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GENERALIZED FUZZY $\mathcal{Z}$ CLOSED SETS IN DOUBLE FUZZY TOPOLOGICAL SPACES

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ABSTRACT. In this paper we introduce (l, k)-generalized fuzzy $\mathcal{Z}$-closed respective border, exterior $\&$ frontier in double fuzzy topological spaces. Some characterizations of these notions are presented.

1. INTRODUCTION AND PRELIMINARIES

In 1986, Atanassov [1] started 'Intuitionistic fuzzy sets'and Coker [2] in 1997, initiated Intuitionistic fuzzy topological space. The term 'double' instead of 'intuitionistic' coined by Garcia and Rodabaugh [3] in 2005. In the previous two decades many analysts accomplishing more applications on double fuzzy topological spaces. From 2011, $\mathcal{Z}$-open sets and maps were introduced in topological spaces by El-Maghrabi and Mubarki [4].

In [6] (l, k)-fuzzy $\mathcal{Z}$-closed sets and study some of their properties were studied in double fuzzy topological spaces.

X denotes a non-empty set, $I_1 = [0, 1)$, $I_0 = (0, 1]$, $I = [0, 1]$, $0 = \underline{0}(X)$, $1 = \underline{1}(X)$, $r \in I_0$ and $\kappa \in I_1$ and always $1 \geq r + \kappa$. I^X is a family of all fuzzy sets on X. In 2002, Double fuzzy topological spaces (briefly, dfts), (X, η, η^*), (r, κ)-fuzzy open (resp. (r, κ)-fuzzy closed) (briefly (r, κ)-fo (resp. (r, κ)-fc)) set were given by Samanta and Mondal [5].

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All other undefined notions are from [4–6] and cited therein.

2. AN (l,k)-GENERALIZED FUZZY $\mathcal{Z}$ CLOSED SETS

Definition 2.1. *Let (X,γ,γ^*) be a $dfts$, $\lambda,\mu\in I^X$, $l\in I_0$ and $k\in I_1 \ni l+k\le 1$ the fs λ is called an (l,k)- generalized fuzzy*

(i) *$\mathcal{Z}$ closed (briefly (l,k)-$gf\mathcal{Z}c$) set if $\mathcal{Z}C_{\gamma,\gamma^*}(\lambda,l,k)\le\mu$ whenever $\lambda\le\mu$, $\gamma(\mu)\ge l\ni\gamma^*(\mu)\le k$.*
(ii) *$\mathcal{Z}$ open (briefly (l,k)-$gf\mathcal{Z}o$) set if $\underline{1}-\lambda$ is an (l,k)-$gf\mathcal{Z}c$ set.*

Example 1. *Let $X=\{u,v,w\}$ and let the fs's α_1, α_2 and α_3 are defined as $\alpha_1(u)=0.3,\ \alpha_2(v)=0.4,\ \alpha_1(w)=0.5$, $\alpha_2(u)=0.6,\ \alpha_2(v)=0.9,\ \alpha_2(w)=0.5$ and $\alpha_3(u)=0.4,\ \alpha_3(v)=0.0,\ \alpha_3(w)=0.5$. Consider the dfts's (X,γ,γ^*) with*

$$\gamma(\mu)=\begin{cases}1, & \text{if } \mu\in\{\underline{0},\underline{1}\},\\ \frac{1}{2}, & \text{if } \mu\in\{\alpha_1,\alpha_2\}\\ 0, & o.w.\end{cases},\quad \gamma^*(\mu)=\begin{cases}0, & \text{if } \mu\in\{\underline{0},\underline{1}\},\\ \frac{1}{2}, & \text{if } \mu\in\{\alpha_1,\alpha_2\}\\ 1, & o.w.\end{cases}.$$

Then the fs α_3 is an $(\frac{1}{2},\frac{1}{2})$-$gf\mathcal{Z}o$ set.

Theorem 2.1. *Let (X,γ,γ^*) be a $dfts$, $l\in I_0$ and $k\in I_1$, $\lambda\in I^X$ is (l,k)-$gf\mathcal{Z}o$ set iff $\xi\le\mathcal{Z}I_{\gamma,\gamma^*}(\lambda,l,k)$ whenever $\xi\le\lambda,\ \gamma(\underline{1}-\xi)\ge l$ and $\gamma^*(\underline{1}-\xi)\le k$.*

Definition 2.2. *Let (X,γ,γ^*) be a $dfts$, $\forall\ \lambda,\xi\in I^X$, $l\in I_0$ and $k\in I_1\ni l+k\le 1$ we define an (l,k)-generalized fuzzy $\mathcal{Z}$ closure operator (l,k)-$G\mathcal{Z}C_{\gamma,\gamma^*}: I^X\times I_0\times I_1\to I^X$ as $G\mathcal{Z}C_{\gamma,\gamma^*}(\lambda,l,k)=\bigwedge\left\{\xi\in I^X|\lambda\le\xi and\xi \text{ is } (l,k)\text{-}gf\mathcal{Z}c\right\}$.*

Remark 2.1. *Every (l,k)-$f\mathcal{Z}c$ set is an (l,k)-$gf\mathcal{Z}c$ set. But not conversely.*

Example 2. *In Example 1, the fs α_3 is an $(\frac{1}{2},\frac{1}{2})$-$gf\mathcal{Z}o$ set but not an $(\frac{1}{2},\frac{1}{2})$- $f\mathcal{Z}o$.*

Theorem 2.2. *Let (X,γ,γ^*) be a $dfts$, $\forall\ \lambda,\xi\in I^X$, $l\in I_0$ and $k\in I_1\ni l+k\le 1$, then the operator (l,k)-$G\mathcal{Z}C_{\gamma,\gamma^*}$ satisfies the statements*

(i) $G\mathcal{Z}C_{\gamma,\gamma^*}(\underline{0},l,k)=\underline{0}$ *and* $G\mathcal{Z}C_{\gamma,\gamma^*}(\underline{1},l,k)=\underline{1}$;
(ii) $\lambda\le G\mathcal{Z}C_{\gamma,\gamma^*}(\lambda,l,k)$;
(iii $G\mathcal{Z}C_{\gamma,\gamma^*}(\lambda\vee\xi,l,k)\ge G\mathcal{Z}C_{\gamma,\gamma^*}(\lambda,l,k)\vee G\mathcal{Z}C_{\gamma,\gamma^*}(\xi,l,k)$;
(iv) $G\mathcal{Z}C_{\gamma,\gamma^*}(G\mathcal{Z}C_{\gamma,\gamma^*}(\lambda,l,k),l,k)=G\mathcal{Z}C_{\gamma,\gamma^*}\ (\lambda,l,k)$;
(v) *If λ is (l,k)-$gf\mathcal{Z}c$ set then $G\mathcal{Z}C_{\gamma,\gamma^*}(\lambda,l,k)=\lambda$;*
(vi) $G\mathcal{Z}C_{\gamma,\gamma^*}(\lambda,l,k)\le\mathcal{Z}C_{\gamma,\gamma^*}(\lambda,l,k)\le C_{\gamma,\gamma^*}(\lambda,l,k)$.

Theorem 2.3. *Let (X, γ, γ^*) be a $dfts$, $\forall\ \lambda, \xi \in I^X$, $l \in I_0$ and $k \in I_1 \ni l + k \leq 1$ we define an (l,k)- generalized fuzzy $\mathcal{Z}$ interior operator (l,k)-$G\mathcal{Z}I_{\gamma,\gamma^*}$: $I^X \times I_0 \times I_1 \to I^X$ as $G\mathcal{Z}I_{\gamma,\gamma^*}(\lambda, l, k) = \bigvee \left\{\xi \in I^X | \lambda \geq \xi \text{ and } \xi \text{ is } (l,k)\text{-}gf\mathcal{Z}o\right\}$, then $G\mathcal{Z}I_{\gamma,\gamma^*}(\underline{1} - \lambda, l, k) = \underline{1} - G\mathcal{Z}C_{\gamma,\gamma^*}(\lambda, l, k)$.*

Proposition 2.1. *Let (X, γ, γ^*) be a $dfts$, $\lambda \in I^X$, $l \in I_0$ and $k \in I_1$.*

(i) *If λ is (l,k)-$gf\mathcal{Z}c$ set and an (l,k)-$f\mathcal{Z}o$ set then λ is an (l,k)-$f\mathcal{Z}c$ set.*
(ii) *If λ is (l,k)-$gf\mathcal{Z}c$ set and an (l,k)-fo set then $\lambda \wedge \xi$ is an (l,k)-$f\mathcal{Z}c$ set whenever $\xi \leq \mathcal{Z}C_{\gamma,\gamma^*}(\lambda, l, k)$.*

Definition 2.3. *Let (X, γ, γ^*) be a $dfts$, $\forall\ \lambda \in I^X$, $l \in I_0$ and $k \in I_1$. A fs λ is called as (l,k)- generalized* fuzzy*

(i) *$\mathcal{Z}$ closed (briefly (l,k)-$g^*f\mathcal{Z}c$) set if $\mathcal{Z}C_{\gamma,\gamma^*}(\lambda, l, k) \leq \xi$ whenever $\lambda \leq \xi$ and ξ is an (l,k)-gfo set in I^X.*
(ii) *$\mathcal{Z}$ open (briefly (l,k)-$g^*f\mathcal{Z}o$) set if $\underline{1} - \lambda$ is (l,k)-$g^*f\mathcal{Z}c$ set.*

Example 3. *In Example 1, the fs α_3 is an $(\frac{1}{2}, \frac{1}{2})$-$g * f\mathcal{Z}o$ set.*

Theorem 2.4. *Let (X, γ, γ^*) be a $dfts$, $\lambda \in I^X$, $l \in I_0$ and $k \in I_1$, then λ is (l,k)-$g^*f\mathcal{Z}o$ set iff $\xi \leq \mathcal{Z}I_{\gamma,\gamma^*}(\lambda, l, k)$ whenever ξ is an (l,k)-gfc set.*

Proposition 2.2. *Let (X, γ, γ^*) be a $dfts$. For each λ and $\xi \in I^X$, $l \in I_0$ and $k \in I_1$,*

(i) *If λ and ξ are (l,k)-$g^*f\mathcal{Z}c$ sets then $\lambda \wedge \xi$ is an (l,k)-$g^*f\mathcal{Z}c$ set,*
(ii) *If λ is (l,k)-$g^*f\mathcal{Z}c$ set and $\gamma(\xi) \geq l, \gamma^*(\xi) \leq k$ then $\lambda \wedge \xi$ is an (l,k)-$g^*f\mathcal{Z}c$ set.*

Proposition 2.3. *Let (X, γ, γ^*) be a $dfts$. $\forall\ \lambda and \xi \in I^X$, $l \in I_0$ and $k \in I_1$.*

(i) *If λ is both (l,k)-gfo and (l,k)-$g^*f\mathcal{Z}c$ set then λ is an (l,k)-$f\mathcal{Z}c$ set.*
(ii) *If λ is (l,k)-$g^*f\mathcal{Z}c$ set and $\lambda \leq \xi \leq \mathcal{Z}C_{\gamma,\gamma^*}(\lambda, l, k)$ then ξ is an (l,k)-$g^*f\mathcal{Z}c$ set.*

Theorem 2.5. *Let $(X, \gamma_1, \gamma_1^*)$ and $(Y, \gamma_2, \gamma_2^*)$ be $dfts$'s. If $\lambda \leq \underline{1}_Y \leq \underline{1}_X$, $\ni \lambda$ is (l,k)-$g^*f\mathcal{Z}c$ set in I^X, $l \in I_0$ and $k \in I_1$, then λ is an (l,k)-$g^*f\mathcal{Z}c$ set relative to Y.*

Theorem 2.6. *Let (X, γ, γ^*) be a $dfts$, $\forall\ \lambda, \xi \in I^X$, $l \in I_0$ and $k \in I_1$, with $\xi \leq \lambda$. If ξ is (l,k)-$g^*f\mathcal{Z}c$ set relative to $\lambda \ni \lambda$ is both (l,k)-gfo and (l,k)-$g^*f\mathcal{Z}c$ set of I^X then ξ is an (l,k)-$g^*f\mathcal{Z}c$ set relative to X.*

3. Properties of (l,k)-generalized fuzzy $\mathcal{Z}$ open sets

Proposition 3.1. *For any λ, ξ in a $dfts$ (X, γ, γ^*),*

(i) $G\mathcal{Z}I_{\gamma,\gamma^*}(\lambda, l, k)$ *is the largest* (l,k)-$gf\mathcal{Z}o$ *set* $\ni G\mathcal{Z}I_{\gamma,\gamma^*}(\lambda, l, k) \leq \lambda$.

(ii) $\lambda = G\mathcal{Z}I_{\gamma,\gamma^*}(\lambda, l, k)$ *iff* λ *is an* (l,k)-$gf\mathcal{Z}o$ *set.*

(iii) $G\mathcal{Z}I_{\gamma,\gamma^*}(G\mathcal{Z}I_{\gamma,\gamma*}(\lambda, l, k), l, k) = G\mathcal{Z}I_{\gamma,\gamma^*}(\lambda, l, k)$.

(iv) $\underline{1} - G\mathcal{Z}I_{\gamma,\gamma^*}(\lambda, l, k) = G\mathcal{Z}C_{\gamma,\gamma^*}(\underline{1} - \lambda, l, k)$.

(v) $\underline{1} - G\mathcal{Z}C_{\gamma,\gamma^*}(\lambda, l, k) = G\mathcal{Z}I_{\gamma,\gamma^*}(\underline{1} - \lambda, l, k)$.

(vi) *If* $\lambda \leq \xi$, *then* $G\mathcal{Z}I_{\gamma,\gamma*}(\lambda, l, k) \leq G\mathcal{Z}I_{\gamma,\gamma*}(\xi, l, k)$.

(vii) *If* $\lambda \leq \xi$, *then* $G\mathcal{Z}C_{\gamma,\gamma*}(\lambda, l, k) \leq G\mathcal{Z}C_{\gamma,\gamma*}(\xi, l, k)$.

$(viii)$ $G\mathcal{Z}I_{\gamma,\gamma^*}(\lambda, l, k) \wedge G\mathcal{Z}I_{\gamma,\gamma*}(\xi, l, k) = G\mathcal{Z}I_{\gamma,\gamma^*}(\lambda \wedge \xi, l, k)$.

(ix) $G\mathcal{Z}I_{\gamma,\gamma^*}(\lambda, l, k) \vee G\mathcal{Z}I_{\gamma,\gamma^*}(\xi, l, k) = G\mathcal{Z}I_{\gamma,\gamma^*}(\lambda \vee \xi, l, k)$.

Definition 3.1. *In any $dfts$ (X, γ, γ^*), the (l,k)-fuzzy $\mathcal{Z}$ border (resp. frontier and exterior) of a fs $\lambda \in I^X$ (briefly $\mathcal{Z}\mathcal{B}_{\gamma,\gamma^*}(\lambda, l, k)$ (resp. $\mathcal{Z}\mathcal{F}_{\gamma,\gamma^*}(\lambda, l, k)$ and $\mathcal{Z}\mathcal{E}_{\gamma,\gamma^*}(\lambda, l, k)$)) is given by $\mathcal{Z}\mathcal{B}_{\gamma,\gamma^*}(\lambda, l, k) = \lambda - \mathcal{Z}I_{\gamma,\gamma^*}(\lambda, l, k)$ (resp. $\mathcal{Z}\mathcal{F}_{\gamma,\gamma^*}(\lambda, l, k) = \mathcal{Z}C_{\gamma,\gamma^*}(\lambda, l, k) - \mathcal{Z}I_{\gamma,\gamma^*}(\lambda, l, k)$ and $\mathcal{Z}\mathcal{E}_{\gamma,\gamma^*}(\lambda, l, k) = \mathcal{Z}I_{\gamma,\gamma^*}(\underline{1} - \lambda, l, k)$).*

Definition 3.2. *In any $dfts$ (X, γ, γ^*), the (l,k)-generalized fuzzy $\mathcal{Z}$ border (resp. frontier and exterior) of a fs $\lambda \in I^X$ (briefly $G\mathcal{Z}\mathcal{B}_{\gamma,\gamma^*}(\lambda, l, k)$ (resp. $G\mathcal{Z}\mathcal{F}_{\gamma,\gamma^*}(\lambda, l, k)$ and $G\mathcal{Z}\mathcal{E}_{\gamma,\gamma^*}(\lambda, l, k)$)) is given by $G\mathcal{Z}\mathcal{B}_{\gamma,\gamma^*}(\lambda, l, k) = \lambda - G\mathcal{Z}I_{\gamma,\gamma^*}(\lambda, l, k)$ (resp. $G\mathcal{Z}\mathcal{F}_{\gamma,\gamma^*}(\lambda, l, k) = G\mathcal{Z}C_{\gamma,\gamma^*}(\lambda, l, k) - G\mathcal{Z}I_{\gamma,\gamma^*}(\lambda, l, k)$ and $G\mathcal{Z}\mathcal{E}_{\gamma,\gamma^*}(\lambda, l, k) = G\mathcal{Z}I_{\gamma,\gamma^*}(\underline{1} - \lambda, l, k)$).*

Proposition 3.2. *For any λ, ξ in a $dfts$ (X, γ, γ^*),*

(i) $G\mathcal{Z}\mathcal{B}_{\gamma,\gamma^*}(\lambda, l, k) \leq \mathcal{Z}\mathcal{B}_{\gamma,\gamma^*}(\lambda, l, k)$.

(ii) *If* λ *is an* $(l,k) - gf\mathcal{Z}o$, *then* $G\mathcal{Z}\mathcal{B}_{\gamma,\gamma^*}(\lambda, l, k) = \underline{0}$.

(iii) $G\mathcal{Z}\mathcal{B}_{\gamma,\gamma^*}(\lambda, l, k) \leq G\mathcal{Z}C_{\gamma,\gamma^*}(\underline{1} - \lambda, l, k)$.

(iv) $G\mathcal{Z}I_{\gamma,\gamma^*}(G\mathcal{Z}\mathcal{B}_{\gamma,\gamma^*}(\lambda, l, k), l, k) \leq \lambda$.

(v) $G\mathcal{Z}\mathcal{B}_{\gamma,\gamma^*}(\lambda \vee \xi) \leq G\mathcal{Z}\mathcal{B}_{\gamma,\gamma^*}(\lambda, l, k) \vee G\mathcal{Z}\mathcal{B}_{\gamma,\gamma^*}(\xi, l, k)$.

(vi) $G\mathcal{Z}\mathcal{B}_{\gamma,\gamma^*}(\lambda \wedge \xi) \geq G\mathcal{Z}\mathcal{B}_{\gamma,\gamma^*}(\lambda, l, k) \wedge G\mathcal{Z}\mathcal{B}_{\gamma,\gamma^*}(\xi, l, k)$.

Proposition 3.3. *For any λ, ξ in a $dfts$ (X, γ, γ^*),*

(i) $G\mathcal{Z}\mathcal{F}_{\gamma,\gamma^*}(\lambda, l, k) \leq \mathcal{Z}\mathcal{F}_{\gamma,\gamma^*}(\lambda, l, k)$.

(ii) $G\mathcal{Z}\mathcal{B}_{\gamma,\gamma^*}(\lambda, l, k) \leq G\mathcal{Z}\mathcal{F}_{\gamma,\gamma^*}(\lambda, l, k)$.

(iii) $G\mathcal{Z}\mathcal{F}_{\gamma,\gamma^*}(\underline{1} - \lambda, l, k) = G\mathcal{Z}\mathcal{F}_{\gamma,\gamma^*}(\lambda, l, k)$.

(iv) $G\mathcal{Z}\mathcal{F}_{\gamma,\gamma^*}(G\mathcal{Z}I_{\gamma,\gamma^*}(\lambda, l, k), l, k) \leq G\mathcal{Z}\mathcal{F}_{\gamma,\gamma^*}(\lambda, l, k)$.

(v) $G\mathcal{Z}\mathcal{F}_{\gamma,\gamma^*}(G\mathcal{Z}\ C_{\gamma,\gamma^*}(\lambda,l,k),l,k) \leq G\mathcal{Z}\mathcal{F}_{\gamma,\gamma^*}(\lambda,l,k)$.
(vi) $\lambda - G\mathcal{Z}\mathcal{F}_{\gamma,\gamma^*}(\lambda,l,k) \leq G\mathcal{Z}I_{\gamma,\gamma^*}(\lambda,l,k)$.
(vii) $G\mathcal{Z}\mathcal{F}_{\gamma,\gamma^*}(\lambda \vee \xi,l,k) \leq G\mathcal{Z}\mathcal{F}_{\gamma,\gamma^*}(\lambda,l,k) \vee G\mathcal{Z}\mathcal{F}_{\gamma,\gamma^*}(\xi,l,k)$.
$(viii)$ $G\mathcal{Z}\mathcal{F}_{\gamma,\gamma^*}(\lambda \wedge \xi,l,k) \geq G\mathcal{Z}\mathcal{F}_{\gamma,\gamma^*}(\lambda,l,k) \wedge G\mathcal{Z}\mathcal{F}_{\gamma,\gamma^*}(\xi,l,k)$.

Proposition 3.4. *For any $dfts$ (X,γ,γ^*), $\forall\ \lambda \in I^X,\ l \in I_0$ and $k \in I_1$, we have:*
(i) $\mathcal{Z}\mathcal{E}_{\gamma,\gamma^*}(\lambda,l,k) \leq G\mathcal{Z}\mathcal{E}_{\gamma,\gamma^*}(\lambda,l,k)$.
(ii) $G\mathcal{Z}\mathcal{E}_{\gamma,\gamma^*}(\lambda,l,k) = \underline{1} - G\mathcal{Z}C_{\gamma,\gamma^*}(\lambda,l,k)$.
(iii) $G\mathcal{Z}\mathcal{E}_{\gamma,\gamma^*}(G\mathcal{Z}\mathcal{E}_{\gamma,\gamma^*}(\lambda,l,k),l,k) = G\mathcal{Z}I_{\gamma,\gamma^*}(G\mathcal{Z}C_{\gamma,\gamma^*}(\lambda,l,k),l,k)$.
(iv) *If $\lambda \leq \xi$, then $G\mathcal{Z}\mathcal{E}_{\gamma,\gamma^*}(\lambda,l,k) \geq G\mathcal{Z}\mathcal{E}_{\gamma,\gamma^*}(\xi,l,k)$.*
(v) $G\mathcal{Z}\mathcal{E}_{\gamma,\gamma^*}(\underline{1},l,k) = \underline{0}$.
(vi) $G\mathcal{Z}\mathcal{E}_{\gamma,\gamma^*}\ (\underline{0},l,k) = \underline{1}$.
(vii) $G\mathcal{Z}I_{\gamma,\gamma^*}(\lambda,l,k) \leq G\mathcal{Z}\mathcal{E}_{\gamma,\gamma^*}(G\mathcal{Z}\mathcal{E}_{\gamma,\gamma^*}(\lambda,l,k),l,k)$.
$(viii)$ $G\mathcal{Z}\mathcal{E}_{\gamma,\gamma^*}\ (\lambda \vee \xi,l,k) = G\mathcal{Z}\mathcal{E}_{\gamma,\gamma^*}(\lambda,l,k) \wedge G\mathcal{Z}\mathcal{E}_{\gamma,\gamma^*}(\xi,l,k)$.
(ix) $G\mathcal{Z}\mathcal{E}_{\gamma,\gamma^*}(\lambda \wedge \xi,l,k) = G\mathcal{Z}\mathcal{E}_{\gamma,\gamma^*}(\lambda,l,k) \vee G\mathcal{Z}\mathcal{E}_{\gamma,\gamma^*}(\xi,l,k)$.

Proposition 3.5. *If λ is an (l,k)-$gf\mathcal{Z}c$ set in a $dfts$ (X,γ,γ^*) then*
(i) $G\mathcal{Z}\mathcal{B}_{\gamma,\gamma^*}(\lambda,l,k) = G\mathcal{Z}\mathcal{F}_{\gamma,\gamma^*}(\lambda,l,k)$.
(ii) $G\mathcal{Z}\mathcal{E}_{\gamma,\gamma^*}(\lambda,l,k) = \underline{1} - \lambda$.

Definition 3.3. *A $dfts$ (X,γ,γ^*) is said to be a generalized* double fuzzy $\mathcal{Z}$-$(\gamma,\gamma^*)_{1/2}$ space (briefly, $g^*df\mathcal{Z}$-$(\gamma,\gamma^*)_{1/2}$), if each (l,k)-$gf\mathcal{Z}c$ set in X is an (l,k)-gfc set.*

Proposition 3.6. *Let (X,γ,γ^*) be a $g^*df\mathcal{Z}$-$(\gamma,\gamma^*)_{1/2}$ space and λ be an (l,k)-$gf\mathcal{Z}c$ set in X. Then the statements*
(i) $G\mathcal{B}_{\gamma,\gamma^*}(\lambda,l,k) = G\mathcal{F}_{\gamma,\gamma^*}(\lambda,l,k)$,
(ii) $G\mathcal{E}_{\gamma,\gamma^*}(\lambda,l,k) = \underline{1} - \lambda$. *are hold.*

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