such that $f_M(W) \subseteq V$.

Thus we obtain $x \in W \subseteq \tau_{s\gamma_s}$-int($s\tau_{s\gamma_s}$-cl($\tau_{s\gamma_s}$-int(W)))$\subseteq$ $\tau_{s\gamma_s}$-int($\tau_{s\gamma_s}$-cl($\tau_{s\gamma_s}$-int($f^{-1}_M(V)$))) and hence $f^{-1}_M(V) \subseteq \tau_{s\gamma}$-int($\tau_{s\gamma}$-cl($\tau_{s\gamma}$-int($f^{-1}_M(V)$))). Then $f^{-1}_M(V)$ is a S α_γ-OS in X_s. Hence $f^{-1}_M(F) = X_s - f^{-1}_M(Y_s - F) = X_s - f^{-1}_M(V)$ is a S α_γ-CS in X_s.

$(iii) \Rightarrow (i)$ Let B be a S β-OS in Y_s. Then $F = Y_s - B$ is a S β-CS in Y_s. By (iii), we have $f^{-1}_M(F)$) is a S α_γ-CS in X_s.

Hence $f^{-1}_M(B) = X_s - f^{-1}_M(Y_s - B) = X_s - f^{-1}_M(F)$ is a S α_γ-OS in X_s. □

Theorem 3.2. *Let* f_M *:*$(X_s, \tau_s, E_s) \to (Y_s, \sigma_s, K_s)$ *be a Ma and* $\beta : \sigma_s \to P(Y_s)$ *be an open operation on* σ*. Then the following statements are equivalent:*

(i) f_M *is a S(*α_γ,β*)- Co Ma;*

(ii) $\tau_{s\gamma}$*-cl(*$\tau_{s\gamma}$*-int(*$\tau_{s\gamma}$*-cl(*f^{-1}_M*(B))))*$\subseteq f^{-1}_M(\sigma_{s\beta}$*- cl(B)) for each* $B \subseteq Y_S$*;*

(iii) $f_M(\tau_{s\gamma}$*-cl(*$\tau_{s\gamma}$*-int(*$\tau_{s\gamma}$*-cl(A))))* $\subseteq \sigma_{s\beta}$*- cl*$(f_M(A))$ *for each* $A \subseteq X_s$*.*

Proof.

$(i) \Rightarrow (ii)$ Let B be any subset of Y_s. Since $\sigma_{s\beta}$-cl(B) is a soft β-CS in Y_s, By Theorem 3.1 (iii) $f_M^{-1}(\sigma_{s\beta}$- cl(B)) is a S α_γ-CS and $X_s - f_M^{-1}(\sigma_{s\beta}$- cl(B)) is a S α_γ-oS. Thus $X_s - f_M^{-1}(\sigma_{s\beta}$- cl(B)) $\subseteq \tau_{s\gamma}$-int($\tau_{s\gamma}$-cl($\tau_{s\gamma}$-int(X_s− $f_M^{-1}(s\sigma_\beta$- cl(B))))) $\subseteq X_s - \tau_{s\gamma}$-cl($\tau_{s\gamma}$-int($\tau_{s\gamma}$-cl($f^{-1}(\sigma_{s\beta}$- cl(B))))).

Hence $\tau_{s\gamma}$-cl($\tau_{s\gamma}$-int($\tau_{s\gamma}$-cl($f_M^{-1}(B)))) \subseteq f_M^{-1}(s\sigma_{s\beta}$-cl(B)) for each B $\subseteq Y_s$.

$(ii) \Rightarrow (iii)$ Let A be any S subset of X_s. By (ii), $\tau_{s\gamma}$-cl($\tau_{s\gamma}$-int($\tau_{s\gamma}$-cl(A))) $\subseteq \tau_{s\gamma}$-cl($\tau_{s\gamma}$-int($\tau_{s\gamma}$-cl($f_M^{-1}(f(A))))) \subseteq f_M^{-1}(\sigma_{s\beta}$- cl$(f_M(A)))$ and hence $f_M(\tau_{s\gamma}$-cl($\tau_{s\gamma}$-int($\tau_{s\gamma}$-cl(A)))) $\subseteq \sigma_{s\beta}$- cl$(f_M(A))$.

$(iii) \Rightarrow (i)$ Let V be any S β-OS of Y_s. Then by (iii) $f_M(\tau_{s\gamma}$- cl($\tau_{s\gamma}$- int($\tau_{s\gamma}$-cl($f_M^{-1}(Y_s - V))))) \subseteq \sigma_{s\beta}$- cl($f_M(f_M^{-1}(Y_s - V))) \subseteq \sigma_{s\beta}$-cl($Y_s$−V) = Y_s−V. Therefore $\tau_{s\gamma}$-cl($\tau_{s\gamma}$-int($\tau_{s\gamma}$-cl($f_M^{-1}(Y_s - V)))) \subseteq f_M^{-1}(Y_s - V) \subseteq X_s - f_M^{-1}(V)$. Consequently we obtain that $f_M^{-1}(V) \subseteq \tau_{s\gamma}$-int($\tau_{s\gamma}$-cl($\tau_{s\gamma}$-int($f_M^{-1}(V))))$. This shows that $f_M^{-1}(V)$ is a S α_γ-OS. Thus f_M is a S (α_γ , β)- Co Ma. □

Theorem 3.3. *For a Ma f_M :$(X_s, \tau_s, E_s) \to (Y_s, \sigma_s, K_s)$ the following statements are equivalent:*

(i) *f is a S(α_γ,β)-Co Ma;*
(ii) *For each subset A of X_S, $f_M(\tau_{s(\alpha_\gamma)}$-cl(A))) $\subseteq \sigma_{s\beta}$-cl($f_M$(A));*
(iii) *For each subset B of Y_s, $\tau_{s((\alpha_\gamma)}$-cl(f_M^{-1}(B)) $\subseteq f_M^{-1}(\sigma_{s\beta}$-cl(B));*
(iv) *For each subset C of Y_s, $f_M^{-1}(\sigma_{s\beta}$-int(C)) $\subseteq \tau_{s(\alpha_\gamma)}$-int($f_M^{-1}$(C))).*

Proof.

$(i) \Rightarrow (ii)$ Let y $\in f_M(\tau_{s(\alpha_\gamma)}$-cl(A)) and V be any S β-OS containing y. Using Theorem 3.2, there exists a point x $\in$ U with f_M(x)= y and $f_M(U) \subseteq$ V. Since x $\in \tau_{s(\alpha_\gamma)}$-cl(A), U $\bigcap V \neq \phi$ and hence $\phi \neq f_M(U \bigcap A) \subseteq f_M(U) \bigcap f_M(A) \subseteq V \bigcap f_M(A)$. This implies that y $\in \sigma_{s\beta}$-cl(f_M(A)).

Therefore $f(\tau_{s(\alpha_\gamma)}$-cl(A)) $\subseteq \sigma_{s\beta}$-cl($f_M$(A)).

$(ii) \Rightarrow (iii)$ Let B be a S β-CS in Y_s. Then $\sigma_{s\beta}$-cl(B)=B. By (ii), $f_M(\tau_{s(\alpha_\gamma)}$-cl($f_M^{-1}(B)) \subseteq \sigma_{s\beta}$-cl($f_M(f_M^{-1}$(B)))$\subseteq \sigma_{s\beta}$-cl(B)=B holds.

$(iii) \Rightarrow (iv)$ Let C be any subset of Y_s. Clearly $\tau_{s(\alpha_\gamma)}$-int(f_M^{-1}(C)) is a S α_γ-OS in X_s. Also, $f_M \tau_{s(\alpha_\gamma)}$-int($f_M^{-1}$(C))$\subseteq f_M(f_M^{-1}(C)) \subseteq$ C.

Hence $f_M^{-1}(\sigma_{s\beta}$-int(C)) $\subseteq \tau_{s(\alpha_\gamma)}$-int($f_M^{-1}$(C)).

$(iv) \Rightarrow (i)$ Let B be any S β-OS in Y_s. Consider V = Y_s− B. Then V is a S β-CS in Y_s. By (iv) f_M^{-1}(V)is a S α_γ-CS in X_s. Hence f_M^{-1}(B)= $X_s - f_M^{-1}(Y_s$−B)= $X_s - f_M^{-1}$(V) is aS α_γ-OS in X_s. Hence f_M is a S (α_γ,β)-Co Ma. □

Remark 3.1. *Every S(γ,β)-Co Ma is a S(α_γ,β)-Co Ma. But the converse need not be true.*

Proof. Proof follows from the Definitions 3.1, 3.2. In the Example 1, $f_M^{-1}\{(G_2,E)\} = \{(F_3,E)\}$ is not a S γ-OS. Hence f_M is not a S(γ, β)-Co Ma. □

Theorem 3.4. *Let $f_M : (X_S, \tau_s, E_s) \to (Y_s, \sigma_s, K_s)$ be a S (α_γ , β)-Co Ma. Then for each subset V of Y_s, $f_M^{-1}(\tau_{s\gamma}$-int(V))$\subseteq$ $\tau_{s\gamma}$-cl(f_M^{-1}(V)).*

Proof. The result $\tau_{s(\gamma}$-int(V) $\subseteq$ V, implies that $f_M^{-1}(\tau_{s\gamma}$-int(V))$\subseteq$ f_M^{-1}(V). Since f_M is a S ($\alpha\gamma$, β)-Co Ma f_M^{-1}(V) $\subseteq$ $\tau_{s\gamma}$-int($\tau_{s\gamma}$-cl($\tau_{s\gamma}$-int(f_M^{-1}(V)))),implies that $f_M^{-1}(\tau_{s\gamma}$-int(V))$\subseteq$ $\tau_{s\gamma}$-int($\tau_{s\gamma}$-cl($\tau_{s\gamma}$-int(f_M^{-1}(V)))) and $f_M^{-1}(\tau_{s\gamma}$-int(V))$\subseteq$ $\tau_{s\gamma}$-int($\tau_{s\gamma}$-cl(f_M^{-1}(V))). Therefore $f_M^{-1}(\tau_{s\gamma}$-int(V))$\subseteq$ $\tau_{s\gamma}$-cl(f_M^{-1}(V)). □

Theorem 3.5. *Let f_M :$(X_s, \tau_s, E_s) \to (Y_s, \sigma_s, K_s)$ be a Bi Ma.Then f_M is a $S(\alpha_\gamma$, β)-Co Ma if and only if $\sigma_{s\beta}$-int(f_M(U))$\subseteq$ $f_M(\tau_{s(\alpha_\gamma)}$-int(U)) for each subset U of X_s.*

Proof. Suppose f_M is a S(α_γ,β)-Co Ma, using Theorem 3.3(iv), $\sigma_{s\beta}$-int($f_M(\tau_{s(\alpha_\gamma)}$-int(U))) for each subset U of X_s.

Conversely, suppose $\sigma_{s\beta}$-int(f_M(U))$\subseteq$ $f_M(\tau_{s(\alpha_\gamma)}$-int(U)) for each subset U of X_S, by Theorem 3.3(iv),(i), f_M is a S (α_γ, β)-Co Ma. □

Definition 3.3. *Let A be a subset of X_S and p be any point in X_S. Then p is called a S α_γ-limit point of A if U $\bigcap$ (A $-\{p\}) \neq \phi$, for any α_γ-OS U containing p. The set of all S α_γ-limit points of A is called S α_γ-derived set of A and is denoted by $d_{s(\alpha_\gamma)}$.*

Theorem 3.6. *Let f_M :$(X_s, \tau_s, E_s) \to (Y_s, \sigma_s, K_s)$ be a Ma and γ be an open operation then the following are equivalent:*

(i) *f_M is a S(α_γ,β)-Co Ma.*
(ii) *$f_M(d_{s(\alpha_\gamma)}(A)) \subseteq \sigma_{s\beta}$-cl($f_M(A)$) for any subset A of X_S.*

Proof.

$(i) \Rightarrow (ii)$ Suppose f_M is a S(α_γ,β)-Co Ma. Let A be any subset in X_S. Since $\sigma_{s\beta}$-cl$f_M(A)$) is a $\sigma_{s\beta}$-CS in Y_S, $f_M^{-1}(\sigma_{s\beta}$-cl(A))is a S α-γ-CS in X_S. Then $A \subseteq f_M^{-1}(f_M(A)) \subseteq f_M^{-1}(\sigma_{s\beta}$-cl($f_M(A)$)) gives

$\tau_{s(\alpha_\gamma}$-cl(A)$\subseteq$ $\tau_{s(\alpha_\gamma)}$-cl($f_M^{-1}(\sigma_{s\beta}$-cl($f_M(A)$)))= $f_M^{-1}(\sigma_{s\beta}$-cl($f_M(A)$)).

Therefore,

$f_M(d_{s(\alpha_\gamma)}(A)) \subseteq f(\tau_{s(\alpha-\gamma)}$-cl(A) $\subseteq f_M(f_M^{-1}(\sigma_{s\beta}$-cl$(f_M$(A)))) $\subseteq \sigma_{s\beta}$-cl$(f_M(A))$.

$(ii) \Rightarrow (i)$ Suppose $f(d_{s(\alpha_\gamma)}(A)) \subseteq \sigma_{s\beta}$-cl$(f_M(A))$ for A $\subseteq$ B. Let B be any $\sigma_{s\beta}$-CS of Y_S. To prove that f_M^{-1}(B)is a S α_γ-CS in X_S. Then
$f_M(d_{s(\alpha_\gamma)}f_M^{-1}(B)) \subseteq \sigma_{s\beta}$-cl$(f_M(f_M^{-1}(B))) \subseteq \sigma_{s\beta}$-cl(B)= B or $f_M(d_{s(\alpha_\gamma)}(f_M^{-1}(B))) \subseteq$ B gives $d_{s(\alpha_\gamma)}(f_M^{-1}(B)) \subseteq f_M^{-1}$(B)) which implies that f_M^{-1}(B)is a Sα_γ-CS in X_S. Hencef_M is a S(α_γ,β)-Co Ma. ☐

Theorem 3.7. *If f_M :$(X_s, \tau_s, E_s) \to (Y_s, \sigma_s, K_s)$ is a S (α_γ,β)-Co Ma and X_0 is a S γ-OS of X_S, then the restriction $f_M/X_0 : X_0 \to Y_s$ is a S(α_γ,β)- Co Ma, where $\gamma : \tau_s \to P(X_s)$ be a regular operation on τ_s.*

Proof. Let V be any S β-OS of Y_S. Since f_M is a S (α_γ , β)- Co Ma f_M^{-1}(V) is a S α_γ-OS in X_S and using Theorem 3.21 [7] $f_M^{-1}(V) \cap X_0 = (f_M/X_0)^{-1}(V) \in \tau_{s(\alpha_\gamma)}$. Since $X_0 \subseteq X$, $(f_M/X_0)^{-1}(V)$ is a S α_γ-OS in X_0. This shows that f_M/X_0 is a S (α_γ , β)-Co Ma. ☐

Theorem 3.8. *Let (X_S, τ_S, E_s) be a STS,$\gamma : \tau_S \to P(X_S)$ be a regular operation on τ_s and $\{V_k : k \in J\}$ be a cover of X_S by S γ-OSs of X_S. A Ma $f_M : (X, \tau) \to (Y, \sigma)$ is a S(α_γ,β)-Co Ma if and only if the restriction $f_M/V_k : V_k \to Y$ is a S(α_γ,β)- Co Ma for each k $\in$ J.*

Proof. Let f_M be a S(α_γ ,β)-Co Ma. By Theorem 3.4, f_M/V_k is a S(α_γ ,β)-Co Ma for each k $\in$ J.Let f_M/V_k is a S(α_γ , β)-Co Ma for each k $\in$ J. For any S β-OS V of Y_S, $(f_M/V_k)^{-1}(V)$ is a S α_γ-OS in V_k for each $k \in$ J and hence $f_M^{-1}(V) = \cup\{(f_M/V_k)^{-1}(V) : k \in J\}$ is a S α_γ-OS in X_S by Theorem 3.4 [7]. This shows that f_M is a S (α_γ , β)-Co Ma. ☐

4. SOFT(γ,α_β)-OPEN MAPPINGS AND SOFT(γ,α_β)-CLOSED MAPPINGS

In this section, we introduce the concept of S(γ,α_β)-Op Mas and S(γ,α_β)-Cl Mas and study some of their properties.

Definition 4.1. *A Ma f_M :$(X_s, \tau_s, E_s) \to (Y_s, \sigma_s, K_s)$ is said to be a S (γ,α_β)-Op(Cl)Ma if the image of each S γ-Op(Cl) set in X_s is a S α_β-OS(CL)set in Y_s.*

Example 2. *Let $X_s = \{x_1, x_2, x_3\}$, $Y_s = \{y_1, y_2, y_3\}$, $\tau_s = \{\varphi, X_s, (F_1, E_s)),$ $(F_2, E_s)), (F_3, E_s)), (F_4, E_s))\}$ and $\sigma_s = \{\varphi, Y_s, (G_1, E_s)), (G_2, E_s)), (G_3, E_s),$ $(G_4, E_s)\}$, where (F, E_s) and (G, K_s) are defined as $F_1(E_s) = (e_1, \{x_1\})$, $F_2(E_s) =$*

$(e_2, \{x_3\})$, $F_3(E_s) = (e_3, \{x_1, x_2\})$, $F_4(E_s) = (e_3, \{x_1, x_3\})$.
$G_1(E_s) = (k_1, \{y_1\})$, $G_2(E_s) = (k_2, \{y_2\})$, $G_3(E_s) = (k_3, \{y_1, y_2\})$, $G_4(E_s) = (k_3, \{y_1, y_3\})$ *and* (X_s, τ_s, E_s) *and* (Y_s, σ_s, K_s) *be STSs.*

Define operations $\gamma_s : \tau_s \to P(X_s)$ and $\beta_s : Y_s \to P(Y_s)$ by $A^{\gamma_s} = cl(A)$ and $B^{\beta_s} = int(cl(B))$. Then $\tau_{s(\alpha_\gamma)} = \{(F_1, E_s), (F_2, E_s), (F_3, E_s), (F_4, E_s)\}$ and $\sigma_{s(\beta)} = \{(G_1, K_s), (G_2, K_s), (G_3, K_s), (G_4, K_s)\}$.

Define $U_M : X_s \to Y_s$ and $P_M : E_s \to K_s$ as u(x_1) = y_1,u(x_2) = y_3 and $u(x_3) = y_2$; p(e_1) = k_1,p(e_2) = k_3 and p(e_3) = k_2. Let $f_{up} : (X_s, \tau_s, E_s) \to (Y_s, \sigma_s, K_s)$ be a S Ma. Then the image of each S γ_s-OS(respectively γ-CS) is a S α_γ-OS(respectively α_γ-CS) under f_M. Hence f_M is a S(γ ,α_β)-OS(respectively (γ , α_β)-CS).

Theorem 4.1. *A Ma f_M :$(X_s, \tau_s, E_s) \to (Y_s, \sigma_s, K_s)$ is a S(γ,α_β)-Op Ma if and only if for each x $\in X$ and each S γ_S-neighbourhood U of x, there exists a S α_β-OS V of Y_s containing $f_M(x)$ such that $V \subseteq f_M(U)$.*

Proof. Suppose that f_M is a S(γ, α_β)-Op Ma . For each x $\in X_s$ and each neighbourhood U of x , there exists a S γ_s-OS U_0 such that x $\in U_0 \subseteq$ U. Since f_M is a S (γ,α_β)-Op Ma, V = $f(U_0)$ is a S α_β-OS and $f_M(x) \in V \subseteq f_M(U)$.
Conversely, let U be a S γ_s-OS of X_s. For each x $\in$ U, there exists a S α_β-OS $V_{f_M(x)}$ such that $f_M(x) \in V_{f_M(x)} \subseteq f_M(U)$. Therefore, we obtain $f_M(U) = \cup \left\{V_{f_M(x)}\ x \in U\right\}$ and by Theorem 3.4 , [8], $f_M(U)$ is a S α_β-OS. This shows that f_M is a S(γ, α_β)-Op Ma. □

Theorem 4.2. *A Ma f_M :$(X_s, \tau_s, E_s) \to (Y_s, \sigma_s, K_s)$ is a S (γ, α_β)-Op Ma if and only if for each subset $W \subseteq Y_S$ and each S γ_s-CS F of X_s containing $f_M^{-1}(W)$, there exists a S α_β-CS H of Y_S containing W such that $f_M^{-1}(H) \subseteq F$.*

Proof. Let $H = Y_s - f_M(X_s - F)$. Since $f_M^{-1}(W) \subseteq F$, we have $f_M(X_S - F) \subseteq Y_S - W$. Since f_M is a S (γ,α_β)-Op Ma, H is a S α_β-CS and $f_M^{-1}(H) = X_s - f_M^{-1}(f_M(X_s - F)) \subseteq X_s - (X_s - F) = F$.
Conversely, let U be any S γ_s-OS of X_s and $W = Y_s - f_M(U)$. Then $f_M^{-1}(W) = X - f_M^{-1}(f_M(U)) \subseteq X_s - U$ and X_s- U is a S γ_s-CS. By the hypothesis, there exists a S α_β-CS H of Y_s containing W such that $f_M^{-1}(H) \subseteq X_s - U$. Then, we have $f_M^{-1}(H) \cap U = \phi$ and $H \cap f_M(U) = \phi$. Therefore , we obtain $Y_s - f_M(U) \supseteq H \supseteq W = Y_s - f_M(U)$ and $f_M(U)$ is a S (γ,α_β)-OS in Y_s. This shows that f_M is a S(γ,α_β)-Op Ma. □

Theorem 4.3. *If a Ma $f_M : (X_s, \tau_s, E_s) \to (Y_s, \sigma_s, F_s)$ is a S(γ,α_β)-Op Ma and $\gamma_s : \tau_s \to P(X_s)$ is an open operation on τ_s.Then the following properties hold:*

(i) *$f_M^{-1}(s\sigma_{s\beta}$-cl($\sigma_{s\beta}$-int($\sigma_{s\beta}$-cl(B)))) $\subseteq \tau_{s\gamma}$-cl($f_M^{-1}(B)$) for each set B $\subseteq Y_s$;*

(ii) *$f_M^{-1}(\sigma_{s\beta}$-cl(V) $\subseteq \tau_{s\gamma}$-cl($f_M^{-1}(V)$) for each S β_σ-OS V of Y_s.*

Proof.

(i) Let B be any subset of Y_s. Then $\tau_{s\gamma}$-cl($f_M^{-1}(B)$) is a S γ_s-CS in X_s. By Theorem 4.2 there exists a S α_β-CS H of Y_s containing B such that $f_M^{-1}(H) \subseteq \tau_{s\gamma}$-cl($f_M^{-1}(B)$). Since Y_s- H is a S α_β-OS, $f_M^{-1}(Y_s - H) \subseteq f_M^{-1}(\sigma_{s\beta}$-int($\sigma_{s\beta}$-cl($\sigma_{s\beta}$-int($Y_s - H$)))) and $X_s - f_M^{-1}(H) \subseteq f_M^{-1}(Y_s - \sigma_{s\beta}$-cl($\sigma_{s\beta}$-int($\sigma_{s\beta}$-cl(H)))) = $X_S - f_M^{-1}(\sigma_{s\beta}$-cl($\sigma_{s\beta}$-int($\sigma_{s\beta}$-cl(H)))). Thus $f_M^{-1}(\sigma_{s\beta}$-cl($\sigma_{s\beta}$-int($\sigma_{s\beta}$-cl(B)))) $\subseteq f_M^{-1}(s\sigma_\beta$-cl($\sigma_{s\beta}$-int($\sigma_{s\beta}$-cl(H)))) $\subseteq f_M^{-1}(H) \subseteq \tau_{s\gamma}$-cl($f_M^{-1}(B)$). Therefore $f_M^{-1}(\sigma_{s\beta}$-cl($\sigma_{s\beta}$-int($\sigma_{s\beta}$-cl(B))))$\subseteq \tau_{s\gamma}$-cl($f_M^{-1}(B)$).

(ii) Let V be any S β_σ-OS of Y_s. Then $\sigma_{s\beta}$-int(V) = V and using (i) , we obtain $f_M^{-1}(\sigma_{s\beta}$-cl(V)) $= f_M^{-1}(\sigma_{s\beta}$-cl($\sigma_{s\beta}$-int($\sigma_{s\beta}$-cl(V)))) $\subseteq \tau_{s\gamma}$-cl($f_M^{-1}(V)$). □

Theorem 4.4. *Suppose $f_M : (X_s, \tau_s, E_s) \to (Y_s, \sigma_s, F_s)$ is a S (γ,α_β)-Op Ma and $\gamma_s : \tau_s \to P(X_s)$ is an open operation on τ_s. Then the following conditions are equivalent:*

(i) *f_M is a S (γ,α_β)-Op Ma.*

(ii) *$f_M(\tau_{s\gamma}$-int(A))$\subseteq \sigma_{s(\alpha_\beta)}$-int($f_M$(A))for $A \subseteq X_s$;*

(iii) *$\tau_{s\gamma}$-int($f_M^{-1}(B)$) $\subseteq f_M^{-1}(\sigma_{s(\alpha_\beta)}$-int(B)) for B $\subseteq Y_s$.*

Proof.

$(i) \to (ii)$. Let A be a subset of X_s. Then $\tau_{s\gamma}$-int(A) is a S γ-OS in X_s. Since f is a S(γ, α_β)-Op Ma ,we have $f_M(\tau_{s\gamma}$-int(A)) is a S α_β-OS in Y_s. Therefore $f_M(\tau_{s\gamma}$-int(A))= $\sigma_{s(\alpha_\beta)}$-int($f_M(\tau_{s\gamma}$-int(A))) $\subseteq \sigma_{s(\alpha_\beta)}$-int($f_M$(A)).

$(ii) \to (iii)$. Let B be a subset of Y_s. Then f_M^{-1}(B) $\subseteq X_s$. By (ii), we have $f_M(\tau_{s\gamma}$-int (f_M^{-1}(B))) = $\sigma_{s(\alpha_\beta)}$-int($f_M(f_M^{-1}$(B))) $\subseteq$ $\sigma_{s(\alpha_\beta)}$-int(B). Thus $\tau_{s\gamma}$-int(f_M^{-1}(B))$\subseteq f_M^{-1}(\sigma_{s\alpha_\beta}$-int(B)).

$(iii) \to (i)$ Let U be a S γ_S-OS in X_s. Then U = $\tau_{s\gamma}$-int(U) $\subseteq \tau_{s\gamma}$-int($f^{-1_M}(f_M(U))$) $\subseteq f_M^{-1}(\sigma_{s\alpha_\beta}$ -int(f_M(U)))by(iii) . This implies that $f_M(U) \subseteq \sigma_{s\alpha_\beta}$-int(f_M(U)). Therefore f_M(U)is a S α_β-OS in Y_s. □

Theorem 4.5. *For any Bi Ma f :$(X_s, \tau_s, E_s) \to (Y_s, \sigma_s, K_s)$, the following conditions are equivalent:*

(i) *$f_M^{-1} : (X_s, \tau_s, E_s) \to (Y_s, \sigma_s, F_s)$ is a S (α_γ,β)-Co Ma;*

(ii) *f_M is a S (γ,α_β)-Op Ma;*
(iii) *f_M is a S (γ,α_β)-Cl Ma.*

Proof. Follows from the Definitions 3.1 and 4.1. □

5. ACKNOWLEDGEMENT

The author wishes to thank Vel Tech Rangarajan Dr. Sagunthala R and D Institute of Science and Technology- VEL TECH SEED FUND for their support in the development of this article.

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