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ANALYZE ON FUZZY INVENTORY MODEL WITH SHORTAGES UTILIZING KUHN-TUCKER TECHNIQUE

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ABSTRACT. In this paper, our own selves dissolve a fuzzy stockpile portrait, fuzzy budget asset can be fuzzified by ordering cost, worn out cost, import cost using quadruple fuzzy numbers. The optimization theory deals with the development of models and methods to find optimal solutions to specified mathematical problems. Kuhn-Tucker conditions are necessary conditions for a solution in non-linear programming for the first order in mathematics. Kuhn-Tucker conditions are necessary under certain specific circumstances as well as sufficient conditions. The use of these mathematical methods of optimization in economics is also introduced in this paper. Here Graded mean integration representation manner is utilized for defuzzification to assess total cost. Numerical illustration is exemplified.

1. INTRODUCTION

Inventory control is very dominant sector for applicants and for the purpose of researchers. In the beginning, the conjecture of stock book prototypes are evaluated as impermance and directed by using contingency theory. The first quantitative prescription of reserve was the basic budgetary order batch model. In 1950 the flash Berkeley forum on Mathematical Statistics and Probability,

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clutched in Berkeley, a Princeton maths legendry, Albert W. Tucker, he was commonly recognized as just a topologist, granted a lecture called ' Nonlinear computing'. It was positioned on a collaborative endeavor by Tucker, Harold W. Kuhn terminated his research at Princeton academy. The conversations were pronounced in a symposium archives. In the mathematical literature [Kuhn and Tucker, 1950] during the foretime the term 'nonlinear computing' the caption Kuhn and Tucker adopt for their concept arose. Tucker and Kuhn popularizes a problem and demonstrated the theory's leading theorem termed kuhn tucker axiom [1]. This axiom fires the doctrine of non precarious programming, which provides the decisive cases for the continuation of an optimal result to a nonlinear prioritizing problem.William Karush's adept thesis was entitled "Several parables along Inequalities as border situations Minimum of Functions" [2]. In Chicago's University mathematical department was entrenched with the university's opening in 1892. The department's first director was Eliakim H. Moore in (1862−1932), who established a mathematical atmosphere in collaboration in (1857−1936) [3], Heinrich Maschke (1853−1903) [5], who instantly grow into the pre eminent bureau of mathematics in United States by Rowe and Parshall [4]. As a thesis advisor, Bolza was very popular, frequently escorting his scholars to effort in the industry was investigating himself at the moment. The emanate was in Chicago called Chicago School of Variation Calculus Rowe and Parshall (1994), he built a solid foundation for science.

2. Preliminaries

Definition 2.1. *Let ψ be a spot of mark with a generic aspect of ψ expressed by c. Hence $\psi = \{c\}$. A fuzzy set S in ψ is characterized by a membership function $V_M(s)$ which accomplice with individual marks in ψ a real number in the interruption [0,1] with the conscience of $V_M(s)$ at c exhibiting the 'grade of enrollment' of c in S. Thus the adjacent value of $V_M(s)$ to integrity,the advanced grade of enrollment of c in S.*

Definition 2.2. *A trapezoidal fuzzy statistic $\chi = \{e, f, g, h\}$ is represented with enrollment function μ_χ as,*

$$\mu_{\tilde{A}}(\delta) = \begin{cases} L(\delta) = \dfrac{\delta - e}{f - e}, & e \leq \delta \leq f; \\ 1, & f \leq \delta \leq g; \\ R(\delta) = \dfrac{h - \delta}{h - g}, & g \leq \delta \leq h; \\ 0, & otherwise \end{cases}.$$

Definition 2.3. *Let $\eta = (\alpha, \beta, \varepsilon, \varphi; \Omega)$ be a hypothesized L-R categorize fuzzy number, L^{-1} and R^{-1} be the contrary objectives of L and R commonly. Then the Graded Mean Integration Representation of η is*

$$P(\eta) = \frac{\displaystyle\int_0^W c(\lambda L^{-1}(x) + (1-\lambda)R^{-1}(x))xv}{\int_0^W cxv},$$

where c is between 0 and w,$e \leq x \leq f$,$g \leq x \leq h$; we call $P(\eta)$ as graded λ-inclination integration depiction of η. So, $P(\eta) = \dfrac{\alpha + \beta + 2\varepsilon + \varphi}{6}$.

3. The fuzzy computation operations under function convention

Presume $\tilde{\gamma} = (\gamma_1, \gamma_2, \gamma_3, \gamma_4)$ and $\tilde{\sigma} = (\sigma_1, \sigma_2, \sigma_3, \sigma_4)$ are pair of four fuzzy numbers.

(1) The addition of $\tilde{\gamma}$ and $\tilde{\sigma}$ is $\tilde{\gamma} \oplus \tilde{\sigma} = (\gamma_1 + \sigma_1, \gamma_2 + \sigma_2, \gamma_3 + \sigma_3, \gamma_4 + \sigma_4)$, Where $(\gamma_1, \gamma_2, \gamma_3, \gamma_4, \sigma_1, \sigma_2, \sigma_3, \sigma_4)$ are any real numbers.

(2) The multiplication of $\tilde{\gamma}$ and $\tilde{\sigma}$ is $\tilde{\gamma} \otimes \tilde{\sigma} = (\gamma_1\sigma_1, \gamma_2\sigma_2, \gamma_3\sigma_3, \gamma_4\sigma_4)$, Where $(\gamma_1, \gamma_2, \gamma_3, \gamma_4, \sigma_1, \sigma_2, \sigma_3, \sigma_4)$ are all non-zero positive figures.

(3) The subtraction of $\tilde{\gamma}$ and $\tilde{\sigma}$ is $\tilde{\gamma} \ominus \tilde{\sigma} = (\gamma_1 - \sigma_4, \gamma_2 - \sigma_3, \gamma_3 - \sigma_2, \gamma_4 - \sigma_1)$, $\tilde{-\sigma} = (-\sigma_1, -\sigma_2, -\sigma_3, -\sigma_4)$ Where $(\gamma_1, \gamma_2, \gamma_3, \gamma_4, \sigma_1, \sigma_2, \sigma_3, \sigma_4)$ are any real numbers.

(4) $\dfrac{1}{\tilde{\sigma}} = \tilde{\sigma}^{-1} = (\dfrac{1}{\sigma_4}, \dfrac{1}{\sigma_3}, \dfrac{1}{\sigma_2}, \dfrac{1}{\sigma_1})$where are all positive real numbers. If $(\gamma_1, \gamma_2, \gamma_3, \gamma_4, \sigma_1, \sigma_2, \sigma_3, \sigma_4)$ are all nonzero positive real numbers, then the division of $\tilde{\gamma}$ and $\tilde{\sigma}$ is $\tilde{\gamma} \oslash \tilde{\sigma} = (\dfrac{\gamma_1}{\sigma_4}, \dfrac{\gamma_2}{\sigma_3}, \dfrac{\gamma_3}{\sigma_2}, \dfrac{\gamma_4}{\sigma_1})$.

(5) Let $\partial \in R$. Then $\partial \geq 0, \partial \otimes \tilde{F} = (\partial f_4, \partial f_3, \partial f_2, \partial f_1)$, $\partial < 0, \partial \otimes \tilde{F} = (\partial f_4, \partial f_3, \partial f_2, \partial f_1)$.

4. NOTATIONS

A-Ordering cost, M-Holding cost, s-Shortage cost, N-Length of the plan, B-Demand with time period, q^*-Order quantity, T_c-Total cost, $\tilde{T_c}$-Fuzzy total cost, $F(Q)$-Defuzzified total cost, $F(Q)^*$-Defuzzified total cost in minimum, Q^*-Optimal order quantity.

5. CRISP SENSE

First, we deal an inventory model with shortages in crisp sense, the economic size obtained by the equation $TC = \dfrac{MNq}{2} + \dfrac{AB}{q}$. Differentiating partially w.r.t q and equate them to 0', we get $q^* = \sqrt{\dfrac{2AB}{MN}}$.

6. FUZZY SENSE

$\tilde{M} = (m_1, m_2, m_3, m_4), \tilde{B} = (b_1, b_2, b_3, b_4), \tilde{N} = (n_1, n_2, n_3, n_4), \tilde{A} = (a_1, a_2, a_3, a_4)$
By Graded mean integration representation method,Differentiate partially w.r.t q and equate it to 0,

$$\frac{\partial T_{\tilde{c}}}{\partial q} = 0$$

$$\Rightarrow \quad \frac{1}{6}\Bigg[\left(\frac{m_1n_1}{2} - \frac{a_1b_1}{q^2}\right) + \left(\frac{m_2n_2}{2} - \frac{a_2b_2}{q^2}\right) + 2\left(\frac{m_3n_3}{2} - \frac{a_3b_3}{q^2}\right) + \left(\frac{m_4n_4}{2} - \frac{a_4b_4}{q^2}\right)\Bigg] = 0$$

$$\Rightarrow \quad q^* = \sqrt{\frac{2(a_1b_1 + 2a_2b_2 + 2a_3b_3 + a_4b_4)}{(m_1n_1 + 2m_2n_2 + 2m_3n_3 + m_4n_4)}}.$$

7. KUHN-TUCKER METHOD

Conditions:
Here we have the conditions for Kuhn-tucker model:

(i) $\lambda \geq 0$,
(ii) $\nabla f_p(T_{\tilde{c}}) - \lambda \nabla z(Q) = 0$,
(iii) $\lambda_i z_i(Q) = 0, i = 0, i = 1, 2, ..., m$,
(iv) $z_i(Q) \geq 0$

$$P(T_{\tilde{c}}) = \frac{1}{6}\Bigg[\left(\frac{m_1n_1q_1}{2} - \frac{a_1b_1}{q_4}\right) + \left(\frac{m_2n_2q_2}{2} - \frac{a_2b_2}{q_3}\right) + 2\left(\frac{m_3n_3q_3}{2} - \frac{a_3b_3}{q_2}\right) + \left(\frac{m_4n_4q_4}{2} - \frac{a_4b_4}{q_1}\right)\Bigg] = 0,$$

with $0 \leq q_1, \leq q_2, \leq q_3, \leq q_4$,can be written as $q_2 - q_1 \geq 0, q_3 - q_2 \geq 0, q_4 - q_3 \geq 0, q_1 \geq 0$.

Condition 1: $\lambda_1, \lambda_2, \lambda_3, \lambda_4 \leq 0$.

Condition 2:

$$\frac{\partial}{\partial q_1}(P(T_{\tilde{c}})) - \lambda_1 \frac{\partial}{\partial q_1} z_1(Q) - \lambda_2 \frac{\partial}{\partial q_2} z_2(Q) - \lambda_3 \frac{\partial}{\partial q_3} z_3(Q) - \lambda_4 \frac{\partial}{\partial q_4} z_4(Q) = 0\,.$$

Differentiate q_1, q_2, q_3, q_4 and equate it to zero.

$$P(T_{\tilde{c}}) = \frac{1}{6}[\left(\frac{m_1n_1q_1}{2} - \frac{a_1b_1}{q_4}\right) + \left(\frac{m_2n_2q_2}{2} - \frac{a_2b_2}{q_3}\right) + 2\left(\frac{m_3n_3q_3}{2} - \frac{a_3b_3}{q_2}\right) + \left(\frac{m_4n_4q_4}{2} - \frac{a_4b_4}{q_1}\right)] + \lambda_1 - \lambda_4 + \lambda_2 - \lambda_1 - \lambda_2 + \lambda_3 - \lambda_3 + \lambda_4 = 0.$$

Condition 3: The condition is $\lambda_1(q_2 - q_1) = 0, \lambda_2(q_3 - q_2) = 0$, $\lambda_3(q_4 - q_3) = 0, \lambda_4 q_1 = 0$.

Condition 4: The condition is $q_2 - q_1 \geq 0, q_3 - q_2 \geq 0, q_4 - q_3 \geq 0, q_1 \geq 0$. Here w.k.t $q_1 \geq 0, \lambda_4 q_1 = 0$,so we get $\lambda_4 = 0$. Then we replace q_2 by q_1 then q_3 by q_2 and q_4 by q_3,then $[q_1 = q_2 = q_3 = q_4]$, by adding condition 2 we get,

$$q^* = \sqrt{\frac{2(a_1b_1 + 2a_2b_2 + 2a_3b_3 + a_4b_4)}{(m_1n_1 + 2m_2n_2 + 2m_3n_3 + m_4n_4)}}\,.$$

8. Numerical example

Crisp sense: $A = 2000, B = 1000, q = 50, M = 10, N = 12$

Case I: $q^* = \sqrt{\frac{2AB}{MN}} = 182.58$

Case II: $TC = \frac{MNq}{2} + \frac{AB}{q} = 21908.9$

Fuzzy sense:

$m_1, m_2, m_3, m_4 = (8, 9, 11, 12), n_1, n_2, n_3, n_4 = (10, 11, 13, 14),$

$a_1, a_2, a_3, a_4 = (1800, 1900, 2100, 2200),$
$b_1, b_2, b_3, b_4 = (800, 900, 1100, 1200)\,.$

Case I:

$$\begin{aligned} P(T_{\tilde{c}}) &= \frac{1}{6}\Bigg[\left(\frac{m_1 n_1 q_1}{2} - \frac{a_1 b_1}{q_4}\right) + \left(\frac{m_2 n_2 q_2}{2} - \frac{a_2 b_2}{q_3}\right) \\ &\quad + 2\left(\frac{m_3 n_3 q_3}{2} - \frac{a_3 b_3}{q_2}\right) + \left(\frac{m_4 n_4 q_4}{2} - \frac{a_4 b_4}{q_1}\right)] \\ &= \text{Rs}.22201.0242\,. \end{aligned}$$

Case II: $q^* = \sqrt{\dfrac{2(a_1 b_1 + 2a_2 b_2 + 2a_3 b_3 + a_4 b_4)}{(m_1 n_1 + 2m_2 n_2 + 2m_3 n_3 + m_4 n_4)}} = 181.97\,.$

9. Conclusion

Here a fuzzy inventory technique with insistence is contemplated and carrying cost,backorder cost and ordering cost are fuzzified adopting trapezoidal fuzzy numbers. Then Graded mean integration representation mechanism is used to defuzzify then to find the total cost.

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