ADV MATH SCI JOURNAL
Advances in Mathematics: Scientific Journal **9** (2020), no.4, 2137–2145
ISSN: 1857-8365 (printed); 1857-8438 (electronic)
https://doi.org/10.37418/amsj.9.4.73 Spec. Issue on NCFCTA-2020

HARMONIC MEAN LABELING OF H-SUPER SUBDIVISION OF PATH GRAPHS

S. MEENA[1] AND M. SIVASAKTHI

ABSTRACT. A graph G with p vertices and q edges is called a harmonic mean graph if it is possible to label the vertices $x \in V$ with distinct labels $f(x)$ from $\{1, 2, ...q + 1\}$ in such a way that each edge $e = uv$ is labeled with

$$f(uv) = \left\lceil \frac{2f(u)f(v)}{f(u) + f(v)} \right\rceil \quad \text{(or)} \quad \left\lfloor \frac{2f(u)f(v)}{f(u) + f(v)} \right\rfloor,$$

then the edge labels are distinct. In this case f is called Harmonic mean labeling of G. In this paper we prove that some families of graphs such as H- super subdivision of path $HSS(P_n)$, $HSS(P_n) \odot K_1$, $HSS(P_n) \odot \overline{K_2}$, $HSS(P_n) \odot K_2$ are harmonic mean graphs.

1. INTRODUCTION

Let $G = (V, E)$ be a (p, q) graph with $p = |V(G)|$ vertices and $q = |E(G)|$ edges, where $V(G)$ and $E(G)$ respectively denote the vertex set and edge set of the graph G. In this paper, we consider the graphs which are simple, finite and undirected. For graph theoretic terminology and notations we refer to S.Arumugam [1].

The concept of graph labeling was introduced by Rosa in 1967. A detailed survey of graph labeling is available in Gallian [3]. The concept of Harmonic mean labeling of graph was introduced by S. Somasundaram, R. Ponraj and S.S.

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[1] *corresponding author*

2010 *Mathematics Subject Classification.* 05078.

Key words and phrases. Harmonic mean graph, H-super subdivision of path $HSS(P_n)$, $HSS(P_n) \odot K_1$, $HSS(P_n) \odot \overline{K_2}$, $HSS(P_n) \odot K_2$.

Sandhya and they investigated the existence of harmonic mean labeling of several family of graph. This was further studied by many authors. We have proved Harmonic mean labeling of subdivision graphs such as $P_n \odot K_1, P_n \odot \overline{K_2}$, H-graph, crown, $C_n \odot K_1, C_n \odot \overline{K_2}$, quadrilateral snake, Triangular snake and also proved Harmonic mean labeling of some graphs such as Triple triangular snake $T(T_n)$, Alternate Triple triangular snake $A[T(T_n)]$, Triple quadrilateral snake $T(Q_n)$, Alternate Triple quadrilateral snake $A[T(Q_n)]$, Twig graph $T(n)$, balloon triangular snake $T_n(C_m)$, and key graph $Ky(m, n)$.

Other valuable references are [4–7].

The following definitions are useful for the present investigation.

Definition 1.1. *[8] A Graph $G = (V, E)$ with p vertices and q edges is called a Harmonic mean graph if it is possible to label the vertices $v \in V$ with distinct labels $f(v)$ from $\{1, 2, ..., q+1\}$ in such a way that when each edge $e = uv$ is labeled with*

$$f(uv) = \left\lceil \frac{2f(u)f(v)}{f(u)+f(v)} \right\rceil \quad \textit{(or)} \quad \left\lfloor \frac{2f(u)f(v)}{f(u)+f(v)} \right\rfloor$$

then the resulting edge labels are distinct. In this case f is called Harmonic mean labeling of G.

Definition 1.2. *[2] Let G be a (p, q) graph. A graph obtained from G by replacing each edge e_i by a H-graph in such a way that the ends e_i are merged with a pendent vertex point in P_2 and a pendent vertex point in P_2' is called H-Super Subdivision of G and it is denoted by $HSS(G)$ where the H-graph is a tree on 6 vertices in which exactly two vertices of degree 3.*

Definition 1.3. *[2] The corona $G_1 \odot G_2$ of two graphs G_1 and G_2 is defined as the graph G obtained by taking one copy of G_1 (which has p_1 vertices) and p_1 copies of G_2 and then joining the i^{th} point of G_1, to every points in the i^{th} copy of G_2.*

In this paper we prove that H- super subdivision of path $HSS(P_n)$, $HSS(P_n) \odot K_1$, $HSS(P_n) \odot \overline{K_2}$, $HSS(P_n) \odot K_2$ are harmonic mean graphs.

2. Harmonic mean labeling of graphs

Theorem 2.1. *The H- super subdivision of path $HSS(P_n)$is a harmonic mean graphs.*

Proof. Let $HSS(P_n)$ be the H-super subdivision of path graph whose vertex set

$$V(G) = \{u_i, v_i, x_i, y_i / 1 \leq i \leq n\} \cup w_{n+1}$$

and the edge set

$$E(G) = \{u_i x_i, u_i v_i, v_i y_i, w_i u_i, v_i w_{i+1} / 1 \leq i \leq n\}.$$

Define a function $f : V \to \{1, 2, ..., q+1\}$ by

$$\begin{aligned}
f(u_i) &= 5i - 3 && \text{for} \quad 1 \leq i \leq n \\
f(v_i) &= 5i && \text{for} \quad 1 \leq i \leq n \\
f(x_1) &= 1 && \\
f(x_i) &= 5i - 2 && \text{for} \quad 2 \leq i \leq n \\
f(y_i) &= 5i - 1 && \text{for} \quad 1 \leq i \leq n \\
f(w_1) &= 3 && \\
f(w_i) &= 5i - 4 && \text{for} \quad 2 \leq i \leq n
\end{aligned}$$

Then the resulting edge labels are distinct. Thus f provides a harmonic mean labeling of graph G.

Hence G is a harmonic mean graph. □

Example 1. *A harmonic mean labeling of graph G obtained by H- super subdivision of path $HSS(P_4)$ are given in Figure 1.*

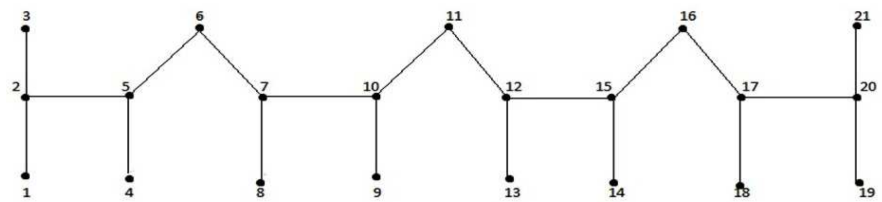

FIGURE 1

Theorem 2.2. *The H- super subdivision of path $HSS(P_n) \odot K_1$ is a harmonic mean graph.*

Proof. Let $HSS(P_n) \odot K_1$ be the H- super subdivision of path graph whose vertex set

$$V(G) = \{u_i, v_i, w_i, r_i, s_i, t_i, x_i, y_i, p_i, q_i / 1 \leq i \leq n\} \cup \{u_{n+1}, v_{n+1}, w_{n+1}, t_{n+1}\}$$

and the edge set

$$E(G) = \{v_iu_{i+1}, v_iw_i, r_iu_{i+1}, v_{i+1}, t_iw_i, x_iv_i, y_iu_{i+1}, x_ip_i, y_iq_i/1 \leq i \leq n\}$$
$$\cup \{w_iu_i/2 \leq i \leq n\} \cup \{u_1v_1, u_nv_n, u_nw_n, w_nt_n\}.$$

Define a labeling $f : V(G) \to \{1, 2, ..., q+1\}$ by

$$f(u_1) = 1$$
$$f(u_i) = 10i - 11 \quad \text{for} \quad 2 \leq i \leq n+1$$
$$f(v_i) = 10i - 6 \quad \text{for} \quad 1 \leq i \leq n$$
$$f(v_{n+1}) = 10n + 2$$
$$f(x_i) = 10i - 5 \quad \text{for} \quad 1 \leq i \leq n$$
$$f(y_i) = 10i - 2 \quad \text{for} \quad 1 \leq i \leq n$$
$$f(p_i) = 10i - 4 \quad \text{for} \quad 1 \leq i \leq n$$
$$f(q_i) = 10i - 3 \quad \text{for} \quad 1 \leq i \leq n$$
$$f(w_1) = 3$$
$$f(w_i) = 10i - 8 \quad \text{for} \quad 2 \leq i \leq n$$
$$f(w_{n+1}) = 10n$$
$$f(r_i) = 10i \quad \text{for} \quad 1 \leq i \leq n$$
$$f(s_i) = 10i + 3 \quad \text{for} \quad 1 \leq i \leq n$$
$$f(t_1) = 2$$
$$f(t_i) = 10i - 9 \quad \text{for} \quad 2 \leq i \leq n$$

Then the resulting edge labels are distinct. Thus f provides a harmonic mean labeling of graph G.

Hence G is a harmonic mean graph. □

Example 2. *A harmonic mean labeling of graph G obtained by H- super subdivision of path $HSS(P_5) \odot K_1$ are given in Figure 2.*

Theorem 2.3. *The H- super subdivision of path $HSS(P_n) \odot \overline{K_2}$ is a harmonic mean graph.*

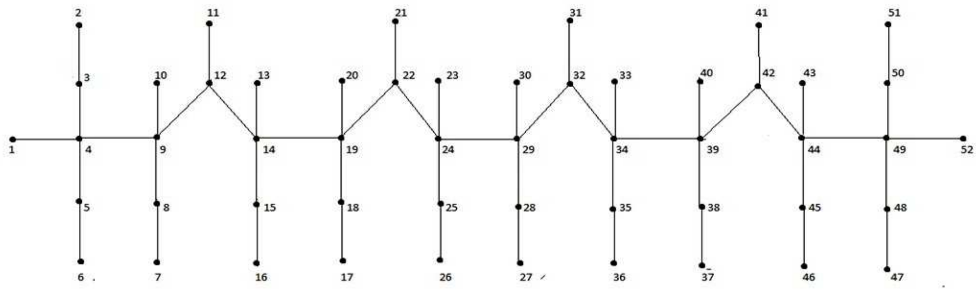

FIGURE 2

Proof. Let $HSS(P_n)\odot\overline{K_2}$ be the H- super subdivision of path graph whose vertex set

$$V(G) = \{u_i, v_i, w_i, p_i, q_i, r_i, s_i, x_i, y_i, z_i, t_ik_i, l_i, g_i, h_i/1 \leq i \leq n\} \cup \{w_{n+1}, x_{n+1}, y_{n+1}\}$$

and the edge set

$$E(G) = \{u_iv_i, u_iw_i, v_iw_{i+1}, u_ip_i, u_iq_i, w_ix_i, w_iy_i, v_ir_i, v_is_i, u_iz_i, v_it_i, z_ik_i, z_il_i, t_i g_i, t_ih_i/1 \leq i \leq n\} \cup \{v_nw_{n+1}, w_{n+1}x_{n+1}, w_{n+1}y_{n+1}\}.$$

Define a function $f : V(G) \rightarrow \{1, 2, ..., q+1\}$ by

$$f(u_i) = 15i - 9 \quad \text{for} \quad 1 \leq i \leq n$$
$$f(v_i) = 15i - 1 \quad \text{for} \quad 1 \leq i \leq n$$
$$f(p_1) = 1$$
$$f(p_i) = 15i - 11 \quad \text{for} \quad 2 \leq i \leq n$$
$$f(q_i) = 15i - 10 \quad \text{for} \quad 1 \leq i \leq n$$
$$f(r_i) = 15i - 2 \quad \text{for} \quad 1 \leq i \leq n$$
$$f(s_i) = 15i \quad \text{for} \quad 1 \leq i \leq n$$
$$f(w_1) = 4$$
$$f(w_i) = 15i - 14 \quad \text{for} \quad 2 \leq i \leq n$$
$$f(x_i) = 15i - 13 \quad \text{for} \quad 1 \leq i \leq n+1$$
$$f(y_i) = 15i - 12 \quad \text{for} \quad 1 \leq i \leq n+1$$
$$f(z_i) = 15i - 8 \quad \text{for} \quad 1 \leq i \leq n$$

$$f(t_i) = 15i - 4 \quad \text{for} \quad 1 \le i \le n$$
$$f(k_i) = 15i - 7 \quad \text{for} \quad 1 \le i \le n$$
$$f(l_i) = 15i - 6 \quad \text{for} \quad 1 \le i \le n$$
$$f(g_i) = 15i - 5 \quad \text{for} \quad 1 \le i \le n$$
$$f(h_i) = 15i - 3 \quad \text{for} \quad 1 \le i \le n$$

Then the resulting edge labels are distinct. Thus f provides a harmonic mean labeling of graph G.

Hence G is a harmonic mean graph. □

Example 3. *A harmonic mean labeling of graph G obtained by H- super subdivision of path $HSS(P_5) \odot \overline{K_2}$ are given in Figure 3.*

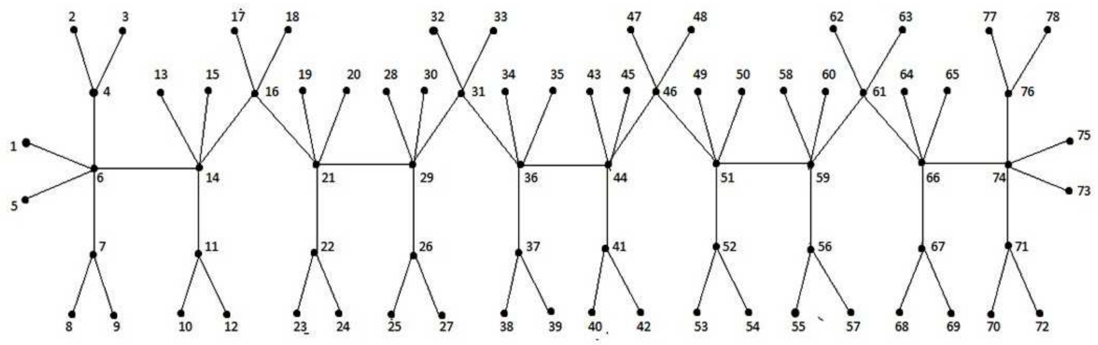

FIGURE 3

Theorem 2.4. *The H- super subdivision of path $HSS(P_n) \odot K_2$ is a harmonic mean graph.*

Proof. Let $HSS(P_n) \odot K_2$ be the H- super subdivision of path graph whose vertex set

$$V(G) = \{u_i, v_i, w_i, p_i, q_i, r_i, s_i, x_i, y_i, z_i, t_i k_i, l_i, g_i, h_i / 1 \le i \le n\}$$
$$\cup \{w_{n+1}, x_{n+1}, y_{n+1}\}$$

and the edge set

$$E(G) = \{u_i v_i, u_i w_i, v_i w_{i+1}, u_i p_i, u_i q_i, p_i q_i, v_i r_i, v_i s_i, r_i s_{i,} w_i x_i, w_i y_i, x_i y_i, u_i z_i,$$
$$v_i t_i, z_i k_i, z_i l_i, k_i l_i, t_i g_i, t_i h_i g_i h_i, /1 \le i \le n\} \cup \{v_n w_{n+1}, w_{n+1} x_{n+1}, w_{n+1} y_{n+1}\}$$

then the resultant graph is a harmonic mean labeling of H- super subdivision of path $HSS(P_n) \odot K_2$ graph.

Define a function $f : V(G) \to \{1, 2, ..., q+1\}$ by

$$
\begin{aligned}
&f(u_1) = 8\\
&f(u_i) = 20i - 13 \quad \text{for} \quad 2 \le i \le n\\
&f(v_i) = 20i - 1 \quad \text{for} \quad 1 \le i \le n\\
&f(w_1) = 3\\
&f(w_i) = 20i - 19 \quad \text{for} \quad 2 \le i \le n\\
&f(p_i) = 20i - 15 \quad \text{for} \quad 1 \le i \le n\\
&f(q_1) = 7\\
&f(q_i) = 20i - 14 \quad \text{for} \quad 2 \le i \le n\\
&f(r_i) = 20i - 3 \quad \text{for} \quad 1 \le i \le n\\
&f(s_i) = 20i - 2 \quad \text{for} \quad 1 \le i \le n\\
&f(x_1) = 1\\
&f(x_i) = 20i - 18 \quad \text{for} \quad 2 \le i \le n\\
&f(y_1) = 2\\
&f(y_i) = 20i - 17 \quad \text{for} \quad 2 \le i \le n\\
&f(z_i) = 20i - 11 \quad \text{for} \quad 1 \le i \le n\\
&f(t_i) = 20i - 5 \quad \text{for} \quad 1 \le i \le n\\
&f(k_i) = 20i - 10 \quad \text{for} \quad 1 \le i \le n\\
&f(l_i) = 20i - 9 \quad \text{for} \quad 1 \le i \le n\\
&f(g_i) = 20i - 8 \quad \text{for} \quad 1 \le i \le n\\
&f(h_i) = 20i - 6 \quad \text{for} \quad 1 \le i \le n
\end{aligned}
$$

Then the resulting edge labels are distinct. Thus f provides a harmonic mean labeling of graph G.

Hence G is a harmonic mean graph. □

Example 4. *A harmonic mean labeling of graph G obtained by H- super subdivision of path* $HSS(P_4) \odot K_2$ *are given in Figure 4.*

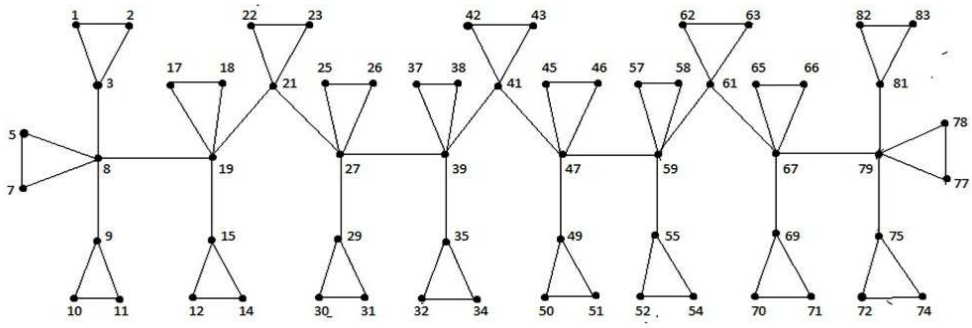

FIGURE 4

3. CONCLUSION

We have presented four new results on Harmonic mean labeling of certain classes of graphs like the H-super subdivision of path $HSS(P_n)$, $HSS(P_n) \odot K_1$, $HSS(P_n) \odot \overline{K_2}$, $HSS(P_n) \odot K_2$. Analogous work can be carried out for other families and in the context of different types of graph labeling techniques

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DEPARTMENT OF MATHEMATICS
GOVERNMENT ARTS COLLEGE
C. MUTLUR, CHIDAMBARAM-608102, TAMIL NADU, INDIA
E-mail address: meenasaravanan14@gmail.com

DEPARTMENT OF MATHEMATICS
KRISHNASAMY COLLEGE OF SCIENCE ARTS AND MANAGEMENT FOR WOMEN
CUDDALORE-607109, TAMIL NADU, INDIA
E-mail address: sivasakthi2203@gmail.com