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A VIEW ON PF DETOUR MEASURES IN PF GRAPHS

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ABSTRACT. A new distance measure in PF graph called PF detour measure distance is introduced which will lead us to a different notation of a connected PF graphs. These concepts are illustrated through examples and some detour distance measure are introduced in theorems, certain characteristic properties are presented in this paper.

1. INTRODUCTION

In our real life situations, all problems are in uncertainty. This phenomena of uncertainty if represented by the method of fuzzy set and its logic by Zadeh in 1965 [6]. Later, the fuzzy graph invented by Rosenfied [5]. In 1982, Atanassov [2] generalized the idea of fuzzy set and introduced a new set theory called the 'Intuitionistic Fuzzy Set'. In the Intuitionistic Fuzzy Set, each element has degrees of membership and non-membership whose sum lies in $[0, 1]$.

Instuitionistic Fuzzy graphs have been successfully applied in solving many problems connected with different areas [1, 3]. But there are many problem in real life cannot be represented adequately by IF graphs. So we need a more general graph theory to tackle such type of situations. The aim of this work is to develop the graph theoretic concept under Pythogorean fuzzy environment. In this paper, we will denote Pythogorean Fuzzy Graphs as $PF\ Graphs$.

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2. Preliminaries

Definition 2.1. *(Intuitionstic Fuzzy Graph) An Intuitionstic Fuzzy Graph (IFG) with underlying set V is a pair $G^* = \left(\tilde{P}, \tilde{Q}\right)$, where $\tilde{P} = \langle \zeta_{\tilde{P}}, \xi_{\tilde{P}} \rangle$ is an IFS in V with $0 \leq \zeta_{\tilde{P}}(m) + \xi_{\tilde{P}}(m) \leq 1$ $\forall m \in V$ and $\tilde{Q} = \left\langle \zeta_{\tilde{Q}}, \xi_{\tilde{P}} \right\rangle$ is an IFS in $E \subseteq V \times V$ such that*

$$\zeta_{\tilde{Q}(r,s)} \leq \min\left(\zeta_{\tilde{Q}(r)}, \zeta_{\tilde{Q}(s)}\right) \text{ and } \xi_{\tilde{Q}(r,s)} \leq \max\left(\xi_{\tilde{P}(r)}, \xi_{\tilde{P}(s)}\right)$$

and

$$0 \leq \zeta_{\tilde{Q}(r,s)} + \xi_{\tilde{Q}(r,s)} \leq 1, \forall (r,s) \in E.$$

Here $\tilde{P}$ and $\tilde{Q}$ represent the intuitionstic fv set of G^* and the intuitionstic fe set of G^*, respectively [4].

Definition 2.2. *(Pythogorean Fuzzy set) A Pythagorean fuzzy set P defined in $R = (r_1, r_2, \cdots, r_n)$ is given by*

$$P = \left\{ \langle x, \zeta_P(r), \xi_P(r) \rangle / r \in R \right\},$$

where

$$\zeta_P(r) : R \in [0,1], \xi_P(u) : R \in [0,1] \text{ and } 0 \leq \zeta_P^2(r) + \xi_P^2(r) \leq 1, \forall r \in R,$$

where the numbers $\zeta_P(r)$ and $\xi_P(r)$ represent the degree of membership and the degree of non-membership, respectively, of $r \in R$ in P [4].

Definition 2.3. *(Set operations on Pythogorean Fuzzy set) Let $P, Q \in PFS(R)$ given by*

$$P = \left\{ \langle r, \zeta_P(r), \xi_P(r) \rangle | r \in R \right\} \text{ and } Q = \left\{ \langle r, \zeta_Q(r), \xi_Q(r) \rangle | r \in R \right\}$$

(i) *$P \subseteq Q$ if and only if $\zeta_P(r) \leq \zeta_Q(r)$ and $\xi_P(r) \geq \xi_Q(r)$, $\forall r \in R$;*
(ii) *$P = Q$ iff $P \subseteq Q$ and $Q \subseteq P$;*
(iii) *$P^C = \left\{ \langle r, \xi_P(r), \zeta_P(r) \rangle | r \in R \right\}$;*
(iv) *$P \cup Q = \left\{ \langle r, \max\left(\zeta_P(r), \zeta_Q(r)\right), \min\left(\xi_P(r), \xi_Q(r)\right) \rangle | r \in R \right\}$;*
(v) *$P \cap Q = \left\{ \langle r, \min\left(\zeta_P(r), \zeta_Q(r)\right), \max\left(\xi_P(r), \xi_Q(r)\right) \rangle | r \in R \right\}$ [4].*

Definition 2.4. *(Pythogorean Fuzzy Graph) A PF graph if a pair, $G^{**} = (P, Q)$, where $P = \langle \zeta_P, \xi_P \rangle$ is a Pythagorean fs in V with $0 \leq \zeta_P^2(r) + \xi_P^2(r) \leq 1$ and $Q = \langle \zeta_Q, \xi_Q \rangle$ is a Pythagorean fs in $e \subseteq V \times V$ such that*

$$\zeta_Q(r,s) \leq \min\left(\zeta_P(r), \zeta_P(s)\right),$$

$$\xi_B(r,s) \le \max\left(\xi_P(r), \xi_P(s)\right)$$

and

$$0 \le \zeta_Q^2(r,s) + \xi_Q^2(r,s) \le 1, \forall (r,s) \in E.$$

Then, P and Q are the Pythagorean fv set and the Pythagorean fe set of G^{**} [4].

3. PF Detour Measure

Definition 3.1. *For connected PF graph G, the PF length of a path $r-s$ is defined by $\ell(P) = \langle \zeta_\ell(P), \xi_\ell(P) \rangle$, where*

$$\zeta_\ell(P) = \frac{1}{\sum \zeta_B\left(r_{i-1}, r_i\right)} \quad \text{and} \quad \xi_\ell(P) = \frac{1}{\sum \xi_B\left(r_{i-1}, r_i\right)}.$$

Definition 3.2. *For connected PF graph G, the PF max-detour distance between the vertices r and s and is denoted by $\Delta(r,s) = \langle \zeta_\Delta(r,s), \xi_\Delta(r,s) \rangle$, where $\zeta_\Delta(r,s) = \max\left[\zeta_\ell(r,s)\right]$ and $\xi_\Delta(r,s) = \min\left[\xi_\ell(r,s)\right]$.*

Definition 3.3. *For connected PF graph G, the PF min-detour distance between the vertices r and s and is denoted by $\delta(r,s) = \langle \zeta_\delta(r,s), \xi_\delta(r,s) \rangle$, where $\zeta_\delta(r,s) = \min\left[\zeta_\ell(P)\right]$ and $\xi_\delta(r,s) = \max\left[\xi_\ell(P)\right]$.*

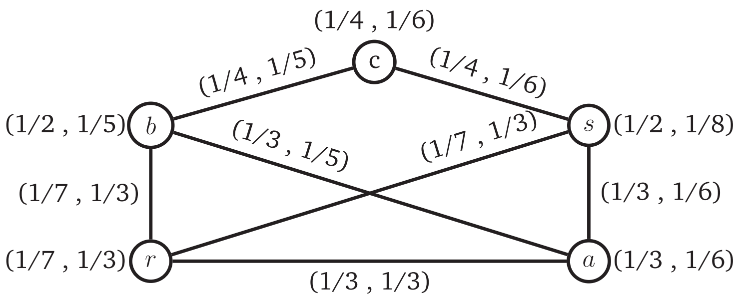

FIGURE 1

$$\Delta(r,s) \Rightarrow u - v \Rightarrow \langle 7, 3 \rangle$$

$$r - a - s \Rightarrow \left.\begin{array}{ll} 3+3 & = 6 \\ 3+6 & = 9 \end{array}\right\} \Rightarrow \langle 6, 9 \rangle$$

$$r - b - c - s \Rightarrow \left.\begin{array}{ll} 7+4+4 & = 15 \\ 3+5+6 & = 14 \end{array}\right\} \Rightarrow \langle 15, 14 \rangle$$

$$r - a - b - c - s \Rightarrow \left.\begin{array}{ll} 3+3+4+4 & = 14 \\ 3+5+5+6 & = 19 \end{array}\right\} \Rightarrow \langle 14, 19 \rangle$$

$$\Delta(r,s) = \langle 15, 3 \rangle$$

$$\delta(r,s) = \langle 6, 19 \rangle$$

Theorem 3.1. *For the PF max-detour distance Δ on a connected PF graph G, $(V(G), \Delta)$ in a metric space and $(V(G), \delta)$ is also metric space.*

Definition 3.4. *For connected PF graph G, the PF detour eccentricity is defined by $e_\Delta(s) = \langle \zeta_{e_\Delta}(s) \xi_{e_\Delta}(s) \rangle$ of a vertex $s \in V$ where $\zeta_{e_\Delta}(s) = \max\{\zeta_\Delta(r,s)/r \in V\}$ and $\xi_{e_\Delta}(s) = \min\{\xi_\Delta(r,t)/r \in V\}$.*

Theorem 3.2. *If r and s are distinct vertices in a connected PF graph $G = (A, B)$. Then*

(i) $|e_\Delta(r) - e_\Delta(s)| \leq \Delta(r,s)$, *i.e.,* $\left|\zeta_{e_{\Delta(r)}} - \zeta_{e_{\Delta(s)}}\right| \leq \zeta_\Delta(r,s)$ *and*

$$\left|\xi_{e_{\Delta(r)}} - \xi_{e_{\Delta(s)}}\right| \leq \xi_\Delta(r,s),$$

(ii) $|e_\delta(r) - e_\delta(s)| \leq \delta(r,s)$.

Proof. By the definition, the PF *detour eccentricity* $e_\Delta(s)$ of a vertex V in a connected PF *graph* G is the maximum PF *detour distance* from V to any vertex other than V.

Let x be a vertex giving maximum PF *detour distance* from V on the $(s-x)$. PF *detour* such that

$$\zeta_{e_{\Delta(s)}} = \zeta_\Delta(s,x),$$

$$\xi_{e_{\Delta(s)}} = \xi_\Delta(s,x).$$

Therefore, by triangle inequality

$$\zeta_{e_{\Delta(s)}} = \zeta_\Delta(s,x) \leq \zeta_{\Delta(s,r)} + \zeta_{\Delta(r,x)}, \tag{3.1}$$

$$\xi_{e_{\Delta(s)}} = \xi_\Delta(s,x) \leq \xi_{\Delta(s,r)} + \xi_{\Delta(r,x)}, \tag{3.2}$$

for any u of G, i.e., $\zeta_{e_{\Delta(s)}} \leq \zeta_{\Delta(s,r)} + \zeta_{\Delta(r,x)}$, $\xi_{e_{\Delta(s)}} \leq \xi_{\Delta(s,r)} + \xi_{\Delta(r,x)}$, since $\zeta_{\Delta(r,x)} \leq \zeta_{e_{\Delta(r)}}$, $\xi_{\Delta(r,x)} \leq \xi_{e_{\Delta(r)}}$. Therefore from (3.1) and (3.2),

$$\zeta_{e_{\Delta(s)}} - \zeta_{e_{\Delta(s)}} \leq \zeta_{\Delta(s,r)} \quad \text{and} \quad \xi_{e_{\Delta(s)}} - \xi_{e_{\Delta(r)}} \leq \xi_{\Delta(s,r)}.$$

Interchanging the roles of r and s, we get

$$\zeta_{e_\Delta(r)} - \zeta_{e_\Delta(s)} \leq \zeta_{\Delta(r,s)} \quad \text{and} \quad \xi_{e_\Delta(r)} - \xi_{e_\Delta(s)} \leq \xi_{\Delta(r,s)} \tag{3.3}$$

$$\text{i.e.,} \; -\zeta_{\Delta(r,s)} \leq \zeta_{e_\Delta(s)} - \zeta_{e_\Delta(r)} \quad \text{and} \quad -\xi_{\Delta(r,s)} \leq \xi_{e_\Delta(s)} - \xi_{e_\Delta(r)}. \tag{3.4}$$

Combination of the above two results (3.3) and (3.4) yields

$$-\zeta_{\Delta(r,s)} \leq \zeta_{e_\Delta(r)} - \zeta_{e_\Delta(s)} \leq \zeta_{\Delta(r,s)} \quad \text{and} \quad -\xi_{\Delta(r,s)} \leq \xi_{e_\Delta(r)} - \xi_{e_\Delta(s)} \leq \xi_{\Delta(r,s)} .$$

Hence $\left|\zeta_{e_\Delta(r)} - \zeta_{e_\Delta(s)}\right| \leq \zeta_{\Delta(r,s)}$ and $\left|\xi_{e_\Delta(r)} - \xi_{e_\Delta(s)}\right| \leq \xi_{\Delta(r,s)}$. □

Theorem 3.3. *For vertices r and s in a connected PF graph G, $0 \leq \zeta_{\delta(r,s)} \leq \zeta_{\Delta(r,s)} < \infty$ and $0 \leq \xi_{\delta(r,s)} \leq \xi_{\Delta(r,s)} < \infty$.*

Theorem 3.4. *For vertices r and s in a connected PF graph G, $\zeta_{\Delta(r,s)} = 0$ and $\zeta_{\Delta(r,s)} = 1$ iff $\xi_{\delta(r,s)} = 0$ and $\xi_{\delta(r,s)} = 1$ iff $r = s$.*

Definition 3.5. *For connected PF graph G, the PF detour radius of G is $rad(G) = \left\langle \zeta_{rad_\Delta(G)}, \xi_{rad_\Delta(G)} \right\rangle$ is the minimum PF detour eccentricity among the vertices of G.*

$$\textit{i.e.,}\; \zeta_{rad_\Delta(G)} = \min\left\{\zeta_{e_\Delta(x)}/x \in V\right\},\; \xi_{rad_\Delta(G)} = \max\left\{\xi_{e_\Delta(x)}/x \in V\right\}.$$

Definition 3.6. *The PF detour diameter of G is the maximum PF detour eccentricity among the vertices of G.*

$$\textit{i.e.,}\; \zeta_{diam\,\Delta(G)} = \max\left\{\zeta_{e_\Delta(x)}/x \in V\right\},\; \xi_{diam\,\Delta(G)} = \min\left\{\xi_{e_\Delta(x)}/x \in V\right\}.$$

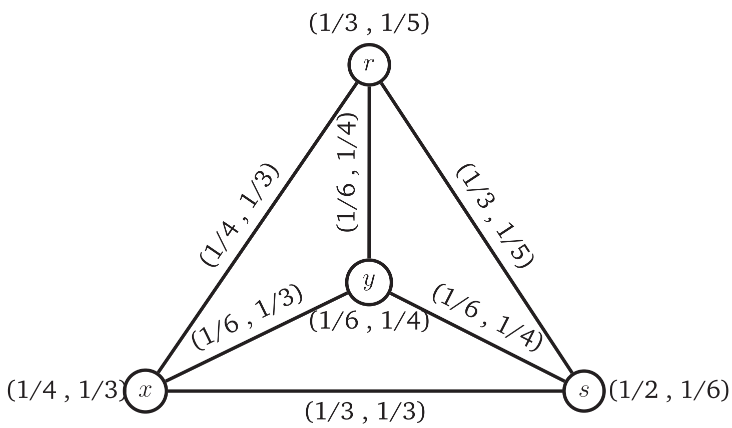

FIGURE 2

For the PF detour distance between the vertices r and s, there are 5 paths between r and s,

$$(1)\ r-s \quad (2)\ r-y-s \quad (3)\ r-x-s \quad (4)\ r-x-y-s \quad (5)\ r-y-x-s$$

and their corresponding lengths are

(1) $r-s \Rightarrow \langle 3,5\rangle$, (2) $r-y-s \Rightarrow \langle 6+6,4+4\rangle \Rightarrow \langle 12,8\rangle$,
(3) $r-x-s \Rightarrow \langle 8,6\rangle$, (4) $r-x-y-s \Rightarrow \langle 16,10\rangle$,
(5) $r-y-x-s \Rightarrow \langle 16,9\rangle$.

$\therefore$ The PF detour distance is the maximum length between the vertices r and s i.e., $\Delta(r,s)=\langle 16,5\rangle$, $\Delta(r,x)=\langle 16,3\rangle$, $\Delta(r,y)=\langle 14,4\rangle$.

Thus, $\zeta_{e_\Delta(r)}=16,\ \xi_{e_\delta(r)}=3$, $e_\Delta(r)=\langle 16,3\rangle$.

PF detour radius and PF detour diameter similarly.

$\Delta(s,r)=\langle 16,5\rangle$, $\Delta(s,y)=\langle 14,4\rangle$ and $\Delta(s,x)=\langle 16,3\rangle$, $\therefore e_\Delta(s)=\langle 16,3\rangle$.
$\Delta(y,r)=\langle 14,4\rangle$, $\Delta(y,s)=\langle 14,4\rangle$ and $\Delta(y,x)=\langle 13,3\rangle$, $\therefore e_\Delta(y)=\langle 14,3\rangle$.
$\Delta(x,r)=\langle 16,3\rangle$, $\Delta(x,s)=\langle 16,3\rangle$ and $\Delta(x,y)=\langle 13,3\rangle$, $\therefore e_\Delta(x)=\langle 16,3\rangle$

$$\begin{aligned}\therefore \text{rad}_\Delta(G) &= \langle 14,3\rangle\\ \text{diam}(G) &= \langle 16,3\rangle\\ \therefore \zeta_{\text{rad}_\Delta(G)} &\le \zeta_{\text{diam}_\Delta(G)} \text{ and } \xi_{\text{rad}_\Delta(G)} \le \xi_{\text{diam}_\Delta(G)}\end{aligned}$$

i.e., PF detour radius and PF detour diameter can be related, which is an analogous result as in crisp graphs.

Theorem 3.5. *For connected PF graph G, $rad_\Delta(G) \le diam_\Delta(G) \le 2rad_\Delta(G)$ i.e., $\zeta_{rad_\Delta(G)} \le \zeta_{diam_\Delta(G)} \le 2\zeta_{rad_\Delta(G)}$, $\xi_{rad_\Delta(G)} \le \xi_{diam_\Delta(G)} \le 2\xi_{rad_\Delta(G)}$.*

Theorem 3.6. *For every vertex V of a connected PF graph G, $diam_\Delta(G)-e_\Delta(s) \ge k$, where k is arbitrary non-negative real number.*

Theorem 3.7. *For any connected graph G, we have*

$$\begin{aligned} |\Delta(r,x)-\Delta(s,x)| &\le \Delta(r,s), \forall x \in G\\ \textit{i.e., } \left|\zeta_{\Delta(r,x)}-\zeta_{\Delta(s,x)}\right| &\le \zeta_{\Delta(r,s)}\\ \textit{and } \left|\xi_{\Delta(r,x)}-\xi_{\Delta(s,x)}\right| &\le \xi_{\Delta(r,s)}.\end{aligned}$$

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