

TOPOLOGICAL INDICES ON PROPERTIES OF LINE GRAPH OF SUBDIVISION OF PLANE GRAPHS

M. C. SHANMUKHA, K. N. ANIL KUMAR¹, N. S. BASAVARAJAPPA, AND K. M. NIRANJAN

ABSTRACT. Graph theory is a discipline of mathematics which consist of various topological indices. In this paper various topological indices are applied on line graph of subdivision of plane graphs to obtain the physical characteristics of those graphs. The graphs used in this paper are Jahangir graph and Banana Tree graph.

1. INTRODUCTION AND TERMINOLOGIES

Mathematical chemistry is the branch of research-domain involved in extended applications of mathematics to chemistry. It deals with the statistical modelling of chemical-compound structures. Vital fields of research in mathematical chemistry consist of chemical graph theory, which describe the topology such as the mathematical study of isomerism and the growth of topological indices which find application in quantitative structure property relationship. Several topological indices are applied on the chemical structures to describe the physical and chemical characteristics. In this complete paper, we use molecular graph where atoms are vertices and chemical bonds are edges. The graphs are connected with no parallel edges.

¹*corresponding author*

2010 *Mathematics Subject Classification.* 05C05, 05C12, 05C35.

Key words and phrases. Multiplicative indices, Gourava indices, Jahangir graph and Banana tree graph.

The line graph will be a simple graph where vertices are the endpoints for an edge. The subdivision graph can be derived through appending an extra vertex to all edge of G . In this paper topological indices are applied on the Jahangir graph and the Banana tree graph [3, 4, 11–13].

Jahangir graph consist of a loop with one supplementary vertex close to vertices at a interval S . Banana tree graph is the graph derived through joining one leaf of each of n copies of a k -star graph with a one root vertex which is different for all the stars. The subdivision consists of an additional vertex between each pair of vertices.

In this paper we carry forward the work on above mentioned graphs. The topological indices like Gourava indices, Multiplicative indices are used.

Consider $\Gamma = (V(\Gamma), E(\Gamma))$ be a (molecular) graph, in which $V(\Gamma)$ and $E(\Gamma)$ are vertices and edges it corresponds to atoms set and chemical bonds set.

Definition 1.1. *The multiplicative first F -index was introduced by Bhanumathi [1] and Ghobadi [2], and can be stated as*

$$F_1II(\Gamma) = \prod_{e=vw \in E(\Gamma)} [(d_{\Gamma}(v))^2 + (d_{\Gamma}(w))^2].$$

Definition 1.2. *The second multiplicative F -index [5], for a graph Γ can be stated as*

$$F_2II(\Gamma) = \prod_{e=vw \in E(\Gamma)} [(d_{\Gamma}(v))^2 \times (d_{\Gamma}(w))^2].$$

Definition 1.3. *The multiplicative First and Second hyper F -indices [5] for a graph Γ are*

$$\begin{aligned} HF_1II(\Gamma) &= \prod_{e=vw \in E(\Gamma)} [(d_{\Gamma}(v))^2 + (d_{\Gamma}(w))^2]^2, \\ HF_2II(\Gamma) &= \prod_{e=vw \in E(\Gamma)} [(d_{\Gamma}(v))^2 \times (d_{\Gamma}(w))^2]^2. \end{aligned}$$

Definition 1.4. *Multiplicative Sum Conectivity and Multiplicative Product Conectivity F -indices [5], for a graph Γ are*

$$\begin{aligned} SFII(\Gamma) &= \prod_{e=vw \in E(\Gamma)} \frac{1}{\sqrt{(d_{\Gamma}(v))^2 + (d_{\Gamma}(w))^2}}, \\ PFII(\Gamma) &= \prod_{e=vw \in E(\Gamma)} \frac{1}{\sqrt{(d_{\Gamma}(v))^2 \times (d_{\Gamma}(w))^2}}. \end{aligned}$$

Definition 1.5. Again to the graph G , general multiplicative First and Second F -indices can be stated [5] like

$$F_1^k II(\Gamma) = \prod_{e=vw \in E(\Gamma)} [(d_\Gamma(v))^2 + (d_\Gamma(w))^2]^k,$$

$$F_2^k II(\Gamma) = \prod_{e=vw \in E(\Gamma)} [(d_\Gamma(v))^2 \times (d_\Gamma(w))^2]^k.$$

Definition 1.6. The first and second Gourava indices [10] for a graph Γ are stated like

$$GO_1(\Gamma) = \sum_{e=vw \in E(\Gamma)} [d_\Gamma(v) + d_\Gamma(w) + d_\Gamma(v) \times d_\Gamma(w)],$$

$$GO_2(\Gamma) = \sum_{e=vw \in E(\Gamma)} [(d_\Gamma(v) + d_\Gamma(w)) (d_\Gamma(v) \times d_\Gamma(w))].$$

Definition 1.7. The first and second hyper Gourava indices [6] for a graph Γ are stated as

$$HGO_1(\Gamma) = \sum_{e=vw \in E(\Gamma)} [d_\Gamma(v) + d_\Gamma(w) + d_\Gamma(v) \times d_\Gamma(w)]^2,$$

$$HGO_2(\Gamma) = \sum_{e=vw \in E(\Gamma)} [(d_\Gamma(v) + d_\Gamma(w)) (d_\Gamma(v) \times d_\Gamma(w))]^2.$$

Definition 1.8. The sum and product connectivity Gourava indices [7, 8] for a graph Γ are stated as

$$SGO(\Gamma) = \sum_{e=vw \in E(\Gamma)} \frac{1}{\sqrt{d_\Gamma(v) + d_\Gamma(w) + d_\Gamma(v) \times d_\Gamma(w)}},$$

$$PGO(\Gamma) = \sum_{e=vw \in E(\Gamma)} \frac{1}{\sqrt{d_\Gamma(v) + d_\Gamma(w) (d_\Gamma(v) \times d_\Gamma(w))}}.$$

Definition 1.9. For a graph Γ , the general first and second hyper Gourava indices [9] are stated as

$$GO_1^k(\Gamma) = \sum_{e=vw \in E(\Gamma)} [d_\Gamma(v) + d_\Gamma(w) + d_\Gamma(v) \times d_\Gamma(w)]^k,$$

$$GO_2^k(\Gamma) = \sum_{e=vw \in E(\Gamma)} [(d_\Gamma(v) + d_\Gamma(w)) (d_\Gamma(v) \times d_\Gamma(w))]^k.$$

In section 2 and 3 main results obtained are highlighted with detail proofs and the calculations of topological indices on Jahangir graph and Banana tree graph.

2. OUTCOMES FOR LINE GRAPH OF SUBDIVISION OF THE JAHANGIR GRAPH

Jahangir graph $J_{q,p}$ composed of a cycle C_{qp} along with one supplementary vertex that is adjacent to p vertices of C_{qp} at interval q to each other on C_{qp} . The subdivision of Jahangir graph has additional vertex between each pair of vertices. The subdivision of $J_{q,p}$ as order $p(2q + 1)$ and size $2p(q + 1)$ as shown in the Figure. 1(b).

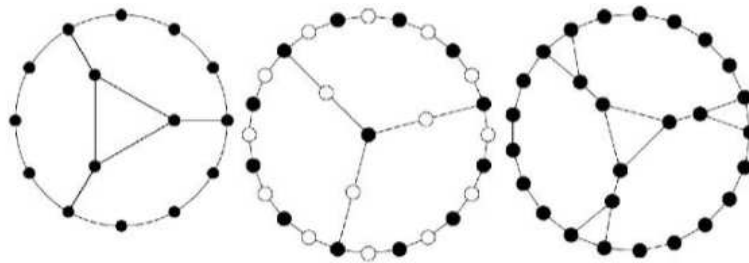


FIGURE 1. a) The Jahangir graph b) subdivision of Jahangir Graph
c) line-graph of subdivision of Jhangir graph $J_{4,3}$.

Lemma 2.1. Consider the Figure1(c), the line graph of subdivision of the Jahangir graph $L(S(J_{q,p}))$ for $q = 4$ and $p = 3$. The order of $L(S(J_{q,p}))$ is $2p(q + 1) + 1$ out of which $2p(q - 1)$ vertices with degree 2, $3p$ vertices with degree 3 and p vertices with degree p . The size of $|L(S(J_{q,p}))|$ is $\frac{p^2 + 4pq + 5p}{2}$. The edge partition of $L(S(J_{q,p}))$ into the edges of the type (d_u, d_v) , where uv is an edge of $L(S(J_{q,p}))$. Then

- (1) The total number of edges with degree $(2, 2)$ are $p(2q - 3)$, represented as E_1 .
- (2) The total number of edges with degree $(2, 3)$ are $2p$, represented as E_2 .
- (3) The total number of edges with degree $(3, 3)$ are $3p$, represented as E_3 .
- (4) The total number of edges with degree $(3, p)$ are p , represented as E_4 .
- (5) The total number of edges with degree (p, p) are $\frac{p(p-1)}{2}$, represented as E_5 .

Theorem 2.1. Consider Γ be the line graph of subdivision of the Jahangir graph $L(S(J_{q,p}))$, Then

$$F_1^K II(\Gamma) = 8^{k(p(2q-3))} \times 13^{k(2p)} \times 18^{k(3p)} \times (9 + p^2)^{k(p)} \times (2p^2)^{k(\frac{p(p-1)}{2})}.$$

Proof. Using the graph representation studies and observations, it is easy to define the general multiplicative first F -index for a graph as

$$\begin{aligned} F_1^k II(\Gamma) &= \prod_{e=vw \in E(\Gamma)} [(d_\Gamma(v))^2 + (d_\Gamma(w))^2]^k \\ &= \prod_{vw \in E_{2,2}} [(2)^2 + (2)^2]^k \times \prod_{vw \in E_{2,3}} [(2)^2 + (3)^2]^k \times \prod_{vw \in E_{3,3}} [(3)^2 + (3)^2]^k \\ &\quad \times \prod_{vw \in E_{3,p}} [(3)^2 + (p)^2]^k \times \prod_{vw \in E_{p,p}} [(p)^2 + (p)^2]^k. \\ F_1^K II(\Gamma) &= 8^{k(p(2q-3))} \times 13^{k(2p)} \times 18^{k(3p)} \times (9 + p^2)^{k(p)} \times (2p^2)^{k(\frac{p(p-1)}{2})}. \end{aligned}$$

□

We get the following results by using Theorem 2.1.

Corollary 2.1. *The multiplicative first F - index For a graph Γ is*

$$F_1 II(\Gamma) = (8)^{p(2q-3)} \times (13)^{2p} \times (18)^{3p} \times (9 + p^2)^p \times (2p^2)^{\frac{p(p-1)}{2}}.$$

Proof. Put $k = 1$ in Theorem 2.1, we gain the required results. □

Corollary 2.2. *For a graph Γ , the Multiplicative First hyper F - index is*

$$HF_1 II(\Gamma) = (8)^{2p(2q-3)} \times (13)^{4p} \times (18)^{6p} \times (9 + p^2)^{2p} \times (2p^2)^{p(p-1)}.$$

Proof. Put $k = 2$ in Theorem 2.1, we gain the desired results. □

Corollary 2.3. *For a graph Γ , the Multiplicative Sum Connectivity F - index is*

$$SF_1 II(\Gamma) = \left(\frac{1}{\sqrt{8}}\right)^{2pq-3p} \times \left(\frac{1}{\sqrt{13}}\right)^{2p} \times \left(\frac{1}{\sqrt{18}}\right)^{3p} \times \left(\frac{1}{\sqrt{(9+p^2)}}\right)^p \times \left(\frac{1}{\sqrt{2p^2}}\right)^{\frac{p(p-1)}{2}}.$$

Proof. Put $k = \frac{-1}{2}$ in Theorem 2.1, we get the desired result. □

Theorem 2.2. *For a graph Γ , the general multiplicative second F - index is*

$$F_2^K II(\Gamma) = (16)^{k(p(2q-3))} \times (36)^{k(2p)} \times (81)^{k(3p)} \times (9p^2)^{k(p)} \times (p)^{2kp(p-1)}.$$

Proof.

$$F_2^k II(\Gamma) = \prod_{e=vw \in E(\Gamma)} [(d_\Gamma(v))^2 \times (d_\Gamma(w))^2]^k.$$

$$\begin{aligned}
&= \prod_{vw \in E_{2,2}} [(2)^2(2)^2]^k \times \prod_{vw \in E_{2,3}} [(2)^2(3)^2]^k \times \prod_{vw \in E_{3,3}} [(3)^2(3)^2]^k \\
&\quad \times \prod_{vw \in E_{3,p}} [(3)^2(p)^2]^k \times \prod_{vw \in E_{p,p}} [(p)^2(p)^2]^k. \\
&= (16)^{k(p(2q-3))} \times (36)^{k(2p)} \times (81)^{k(3p)} \times (9p^2)^{k(p)} \times (p)^{2kp(p-1)}.
\end{aligned}$$

□

The following results are obtained by using Theorem 2.2.

Corollary 2.4. For a graph Γ , the Multiplicative Second F - index is

$$F_2II(\Gamma) = (4)^{2p(2q-3)} \times (6)^{4p} \times (9)^{6p} \times (3p)^{2p} \times (p)^{2p(p-1)}.$$

Proof. Put $k = 1$ in Theorem 2.2, we get the required result. □

Corollary 2.5. For a graph Γ , the Multiplicative Second Hyper F - index is

$$HF_2II(\Gamma) = (4)^{4p(2q-3)} \times (6)^{8p} \times (9)^{12p} \times (3p)^{4p} \times (p)^{4p(p-1)}.$$

Proof. Put $k = 2$ in Theorem 2.2, we obtain the required result. □

Corollary 2.6. For a graph Γ , the Multiplicative Product Connectivity F - index is

$$PFII(\Gamma) = \left(\frac{1}{4}\right)^{p(2q-3)} \times \left(\frac{1}{6}\right)^{2p} \times \left(\frac{1}{9}\right)^{3p} \times \left(\frac{1}{3p}\right)^p \times \left(\frac{1}{p}\right)^{p(p-1)}.$$

Proof. Put $k = \frac{-1}{2}$ in Theorem 2.2, we acquire the correct result. □

Theorem 2.3. For a graph Γ , the general first Gourava index is

$$p(2q-3)(8)^k + (2p)(11)^k + (3p)(15)^k + p(4p+3)^k + \frac{p(p-1)}{2}(p^2+2p)^k.$$

Proof.

$$\begin{aligned}
GO_1^k(\Gamma) &= \sum_{vw \in (G)} [d_\Gamma(v) + d_\Gamma(w) + d_\Gamma(v) \times d_\Gamma(w)]^k \\
&= p(2q-3)(8)^k + (2p)(11)^k + (3p)(15)^k + p(4p+3)^k \\
&\quad + \frac{p(p-1)}{2}(p^2+2p)^k.
\end{aligned}$$

□

Corollary 2.7. For a graph Γ , the first Gourava index is

$$GO_1(\Gamma) = \frac{1}{2}(p^4 + p^3 + 6p^2 + 32pq + 92p).$$

Proof. put $k=1$ in Theorem 2.3, we get the required result. $\square$

Corollary 2.8. *The first hyper Gourava index of a graph Γ is*

$$HGO_1(\Gamma) = \frac{1}{2}(p^6 + 3p^5 + 28p^3 + 48p^2 + 256pq + 1268p).$$

Proof. put $k=2$ in Theorem 2.3, we gain the required result. $\square$

Corollary 2.9. *For a graph Γ , the sum connectivity Gourava index is*

$$\begin{aligned} SGO_1(G) = & p(2q-3) \left(\frac{1}{\sqrt{8}} \right) + 2p \left(\frac{1}{\sqrt{11}} \right) + 3p \left(\frac{1}{\sqrt{15}} \right) + p \left(\frac{1}{\sqrt{4p+3}} \right) \\ & + \frac{p(p-1)}{2} \left(\frac{1}{\sqrt{p^2+2p}} \right). \end{aligned}$$

Proof. put $k = \frac{-1}{2}$ in Theorem 2.3, we obtain the required result. $\square$

Theorem 2.4. *The general second Gourava index of a graph Γ is*

$$p(2q-3)(16)^k + 2p(30)^k + 3p(54)^k + p(3p^2+9p)^k + \frac{p(p-1)}{2}(2p^3)^k.$$

Proof.

$$\begin{aligned} GO_2^k(\Gamma) &= \sum_{vw \in (G)} [d_\Gamma(v) + d_\Gamma(w) (d_\Gamma(v) \times d_\Gamma(w))]^k \\ &= p(2q-3)(16)^k + 2p(30)^k + 3p(54)^k + p(3p^2+9p)^k \\ &\quad + \frac{p(p-1)}{2}(2p^3)^k. \end{aligned}$$

$\square$

Corollary 2.10. *For a graph Γ , the second Gourava index is*

$$GO_1(\Gamma) = 3p^3 + 10p^2 + 32pq + 173p.$$

Proof. put $k=1$ in Theorem 2.4, we get the required result. $\square$

Corollary 2.11. *The second hyper Gourava index of a graph Γ is*

$$HGO_1(\Gamma) = 2p^8 - 2p^7 + 9p^5 + 54p^4 + 81p^3 + 512pq + 9780p.$$

Proof. put $k=2$ in Theorem 2.4, we gain the required result. $\square$

Corollary 2.12. For a graph Γ , the sum connectivity Gourava index is

$$\begin{aligned} SGO_1(G) = & p(2q - 3) \left(\frac{1}{\sqrt{16}} \right) + 2p \left(\frac{1}{\sqrt{30}} \right) + 3p \left(\frac{1}{\sqrt{54}} \right) + p \left(\frac{1}{\sqrt{9p + 3p^2}} \right) \\ & + \frac{p(p - 1)}{2} \left(\frac{1}{\sqrt{2p^3}} \right). \end{aligned}$$

Proof. put $k = \frac{-1}{2}$ in Theorem 2.4, we get the required result. $\square$

3. OUTCOMES FOR LINE GRAPH OF SUBDIVISION OF BANANA TREE GRAPH

The Banana tree graph $B_{q,p}$ is derived by joining one leaf of each of q copies of an p -star graph with a one root vertex which is different from all the stars. The subdivision of $B_{q,p}$ consist of an additional vertex between each pair of vertices, has order $2pq + 1$ and size $2pq$ as shown in Fig.3.

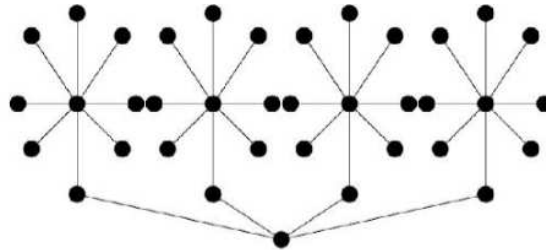


FIGURE 2. The Banana Tree grph $J_{7,4}$

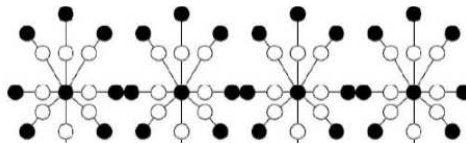
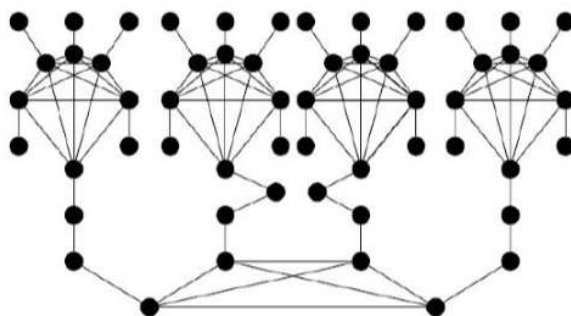


FIGURE 3. The subdivision of Banana Tree grph $J_{7,4}$

Lemma 3.1. Consider the Figure 4, the line graph of subdivision of the Banana tree graph $L(S(B_{q,p}))$ for $q = 7$ and $p = 4$. The order of $L(S(B_{q,p}))$ is $2pq$ out of which $p(q - 2)$ vertices with degree 1, $p(q - 1)$ vertices with degree $q - 1$, p vertices with


 FIGURE 4. The line graph of subdivision of Banana Tree graph $J_{7,4}$

degree p and $2p$ vertices with degree 2. The size of $L(S(B_{q,p}))$ is $\frac{p^2 + 3p + pq^2 - pq}{2}$. The edge partition of $L(S(B_{q,p}))$ into the edges of type (d_u, d_v) , where uv is an edge of $L(S(B_{q,p}))$. Then

- (1) The total number of edges with degree $(1, q - 1)$ are $p(q - 2)$, represented as E_1 .
- (2) The total number of edges with degree $(2, q - 1)$ are p , represented as E_2 .
- (3) The total number of edges with degree $(2, p)$ are p , represented as E_3 .
- (4) The total number of edges with degree $(2, 2)$ are p , represented as E_4 .
- (5) The total number of edges with degree (p, p) are $\frac{p(p-1)}{2}$, represented as E_5 .
- (6) The total number of edges with degree $(q - 1, q - 1)$ are $\frac{p(q-1)(q-2)}{2}$, represented as E_6 .

Theorem 3.1. Consider Γ be the line graph of subdivision of the Banana tree graph $L(S(B_{q,p}))$. Then

$$F_1^K II(\Gamma) = (q^2 - 2q)^{k(p(q-2))} \times (q^2 - 2q + 5)^{k(p)} \times (p^2 + 4)^{k(p)} \times (8)^{k(p)} \\ \times (2p^2)^{k(\frac{p(p-1)}{2})} \times (2q^2 - 4q + 2)^{k(\frac{p(q-1)(q-2)}{2})}.$$

Proof. Using the graph representation studies and observations, it is easy to define the general multiplicative first F - index for a graph as

$$F_1^k II(\Gamma) = \prod_{e=vw \in E(\Gamma)} [(d_\Gamma(v))^2 + (d_\Gamma(w))^2]^k.$$

$$\begin{aligned}
&= \prod_{vw \in E_{1,q-1}} [(1)^2 + (q-1)^2]^k \times \prod_{vw \in E_{2,q-1}} [(2)^2 + (q-1)^2]^k \\
&\times \prod_{vw \in E_{2,p}} [(2)^2 + (p)^2]^k \times \prod_{vw \in E_{2,2}} [(2)^2 + (2)^2]^k \\
&\times \prod_{vw \in E_{p,p}} [(p)^2 + (p)^2]^k \times \prod_{vw \in E_{q-1,q-1}} [(q-1)^2 + (q-1)^2]^k. \\
F_1^K II(\Gamma) &= (q^2 - 2q)^{k(p(q-2))} \times (q^2 - 2q + 5)^{k(p)} \times (p^2 + 4)^{k(p)} \times (8)^{k(p)} \\
&\times (2p^2)^{k(\frac{p(p-1)}{2})} \times (2q^2 - 4q + 2)^{k(\frac{p(q-1)(q-2)}{2})}.
\end{aligned}$$

□

We get the below results by using Theorem 3.1.

Corollary 3.1. *The multiplicative first F - index for a graph G is*

$$\begin{aligned}
F_1 II(\Gamma) &= (q^2 - 2q)^{p(q-2)} \times (q^2 - 2q + 5)^p \times (p^2 + 4)^p \times (8)^p \\
&\times (2p^2)^{\frac{p(p-1)}{2}} \times (2q^2 - 4q + 2)^{\frac{p(q-1)(q-2)}{2}}.
\end{aligned}$$

Proof. Put $k = 1$ in Theorem 3.1, we gain the desired results. □

Corollary 3.2. *For a graph Γ , the Multiplicative First hyper F - index is*

$$\begin{aligned}
HF_1 II(\Gamma) &= (q^2 - 2q)^{2p(q-2)} \times (q^2 - 2q + 5)^{2p} \times (p^2 + 4)^{2p} \times (8)^{2p} \\
&\times (2p^2)^{p(p-1)} \times (2q^2 - 4q + 2)^{p(q-1)(q-2)}.
\end{aligned}$$

Proof. Put $k = 2$ in Theorem 3.1, we gain the desired results. □

Corollary 3.3. *The Multiplicative Sum Connectivity F - index for a graph Γ is*

$$\begin{aligned}
SFII(\Gamma) &= \left(\frac{1}{\sqrt{q^2 - 2q}} \right)^{p(q-2)} \times \left(\frac{1}{\sqrt{q^2 - 2q + 5}} \right)^{2p} \times \left(\frac{1}{\sqrt{p^2 + 4}} \right)^p \\
&\times \left(\frac{1}{\sqrt{2}} \right)^{3p} \times \left(\frac{1}{\sqrt{2p^2}} \right)^{\frac{p(p-1)}{2}} \times \left(\frac{1}{\sqrt{2q^2 - 4q + 2}} \right)^{\frac{p(q-1)(q-2)}{2}}.
\end{aligned}$$

Proof. Put $k = \frac{-1}{2}$ in Theorem 3.1, we get the desired result. □

Theorem 3.2. *For a graph Γ , the general multiplicative second F - index is*

$$\begin{aligned}
F_2^K II(\Gamma) &= ((q-1)^2)^{k(p(q-2))} \times (4(q-1)^2)^{k(p)} \times (4p^2)^{k(p)} \times (16)^{k(p)} \\
&\times (p^4)^{k(\frac{p(p-1)}{2})} \times ((q-1)^4)^{k(\frac{p(q-1)(q-2)}{2})}.
\end{aligned}$$

Proof.

$$\begin{aligned}
 F_2^k II(\Gamma) &= \prod_{e=vw \in E(\Gamma)} [(d_\Gamma(v))^2 \times (d_\Gamma(w))^2]^k \\
 &= \prod_{vw \in E_{1,q-1}} [(1)^2(q-1)^2]^k \times \prod_{vw \in E_{2,q-1}} [(2)^2(q-1)^2]^k \\
 &\quad \times \prod_{vw \in E_{2,p}} [(2)^2(p)^2]^k \\
 &\quad \times \prod_{vw \in E_{2,2}} [(2)^2(2)^2]^k \times \prod_{vw \in E_{p,p}} [(p)^2(p)^2]^k \times \prod_{vw \in E_{q-1,q-1}} [(q-1)^2(q-1)^2]^k. \\
 &= ((q-1)^2)^{k(p(q-2))} \times (4(q-1)^2)^{k(p)} \times (4p^2)^{k(p)} \times (16)^{k(p)} \times (p^4)^{k(\frac{p(p-1)}{2})} \\
 &\quad \times ((q-1)^4)^{k(\frac{p(q-1)(q-2)}{2})}.
 \end{aligned}$$

□

The following results are obtained by using Theorem 3.2.

Corollary 3.4. *For a graph Γ , the Multiplicative Second F -index is*

$$\begin{aligned}
 F_2 II(\Gamma) &= (q-1)^{2p(q-2)} \times (2(q-1))^{2p} \times (2p)^{2p} \\
 &\quad \times (16)^p \times (p)^{2p(p-1)} \times (q-1)^{2p(q-1)(q-2)}.
 \end{aligned}$$

Proof. Put $k = 1$ in Theorem 3.2, we gain the desired result. □

Corollary 3.5. *For a graph Γ , the Multiplicative Second Hyper F -index is*

$$\begin{aligned}
 HF_2 II(\Gamma) &= (q-1)^{4p(q-2)} \times (2(q-1))^{4p} \times (2p)^{4p} \\
 &\quad \times (16)^{2p} \times (p)^{4p(p-1)} \times (q-1)^{4p(q-1)(q-2)}.
 \end{aligned}$$

Proof. Put $k = 2$ in Theorem 3.2, we gain the required result. □

Corollary 3.6. *The Multiplicative Product Connectivity F -index for a graph Γ is*

$$\begin{aligned}
 PFII(\Gamma) &= \left(\frac{1}{q-1}\right)^{p(q-2)} \times \left(\frac{1}{2(q-1)}\right)^p \times \left(\frac{1}{2p}\right)^p \\
 &\quad \times \left(\frac{1}{4}\right)^p \times \left(\frac{1}{p}\right)^{p(p-1)} \times \left(\frac{1}{q-1}\right)^{p(q-1)(q-2)}.
 \end{aligned}$$

Proof. Put $k = \frac{-1}{2}$ in Theorem 3.2, we obtain the desired result. □

Theorem 3.3. For a graph Γ , the general first Gourava index is

$$p(q-2)((2q+1))^k + p(3q-1)^k + p(3q+2)^k + p(8)^k \\ + \frac{p(p-1)}{2}(p^2+2p)^k + \frac{p(q-1)(q-2)}{2}(q^2-1)^k.$$

Proof.

$$GO_1^k(\Gamma) = \sum_{vw \in (G)} [d_\Gamma(v) + d_\Gamma(w) + d_\Gamma(v) \times d_\Gamma(w)]^k \\ = p(q-2)(2q+1)^k + p(3q-1)^k + p(3q+2)^k + p(8)^k \\ + \frac{p(p-1)}{2}(p^2+2p)^k + \frac{p(q-1)(q-2)}{2}(q^2-1)^k.$$

□

Corollary 3.7. For a graph Γ , the first Gourava index is

$$GO_1(\Gamma) = p(q-2)((2q+1)) + p(3q-1) + p(3q+2) + p(8) \\ + \frac{p(p-1)}{2}(p^2+2p) + \frac{p(q-1)(q-2)}{2}(q^2-1).$$

Proof. put $k=1$ in Theorem 3.3, we gain the required result.

□

Corollary 3.8. For a graph Γ , the first hyper Gourava index is

$$HGO_1(\Gamma) = p(q-2)(4q^2+4q+1) + p(18q^2+6q+69) \\ + \frac{p(p-1)}{2}(p^4+4p^3+4p^2) + \frac{p(q-1)(q-2)}{2}(q^4-2q^2+1).$$

Proof. put $k=2$ in Theorem 3.3, we gain the required result.

□

Corollary 3.9. For a graph Γ , the sum connectivity Gourava index is

$$SGO_1(\Gamma) = p(q-2) \left(\frac{1}{\sqrt{2q+1}} \right) + p \left(\frac{1}{\sqrt{3q-1}} \right) + p \left(\frac{1}{\sqrt{3q+2}} \right) \\ + p \left(\frac{1}{\sqrt{8}} \right) + \frac{p(p-1)}{2} \left(\frac{1}{\sqrt{p^2+2p}} \right) + \frac{p(q-1)(q-2)}{2} \left(\frac{1}{\sqrt{q^2-1}} \right).$$

Proof. put $k = \frac{-1}{2}$ in Theorem 3.3, we get the required result.

□

Theorem 3.4. *The general second Gourava index of a graph Γ is*

$$p(q-2)(q(q-1))^k + p(2(q-1)(q+1))^k + p(2q(q+2))^k \\ + p(16)^k + \frac{p(p-1)}{2}(2p^3)^k + \frac{p(q-1)(q-2)}{2}(2(q-1)^3)^k.$$

Proof.

$$GO_2^k(\Gamma) = \sum_{vw \in (G)} [d_\Gamma(v) + d_\Gamma(w) (d_\Gamma(v) \times d_\Gamma(w))]^k \\ = p(q-2)(q(q-1))^k + p(2(q-1)(q+1))^k + p(2q(q+2))^k \\ + p(16)^k + \frac{p(p-1)}{2}(2p^3)^k + \frac{p(q-1)(q-2)}{2}(2(q-1)^3)^k.$$

□

Corollary 3.10. *For a graph Γ , the second Gourava index is*

$$GO_1(\Gamma) = pq(q-1)(q-2) + 2p(q-1)(q+1) + 2pq(q+2) \\ + 16p + p^4(p-1) + p(q-1)^4(q-2).$$

Proof. put $k=1$ in Theorem 3.4, we obtain the desired result.

□

Corollary 3.11. *For a graph Γ , the second hyper Gourava index is*

$$HGO_1(\Gamma) = p(q-2)q^2(q-1)^2 + 4p(q-1)^2(q+1)^2 + 4pq^2(q+2)^2 \\ + 256p + 2p^7(p-1) + 2p(q-1)^7(q-2).$$

Proof. put $k=2$ in Theorem 3.4, we gain the required result.

□

Corollary 3.12. *For a graph Γ , the sum connectivity Gourava index is*

$$SGO_1(\Gamma) = p(q-2) \left(\frac{1}{\sqrt{q(q-1)}} \right) + p \left(\frac{1}{\sqrt{2(q-1)(q+1)}} \right) \\ + p \left(\frac{1}{\sqrt{2q(q+2)}} \right) + p \left(\frac{1}{\sqrt{16}} \right) \\ + \frac{p(p-1)}{2} \left(\frac{1}{\sqrt{2p^3}} \right) + \frac{p(q-1)(q-2)}{2} \left(\frac{1}{\sqrt{2(q-1)^3}} \right).$$

Proof. put $k = \frac{-1}{2}$ in Theorem 3.4, we get the desired result.

□

4. CONCLUSION

The objective of this work is to use various topological indices on different plane graphs. Few indices are defined and represented as expressions to link the graphs with physical characteristics. The results obtained here helps the experts to decide the characteristics of line graph of subdivision of various graphs. Using topological indices concept to determine physical properties of the jahangir graph, the banana tree graph and Firecracker graph has many advantages. The results obtained play a very important role in industries and pharmacy.

REFERENCES

- [1] M. BHANUMATHI, K. EASU JULIA RANI: *The multiplicative F -index*, Int. J. Recent. Trend. Life. Sci. Math. **4**(9), (2017), 1–8.
- [2] S. GHOBADI, M. GHORBANINEJAD: *On F -polynomial, Multiple and hyper F -index of sum molecular graphs*, Bull. Math. Sci. Appl., **20** (2018), 36–43.
- [3] I. GUTMAN, Z. TOMOVIC: *On the application of line graphs in quantitative structure-property studies*, J. Serb. Chem. Soc., **65**(8) (2000), 577–580.
- [4] Y. HUO, JB. LIU, Z. ZAHID, S. ZAFAR, M. R. FARAHANI, M. F. NADEEM: *On certain topological indices of the line graph of $CNCK[n]$ nanocones*, J. Comput. Theor. Nanosci., **5** (2000), 4318–4322.
- [5] V. R. KULLI: *On multiplicative F -indices and multiplicative connectivity F -indices of chemical networks*, Int. J. Curren. Res. Sci. Tech., **5**(2) (2019), 1–10 .
- [6] V. R. KULLI: *On Hyper Gourava Indices and Co-indices*, Int. J. Math. Archiv., **8**(12) (2017), 116–120.
- [7] V. R. KULLI: *On the sum connectivity Gourava Index*, Int. J. Math. Archiv., **8**(6) (2017), 235–242.
- [8] V. R. KULLI: *On the product connectivity Gourava Index*, J. Comp. Math. Sci., **8**(6) (2017), 211–217.
- [9] V. R. KULLI: *Leap Gourava Indices of certain windmill graphs*, Int. J. Math. Archiv., **10**(11) (2019), 7–14. .
- [10] V. R. KULLI: *The Gourava Indices and Coindices of Graphs*, Annal. Pure. Appl. Math., **14** (2017), 33–38.
- [11] M. F. NADEEM, Z. SOHIL, Z. ZOHIB: *On certain topological indices of the line graphs of subdivision graphs*, Appl. Math. Comput., **271** (2015), 790–794.
- [12] P. S. RANJINI, V. LOKESHA, M. A. RAJAN: *On Wiener Polynomial of the Subdivision Graphs*, Chine. J. Engg. Math., **3** (2011), 411–418.
- [13] X. LI, F. ASIF, Z. ZAHID, S. ZAFAR: *On topological properties of Plane graphs by using line operator on their subdivisions*, Int. J. Appl. Math., **32**(6) (2019), 1023–1036.

DEPARTMENT OF MATHEMATICS
JAIN INSTITUTE OF TECHNOLOGY
DAVANAGERE-577003, KARNATAKA, INDIA
E-mail address: mcshanmukha@gmail.com

DEPARTMENT OF MATHEMATICS
BAPUJI INSTITUTE OF ENGINEERING AND TECHNOLOGY
DAVANAGERE-577004, KARNATAKA, INDI.
E-mail address: kn.anil5340@gmail.com

DEPARTMENT OF MATHEMATICS
BAPUJI INSTITUTE OF ENGINEERING AND TECHNOLOGY
DAVANAGERE-577004, KARNATAKA, INDIA
E-mail address: dr.nsbasavaraj@gmail.com

DEPARTMENT OF MATHEMATICS
UBDT COLLEGE OF ENGINEERING
DAVANAGERE-577004, KARNATAKA, INDIA
E-mail address: niranjankm64@gmail.com