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SOME RESULTS ON ANALYTIC EVEN MEAN LABELING OF GRAPH

T. SAJITHA KUMARI[1], M. REGEES, AND S. CHANDRA KUMAR

ABSTRACT. Let $G(V, E)$ be a graph with p vertices and q edges. $A(p, q)$- graph G is called an analytic even mean graph if there exist an injective function $f : V \to \{0, 2, 4, 6, ..., 2q\}$ with an induced edge labeling $f^* : E \to Z$ such that when each edge $e = uv$ with $f(u) < f(v)$ is labeled with $f^*(uv) = \left\lceil \frac{f(v)^2-(f(u)+1)^2}{2} \right\rceil$ if $f(u) \neq 0$ and $f^*(uv) = \left\lceil \frac{f(v)^2}{2} \right\rceil$ if $f(u) = 0$, all the edge labels are even and distinct. In this paper we show the analytic even mean labeling of coconut tree graph, fire cracker graph and some other results.

1. INTRODUCTION

By a graph $G = (V, E)$ with p vertices and q edges we mean a simple and undirected graph. The idea of graph labeling was bring in by Rosa in 1967 [1]. Somasundaram and Ponraj [2] have set up the conception of mean labeling of graphs. A detailed survey of graph labeling can be found in [3]. P. Jeyanthi, R. Gomathy and Gee-Choon Lau [4] called a graph G is analytic odd mean if there exist an injective function $f : V \to \{0, 1, 3, 5, ..., 2q - 1\}$ with an induce edge

[1]*corresponding author*

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labeling $f^* : E \to Z$ such that for every edge uv with $f(u) < f(v)$,

$$f^*(uv) = \begin{cases} \left\lceil \frac{f(v)^2-(f(u)+1)^2}{2} \right\rceil & \text{if} \quad f(u) \neq 0 \\ \left\lceil \frac{f(v)^2}{2} \right\rceil & \text{if} \quad f(u) = 0 \end{cases}.$$

A (p,q) - graph G is called an analytic even mean graph if there exist an injective function $f : V \to \{0, 2, 4, 6, ..., 2q\}$ with an induced edge labeling $f^* : E \to Z$ such that when each edge $e = uv$ with $f(u) < f(v)$ is labeled with $f^*(uv) = \left\lceil \frac{f(v)^2-(f(u)+1)^2}{2} \right\rceil$ if $f(u) \neq 0$ and $f^*(uv) = \left\lceil \frac{f(v)^2}{2} \right\rceil$ if $f(u) = 0$, all the edge labels are even and distinct. This labeling f is called an analytic even mean labeling [5]. The coconut tree is having the vertices $v_0, v_1, v_2, ..., v_i$ of path $(i \geq 1)$ and the pendant vertices $v_{i+1}, v_{i+2}, v_{i+3}, ..., v_{i+n}$, being adjacent with v_0. The fire cracker is constructed as follows: Let $a_0, a_1, a_2, ..., a_{k-1}$ be the vertices of the path P_k and b_j be the vertex adjacent to a_j for $1 \leq j \leq k$. Let $b_{j1}, b_{j2}, b_{j3}, ..., b_{jn}$ be the pendant vertices adjacent to b_j for $1 \leq j \leq k$ [6].

2. Main Results

In this section we prove the analytic even mean labeling of coconut tree graph fire cracker graph and some other results in analytic even mean labeling.

Theorem 2.1. *The coconut tree graph G is an analytic even mean graph.*

Proof. Let $V(G) = \{v_i, v_{n+j}, 1 \leq j \leq m\}$ and

$$E(G) = \{v_i v_{i+1}; 1 \leq i \leq n-1\} \cup \{v_i, v_{n+j}, 1 \leq j \leq m\}.$$

Here $|V(G)| = n + m$ and $|E(G)| = m + n - 1$. We define an injective map $f : V(G) \to \{0, 2, 4, ..., 2(m + n - 1)\}$ by

$$f(v_i) = 2i - 2; \quad 1 \leq i \leq n,$$

$$f(v_{n+j}) = 2n - 2 + 2j; \quad 1 \leq j \leq m.$$

Let f^* be the generated edge labeling of f, given by

$$f^*(v_i v_{i+1}) = \left\lceil \frac{4i-1}{2} \right\rceil; \quad 1 \leq i \leq n-1,$$

$$f^*(v_1 v_{n+j}) = 22n^2 + 2j^2 + 2 - 4n - 4j + 4nj; \quad 1 \leq j \leq m.$$

Hence we can see that the edge labels of path increases by 2 as i increases and pendent edge labels increase by $4n + 2, 4n + 6, 4n + 10, \ldots$ as j increases from 1

to n. So all the edge labels are even and distinct. Hence the coconut tree graph is an analytic even mean graph. $\square$

Example 1. *The analytic even mean labeling of the coconut tree graph with $n = 5$ and $m = 8$ is shown in the following figure.*

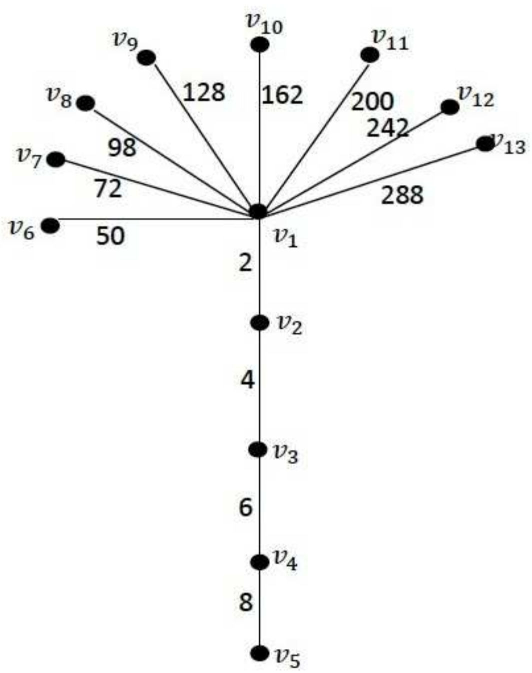

FIGURE 1

Theorem 2.2. *The fire cracker graph is an analytic even mean graph.*

Proof. Let G be the fire cracker graph. The vertex set and edge set are given by

$$V(G) = \{a_i, b_j, b_{jk}, 1 \leq j \leq n-1, 1 \leq j \leq n, \text{ and } 1 \leq k \leq m\}$$

and

$$E(G) = \{a_i a_{i+1}; 0 \leq i \leq n-2\} \cup \{a_i, b_j, 0 \leq i \leq n-2, 1 \leq j \leq n\}$$
$$\cup \{b_j b_{jk}; 1 \leq j \leq n, 1 \leq k \leq m\}.$$

We define an injective map $f : V(G) \to \{0, 2, 4, \ldots, 2n + nm\}$ by

$$f(a_0) = 0; f(a_i) = 2i; \quad 1 \leq i \leq n-1,$$

$$f(b_j) = 2n - 2 + 2j; \quad 1 \leq j \leq n,$$

$$f(b_{jk}) = 4n - 2 + 2m(j-1) + 2k; 1 \le j \le n, 1 \le k \le m.$$

Let f^* be the generated edge labeling of f, given by

$$f^*(a_{i-1}a_i) = \left\lceil \frac{4i-1}{2} \right\rceil; \quad 1 \le i \le n,$$

$f^*(a_0 b_1) = \lceil 2n^2 \rceil$,

$$f^*(a_i b_j) = \left\lceil \frac{4n^2 + 4j^2 - 8n - 8j + 8nj - 4i^2 - 4i + 3}{2} \right\rceil;$$

$0 \le i \le n-2, 1 \le j \le n,$

$$f^*(b_j b_{jk}) = \left\lceil \frac{(4n - 2 + 2m(j-1) + 2k^2) - (2n - 1 + 2j)^2}{2} \right\rceil;$$

$1 \le k \le m, 1 \le j \le n$. Clearly all the edge labels are even and distinct. Hence the fire cracker graph is an analytic even mean graph. □

Example 2. *The analytic even mean labeling of the fire cracker graph for $n = 3$ and $m = 5$ is shown in the following figure.*

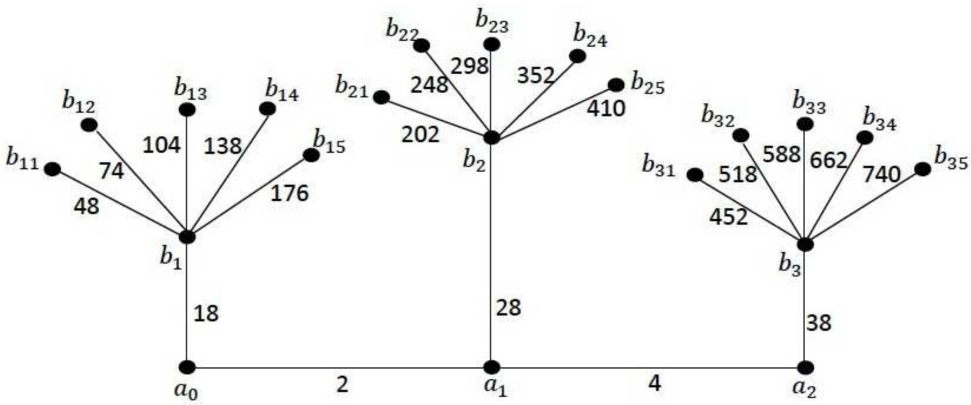

FIGURE 2

Theorem 2.3. *Let $S_1, S_2, S_3, \ldots, S_{k+1}$ be the disjoint copies of the K-star $K_{1,k}$ with vertex set $V(S_i) = \{v_i, v_{ir}; 1 \le r \le k\}$ and the edge set $E(S_i) = \{v_i, v_{ir}; 1 \le r \le k\}$ for $1 \le i \le k+1$. Let G be the graph obtained by joining a new vertex v to the centre of each $k+1$ star. Then G is an analytic even mean graph.*

Proof. Let $V(G) = \{v, v_i, v_{ir}, 0 \leq i \leq k+1$ and $1 \leq r \leq k\}$ and

$$E(G) = \{vv_i, v_iv_{ir}; 1 \leq i \leq k+1, 1 \leq r \leq k\}.$$

We define an injective map $f : V(G) \to \{0, 2, 4, \ldots, 2k^2 + 4k + 2\}$ by

$$f(v) = 0; f(v_i) = 2i; \quad 1 \leq i \leq k+1,$$

$$f(v_{ir}) = 2ki + 2r + 2; \quad 1 \leq i \leq k+1, 1 \leq r \leq k,$$

$$f(b_{jk}) = 4n - 2 + 2m(j-1) + 2k; \quad 1 \leq j \leq n, 1 \leq k \leq m.$$

Let f^* be the generated edge labeling of f, given by

$$f^*(vv_i) = \left\lceil 2i^2 \right\rceil; \quad 1 \leq i \leq k+1,$$

$$f^*(v_iv_{ir}) = \left\lceil \frac{4k^2i^2 + 4r^2 + 8kir + 8r + 8ki - 4i^2 - 4i + 3}{2} \right\rceil;$$

$1 \leq r \leq k, 1 \leq i \leq k+1$. Clearly all the edge labels are even and distinct. Hence the graph G is an analytic even mean graph. □

Example 3. *The analytic even mean labeling of the above graph G with $k = 4$ is shown in the following figure.*

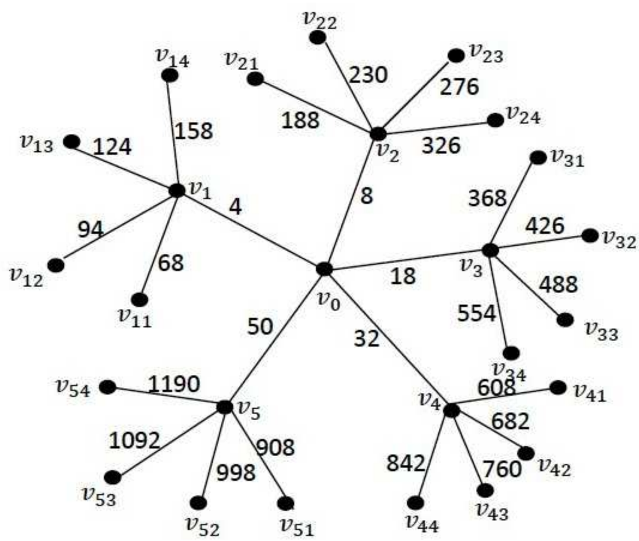

FIGURE 3

Theorem 2.4. *Let $S_1, S_2, S_3, \ldots, S_k$ be the disjoint copies of the K-star $K_{1,k}$ with vertex set $V(S_i) = \{v_i, v_{ir}; 1 \leq r \leq k\}$ and the edge set $E(S_i) = \{v_i, v_{ir}; 1 \leq r \leq k\}$ and G be the graph obtained by joining a new vertex v with $v_{11}, v_{21}, v_{31}, \ldots, v_{k1}$. Then G is an analytic even mean graph.*

Proof. Let G be the graph. Let

$$V(G) = \{v, v_i, v_{ir}, 0 \leq i \leq k+1 \text{ and } 1 \leq r \leq k\}$$

and

$$E(G) = \{vv_i, v_iv_{ir}; 1 \leq i \leq k+1, 1 \leq r \leq k\}.$$

We define an injective map $f : V(G) \to \{0, 2, 4, \ldots 2k^2 + 2k\}$ by

$$f(v) = 0; f(v_i) = 2i; \quad 1 \leq i \leq k,$$

$$f(v_{ir}) = 2ki + 2r; \quad 1 \leq i \leq k+1, 1 \leq r \leq k,$$

Let f^* be the generated edge labeling of f, given by

$$f^*(vv_{i1}) = 2k^2i^2 + 4ki + 2; \quad 1 \leq i \leq k,$$

$$f^*(v_iv_{ir}) = \left\lceil \frac{4k^2i^2 + 4r^2 + 8kir + 8r - 4i^2 - 4i - 1}{2} \right\rceil; \quad 1 \leq r \leq k, 1 \leq i \leq k.$$

Clearly all the edge labels are even and distinct. Hence the graph G is an analytic even mean graph. □

Example 4. *The analytic even mean labeling of the above graph G with $k = 6$ is shown in the figure 4.*

3. Conclusion

Here we proved the analytic even mean labeling of coconut tree graph, fire cracker graph and some other results in analytic even mean labeling. In future, we can construct many analytic even mean graphs using these ideas.

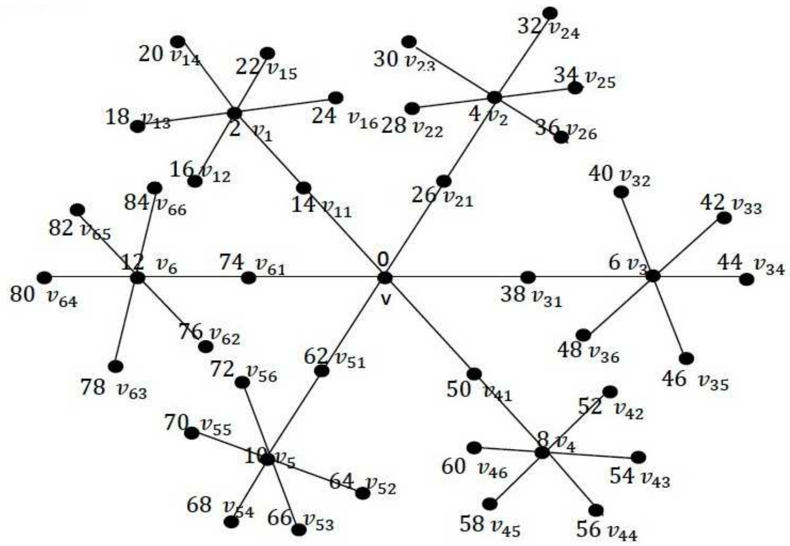

FIGURE 4

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DEPARTMENT OF MATHEMATICS
SCOTT CHRISTIAN COLLEGE(AUTONOMOUS)
NAGERCOIL-629001, TAMIL NADU, INDIA
E-mail address: sajithasabu09@gmail.com

DEPARTMENT OF MATHEMATICS
MALANKARA CATHOLIC COLLEGE
MARIAGIRI, KALIAKAVILAI-629153
TAMILNADU,INDIA

DEPARTMENT OF MATHEMATICS
SCOTT CHRISTIAN COLLEGE(AUTONOMOUS)
NAGERCOIL- 629001, KANYAKUMARI
TAMILNADU, INDIA