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SOME NEW PROPERTIES OF E^*-OPEN SETS IN FUZZY TOPOLOGICAL SPACES

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ABSTRACT. In this paper, we introduce fuzzy e^*-open and closed sets, fuzzy e^*-interior and respective closure operators and investigate some of their properties. Also we discuss the relationship between the existing sets.

1. Introduction and Preliminaries

The concepts of fuzzy sets and fuzzy topology were firstly given by Zadeh in [18] and Chang in [4], and after then there have been numerous improvements on characterizing dubious circumstances and relations in progressively reasonable manner. The fuzzy topology theory has quickly started to assume a significant job in a wide range of logical territories, for example, financial matters, quantum material science and geographic data framework. For instance, Wenzheng Shi and Kimfung Liu referenced that the fuzzy topology hypothesis can conceivably give a progressively sensible depiction of unsure spatial items and questionable relations in [17]. Furthermore, the ideas of fuzzyy topology and fuzzy sets have significant applications on molecule physic regarding string hypothesis and ϵ^{∞} theory studied by El-Naschie [10–12]. In 2008 (resp. 2014), Ekici (resp. Seenivasan) [5, 6] (resp. [14]) initiated $e$open sets and e^*open (resp. $e$open sets and maps) sets in general topological (resp. fuzzy topological) spaces.

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$X,\ Y$ etc. represents nonempty sets, $I = [0,\ 1]$ and $I_0 = (0,\ 1]$ and the undefined fuzzy notions are from [1]- [18].

2. Fuzzy e^*-open sets

Definition 2.1. *In a fts $(X,\ \tau)$, $\gamma,\ \mu \in I^X$,*

(i) *γ is called a fuzzy e^*-open (resp. fuzzy e^*-closed) (briefly, fe^*o (resp. fe^*c)) set if $\gamma \leq Cl(Int(\delta Cl(\gamma)))$ (resp. $Int(Cl(\delta Int(\gamma))) \leq \gamma$).*

(ii) *$e^*Int(\gamma) = \bigvee\{\mu \in I^X : \mu \leq \gamma,\ \mu$ (resp. $e^*Cl(\gamma) = \bigwedge\{\mu \in I^X : \mu \geq \gamma,\ \mu$ is a fe^*c set $\}$) is a fe^*o set $\}$ is called the fuzzy e^*-interior (resp. e^*-closure) of γ.*

Obviously, $e^*Int(\gamma)$ is the largest fe^*o set which is contained in γ and $e^*Cl(\gamma)$ is the smallest fe^*c set which contains γ. Also $e^*Cl(\gamma) = \gamma$ for any fe^*c set γ and $e^*Int(\gamma) = \gamma$ for any fe^*o set γ.

Hence, we have

$$Int(\gamma) \leq \delta sInt(\gamma) \leq eInt(\gamma) \leq \beta Int(\gamma) \leq e^*Int(\gamma) \leq \gamma.$$
$$\gamma \leq e^*Cl(\gamma) \leq \beta Cl(\gamma) \leq eCl(\gamma) \leq \delta sCl(\gamma) \leq Cl(\gamma).$$

and

$$Int(\gamma) \leq \delta pInt(\gamma) \leq eInt(\gamma) \leq \beta Int(\gamma) \leq e^*Int(\gamma) \leq \gamma.$$
$$\gamma \leq e^*Cl(\gamma) \leq \beta Cl(\gamma) \leq eCl(\gamma) \leq \delta pCl(\gamma) \leq Cl(\gamma).$$

Lemma 2.1. *A subset γ of a fts X. Then*

(i) *$e^*Cl(\gamma)$ is fe*c*

(ii) $1 - e^*Cl(\gamma)$=e^*Int $(1-\gamma)$

are hold.

Theorem 2.1. *A subset γ of a fts X. Then*

(i) *γ is $fe^*o \Leftrightarrow \gamma = \gamma \wedge Cl(Int(\delta Cl(\gamma)))$*

(ii) *γ is $fe^*c \Leftrightarrow \gamma = \gamma \vee Int(Cl(\delta Int(\gamma)))$*

(iii) $e^*Cl(\gamma) = \gamma \vee Int(Cl(\delta Int(\gamma)))$

(iv) $e^*Int(\gamma) = \gamma \wedge Cl(Int(\delta Cl(\gamma)))$

are hold.

Theorem 2.2. *Let $\gamma \in I^X$. Then the following hold.*

(i) $e^*Cl(\delta Int(\gamma)) = Int(Cl(\delta Int(\gamma)))$.

(ii) $\delta Int(e^*Cl(\gamma)) = Int(Cl(\delta Int(\gamma)))$.

(iii) $e^*Int(\delta Cl(\gamma)) = \delta Cl(e^*Int(\gamma)) = Cl(Int(\delta Cl(\gamma)))$.

(iv) $e^*Int(eCl(\gamma)) = \delta sI(\delta sCl(\gamma)) \wedge \delta pCl(\gamma)$.

(v) $e^*Cl(eInt(\gamma)) = \delta sCl(\delta sInt(\gamma)) \vee \delta pInt(\gamma)$.

(vi) $eCl(e^*Int(\gamma)) = \delta sInt(\delta sCl\ (\gamma)) \wedge \delta pCl(\gamma)$.

(vii) $eInt(e^*Cl(\gamma)) = \delta sCl(\delta sInt(\gamma)) \vee \delta pInt(\gamma)$. $e^*Int(\gamma) = \gamma \wedge Cl(Int(\delta Cl(\gamma)))$

Remark 2.1. *From the above definitions, obviously, the accompanying ramifications are valid.*

$$
\begin{tikzcd}
 & f\delta so \arrow[dr] & & fe^*o \\
fo \arrow[ur] \arrow[dr] & & feo \arrow[ur] \arrow[dr] & \\
 & f\delta po \arrow[ur] & & f\beta o \arrow[uu]
\end{tikzcd}
$$

FIGURE 1

Clearly (i) every $f\delta po$ is feo, (ii) every $f\delta so$ is feo, (iii) every feo set is $f\beta o$ set and fe^*o set, (iv) every $f\beta o$ set is fe^*o set. The converse need not be valid in all.

3. EXAMPLES

Example 1. *Let γ_1, γ_2, γ_3 & γ_4 be fuzzy subsets of $X = \{a,\ b\}$ defined as $\gamma_1(a) = 0.2$, $\gamma_1(b) = 0.1$; $\gamma_2(a) = 0.3$, $\gamma_2(b) = 0.5$; $\gamma_3(a) = 0.7$, $\gamma_3(b) = 0.7$; $\gamma_4(a) = 0.2$, $\gamma_4(b) = 0.8$. Then $\tau : I^X \to I$ defined as $\tau = \{0,\ 1, \gamma_1,\ \gamma_2,\ \gamma_3\}$. Clearly, τ is a ft. Then γ_4 is $f\beta o$ but it is not feo set.*

Example 2. *Let $X = \{a,\ b,\ c\}$, $\gamma, \mu \in I^X$ be defined as $\gamma_1(a) = 0.4$, $\gamma_1(b) = 0.5$, $\gamma_1(c) = 0.5$; $\gamma_2(a) = 0.4$, $\gamma_2(b) = 0.5$, $\gamma_2(c) = 0.4$; $\gamma_3(a) = 0.5$, $\gamma_3(b) = 0.3$, $\gamma_3(c) = 0.2$; $\gamma_4(a) = 0.5$, $\gamma_4(b) = 0.4$, $\gamma_5(c) = 0.4$. $\gamma_5(a) = 0.3$, $\gamma_5(b) = 0.5$, $\gamma_5(c) = 0.2$; $\gamma_6(a) = 0.7$, $\gamma_6(b) = 0.4$, $\gamma_6(c) = 0.8$. $\gamma_7(a) = 0.4$, $\gamma_7(b) = 0.5$, $\gamma_7(c) = 0.2$; $\gamma_8(a) = 0.5$, $\gamma_8(b) = 0.4$, $\gamma_8(c) = 0.7$. Then $\tau_1, \tau_2, \tau_3, \tau_4 : I^X \to I$ defined as $\tau_1 = \{0, 1, \gamma_1\}$, $\tau_2 = \{0, 1, \gamma_3\}$, $\tau_3 = \{0, 1, \gamma_5, \gamma_1\}$, $\tau_4 = \{0, 1, \gamma_7\}$. Clearly, τ_1, τ_2, τ_3, τ_4 are a ft. Then γ_2 is feo set but it is not $f\delta so$ set; γ_4 is feo set but it is not $f\delta po$ set; δ_6 is fe^*o set but it is not $f\beta o$ set; δ_8 is fe^*o set but it is not feo set. Also δ_8 is not fo set.*

Theorem 3.1. *Any union (resp. intersection) of fe^*o (resp. fe^*c) sets is an fe^*o (resp. fe^*c) set.*

Theorem 3.2. *Let $(X,\ \tau)$ be a fts & $\gamma,\ \mu \in I^X$, then, If μ (resp. $1-\mu$) is a fo set of X, where μ is a crisp subset and γ is an fe^*o (resp. fe^*c) set, then $\gamma \wedge \mu$ is an fe^*o (resp. fe^*c) set.*

Theorem 3.3. *Let $(X,\ \tau)$ be a fts & $\gamma,\ \mu \in I^X$.*

(i) *If γ is fe^*o with $1-\gamma$ is fo, then γ is $f\delta po$.*
(ii) *If γ is fe^*c with γ is fo then $f\delta pc$.*

Theorem 3.4. *Let $(X,\ \tau)$ be a fts & $\gamma,\ \mu \in I^X$.*

(i) *γ is fe^*o iff $1-\gamma$ is fe^*c.*
(ii) *If γ is fo set, then γ is fe^*o set.*
(iii) *$Int(\gamma)$ is an fe^*o set.*
(iv) *$Cl(\gamma)$ is an fe^*c set.*

Theorem 3.5. *Let $\gamma \in I^X$.*

(i) *γ is fe^*o iff $\gamma = e^*Int(\gamma)$.*
(ii) *γ is fe^*c iff $\gamma = e^*Cl(\gamma)$.*

Theorem 3.6. *Let $\gamma \in I^X$, then*

(i) *$e^*Cl(0) = 0$ and $e^*Int(1) = 1$.*
(ii) *$Int(\gamma) \leq e^*Int(\gamma) \leq \gamma \leq e^*Cl(\gamma) \leq Cl(\gamma)$.*
(iii) *$\gamma \leq \mu \Rightarrow e^*Int(\gamma) \leq e^*Int(\mu)$ and $e^*Cl(\gamma) \leq e^*Cl(\mu)$.*
(iv) *$e^*Cl(\gamma) \vee e^*Cl(\mu) \leq e^*Cl(\gamma \vee \mu)$.*
(v) *$e^*Cl(e^*Cl(\gamma)) = e^*Cl(\gamma)$ and $e^*Int(e^*Int(\gamma)) = e^*Int(\gamma)$.*
(vi) *$Cl(e^*Cl(\gamma)) = e^*Cl(Cl(\gamma)) = Cl(\gamma)$.*

Theorem 3.7. *Let $\gamma \in I^X$, we have*

(i) *$e^*Int(1-\gamma) = 1 - e^*Cl(\gamma)$.*
(ii) *$e^*Cl(1-\gamma) = 1 - e^*Int(\gamma)$.*

Theorem 3.8. *Let $\gamma,\ \mu \in I^X$.*

(i) *If γ is $f\beta o$ set, $1-\gamma$ is fo and γ is fδc then γ is feo.*
(ii) *If γ is $f\beta c$ set, γ is fo and γ is $f\delta o$ then γ is fec.*

Theorem 3.9. *Let $\gamma,\ \mu \in I^X$.*

(i) *If* γ *is* fe^*o *with* $1-\gamma$ *is* fo*, then* γ *is* feo *set.*
(ii) *If* γ *is* fe^*c *with* γ *is* fo*, then* γ *is* fec *set.*

Theorem 3.10. *Let* $\gamma,\ \mu \in I^X$.

(i) *If* γ *is fe***o*, $1-\gamma$ *is* fo *and* γ *is* $f\delta c$, *then* γ *is* $f\beta o$ *set.*
(ii) *If* γ *is* fe^*c*,* γ *is* fo *and* γ *is* $f\delta o$, *then* γ *is* $f\beta c$ *set.*

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