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SEVERAL MAPPINGS VIA (L, K)-$\mathcal{Z}$-GENERALIZED OPEN SETS IN DOUBLE FUZZY TOPOLOGICAL SPACES

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ABSTRACT. We introduce some new class of generalized sets namely (l, k)-fuzzy $\mathcal{Z}$- generalized closed sets in double fuzzy topological spaces. Using them we investigate the class of mappings called double fuzzy $\mathcal{Z}$-generalized continuous and irresolute maps. Furthermore we study double fuzzy $\mathcal{Z}$-generalized homeomorphisms and pre double fuzzy $\mathcal{Z}$-generalized homeomorphisms. Also, some of their fundamental properties are studied.

1. INTRODUCTION AND PRELIMINARIES

In 1986, Atanassov [1] started 'Intuitionistic fuzzy sets' and Coker [2] in 1997, initiated Intuitionistic fuzzy topological space. The term 'double' instead of 'intuitionistic' coined by Garcia and Rodabaugh [4] in 2005. In the previous two decades many analysts accomplishing more applications on double fuzzy topological spaces. From 2011, $\mathcal{Z}$-open sets and maps were introduced in topological spaces by El-Maghrabi and Mubarki [3].

X denotes a non-empty set, $I_1 = [0, 1)$, $I_0 = (0, 1]$, $I = [0, 1]$, $0 = \underline{0}(X)$, $1 = \underline{1}(X)$, $r \in I_0$ & $\kappa \in I_1$ and always $1 \geq r + \kappa$. I^X is a family of all fuzzy sets on X. In 2002, Double fuzzy topological spaces (briefly, dfts), (X, η, η^*), (r, κ)-fuzzy

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open (resp. (r,κ)-fuzzy closed) (briefly (r,κ)-fo (resp. (r,κ)-fc)) set were given by Samanta and Mondal [5].

All other undefined notions are from [5–7] and cited therein.

2. (l,k)-FUZZY $\mathcal{Z}$-GENERALIZED CLOSED SETS

Definition 2.1. *A fuzzy subset γ in a $dfts$ (X,α,α^*) is called an (l,k)-fuzzy $\mathcal{Z}$-generalized*

(i) *closed (briefly, (l,k)-$f\mathcal{Z}gc$) set if $\mathcal{Z}C_{\alpha,\alpha^*}(\gamma,l,k)\leq\sigma$, whenever $\gamma\leq\sigma$ & σ is (l,k)-$f\mathcal{Z}o$ set.*

(ii) *open (briefly, (l,k)-$f\mathcal{Z}go$) set if $\sigma\leq\mathcal{Z}I_{\alpha,\alpha^*}(\gamma,l,k))$ whenever $\sigma\leq\gamma$ & σ is (l,k)-$f\mathcal{Z}c$ set.*

Also the complement of an (l,k)-$f\mathcal{Z}gc$ set is called as (l,k)-$f\mathcal{Z}go$ set.

Definition 2.2. *Let (X,α,α^*) be a $dfts$. For γ, $\sigma\in I^X$, the (l,k)-fuzzy $\mathcal{Z}$-generalized closure (interior) of γ and is $\mathcal{Z}gC_{\alpha,\alpha^*}(\gamma,l,k)=\bigwedge\{\sigma\in I^X:\sigma\geq\gamma,\ \sigma$ is a (l,k)-$f\mathcal{Z}gc$ set $\}$ ($\mathcal{Z}gI_{\alpha,\alpha^*}(\gamma,l,k)=\bigvee\{\sigma\in I^X:\sigma\leq\gamma,\ \sigma$ is a (l,k)-$f\mathcal{Z}go$ set $\}$).*

Theorem 2.1. *Every (l,k)-fc set in (X,α,α^*) is (l,k)-$f\mathcal{Z}gc$ set. But not conversely.*

Theorem 2.2. *Every (l,k)-$f\mathcal{Z}c$ in (X,α,α^*) is (l,k)-$f\mathcal{Z}gc$, But not conversely.*

Theorem 2.3. *Let γ be any fuzzy subset of X. Then*

(i) *γ is (l,k)-$f\mathcal{Z}gc$ if $\gamma=\mathcal{Z}gC_{\alpha,\alpha^*}(\gamma,l,k)$.*

(ii) *$\mathcal{Z}gC_{\alpha,\alpha^*}(\gamma,l,k)$ is (l,k)-$f\mathcal{Z}gc$ in X.*

Theorem 2.4. *A finite union of (l,k)-$f\mathcal{Z}go$ sets is an (l,k)-$f\mathcal{Z}go$ set.*

Remark 2.1. *Union of two (l,k)-$f\mathcal{Z}gc$ sets need not be an (l,k)-$f\mathcal{Z}gc$ set.*

Theorem 2.5. *A finite intersection of (l,k)-$f\mathcal{Z}gc$ is an (l,k)-$f\mathcal{Z}gc$ set.*

Remark 2.2. *Intersection of two (l,k)-$f\mathcal{Z}go$ sets need not be an (l,k)-$f\mathcal{Z}go$ set.*

Theorem 2.6. *If γ is (l,k)-$f\mathcal{Z}gc$ set and (l,k)-$f\mathcal{Z}o$ set in (X,α,α^*), then γ is (l,k)-$f\mathcal{Z}c$ in (X,α,α^*).*

Theorem 2.7. *If γ is (l,k)-$f\mathcal{Z}gc$ set in (X,α,α^*) & $\gamma\leq\sigma\leq\mathcal{Z}C_{\alpha,\alpha^*}(\gamma,l,k)$, then σ is (l,k)-$f\mathcal{Z}gc$ set in (X,α,α^*).*

Theorem 2.8. *If γ is (l,k)-$f\mathcal{Z}go$ set in (X,α,α^*) & $\mathcal{Z}I_{\alpha,\alpha^*}(\gamma,l,k) \leq \sigma \leq \gamma$, then σ is (l,k)-$f\mathcal{Z}go$ set in (X,α,α^*).*

Theorem 2.9. *Let (X,α,α^*) be the $dfts$ & γ be a fs of X. Then γ is (l,k)-$f\mathcal{Z}gc$ set iff $\gamma\ \overline{q}\ \sigma$ implies $\mathcal{Z}C_{\alpha,\alpha^*}(\gamma,l,k)\ \overline{q}\ \sigma$, $\forall$ (l,k)-$f\mathcal{Z}c$ set σ of X.*

Theorem 2.10. *Let γ be (l,k)-$f\mathcal{Z}gc$ set in (X,α,α^*) & x_p be a fuzzy point (briefly, fp) of $(X,\alpha,\alpha^*) \ni x_p\ q\ \mathcal{Z}C_{\alpha,\alpha^*}(\gamma,l,k)$ then $\mathcal{Z}C_{\alpha,\alpha^*}(x_p,l,k)\ q\ \gamma$.*

Theorem 2.11. *Let $(Y,\ \alpha_Y,\alpha_Y^*)$ be a subspace of (X,α,α^*) & γ be a fs of Y. If γ is (l,k)-$f\mathcal{Z}gc$ set in X, then γ is (l,k)-$f\mathcal{Z}gc$ set in Y.*

Definition 2.3. *Let γ be a fs in $dfts\ X$ & x_p be a fp of X, then γ is called (l,k)-fuzzy $\mathcal{Z}$-generalized neighbourhood (resp. q-neighbourhood) (briefly, (l,k)-$f\mathcal{Z}g$-nbhd (resp. (l,k)-$f\mathcal{Z}g$q-nbhd)) of x_p if $\exists$ a (l,k)-$f\mathcal{Z}go$ set σ of $X \ni x_p \in \sigma \leq \gamma$ (resp. $x_p\ q\ \sigma \leq \gamma$.)*

Theorem 2.12. *γ is (l,k)-$f\mathcal{Z}go$ set in X iff $\forall$ fp $x_p \in \gamma$, γ is a (l,k)-$f\mathcal{Z}g$-nbhd of x_p.*

Theorem 2.13. *If γ & σ are (l,k)-$f\mathcal{Z}g$-nbhd of x_p, then $\gamma \wedge \sigma$ is also a (l,k)-$f\mathcal{Z}g$-nbhd of x_p.*

Theorem 2.14. *Let γ be a fs of a $dfts\ X$. Then a fp $x_p \in \mathcal{Z}C_{\alpha,\alpha^*}(\gamma,l,k)$ iff every (l,k)-$f\mathcal{Z}g$q-nbhd of x_p is quasi-coincident with γ.*

Definition 2.4. *A $dfts\ X$ is $df\mathcal{Z}$-generalized $T_{\frac{1}{2}}$-space (briefly, $DF\mathcal{Z}gT_{\frac{1}{2}}$-space) if every (l,k)-$f\mathcal{Z}gc$ set in X is (l,k)-fc set in X.*

Theorem 2.15. *A $dfts$ (X,α,α^*) is $DF\mathcal{Z}T_{\frac{1}{2}}$-space iff every fs in (X,α,α^*) is both (l,k)-$f\mathcal{Z}o$ & (l,k)-$f\mathcal{Z}go$.*

3. $DF\mathcal{Z}g$-CONTINUOUS AND $DF\mathcal{Z}g$-IRRESOLUTE MAPPINGS

Definition 3.1. *Let $f : (X,\alpha_1,\alpha_1^*) \to (Y,\alpha_2,\alpha_2^*)$ be a mapping from an $dfts$ $(X,\ \alpha_1,\ \alpha_1^*)$ to another $dfts$ (Y,α_2,α_2^*). Then f is called double fuzzy $\mathcal{Z}$-generalized continuous (resp. irresolute) (briefly, $DF\mathcal{Z}gCts$ (resp. $DF\mathcal{Z}gIrr$)) mapping if $f^{-1}(\gamma)$ is (l,k)-$f\mathcal{Z}go$ set in X $\forall$ (l,k)-fo (resp. (l,k)-$f\mathcal{Z}go$) set γ in Y.*

Theorem 3.1. (i) *Every $DFCts$ map is $DF\mathcal{Z}gCts$.*

(ii) *Every $DF\mathcal{Z}gIrr$ map is $DF\mathcal{Z}gCts$. But not conversely.*

Theorem 3.2. *If $f : (X, \alpha_1, \alpha_1^*) \to (Y, \alpha_2, \alpha_2^*)$ is $DF\mathcal{Z}gCts$ iff the inverse image of (l, k)-fo set of Y is (l, k)-$f\mathcal{Z}go$ set of X.*

Theorem 3.3. *If $f : (X, \alpha, \alpha^*) \to (Y, \beta, \beta^*)$ is $DF\mathcal{Z}gCts$, then (i) $\forall$ fp x_p of X & each $\gamma \in Y \ni f(x_p) \in \gamma$, $\exists$ a (l, k)-$f\mathcal{Z}go$ set σ of $X \ni x_p \in \sigma$ & $f(\sigma) \leq \gamma$. (ii) $\forall$ fp x_p of X & each $\gamma \in Y \ni f(x_p)\, q\, \gamma$, $\exists$ a (l, k)-$f\mathcal{Z}go$ set σ of $X \ni x_p\, q\, \sigma$ & $f(\sigma) \leq \gamma$.*

Theorem 3.4. *Let $f : (X, \alpha_1, \alpha_1^*) \to (Y, \alpha_2, \alpha_2^*)$ be $DF\mathcal{Z}gCts$ mapping and if X is $DF\mathcal{Z}T_{\frac{1}{2}}$-space, then f is $DF\mathcal{Z}Cts$.*

Definition 3.2. *Let $(X, \alpha_1, \alpha_1^*)$ & $(Y, \alpha_2, \alpha_2^*)$ be two $dfts$'s. The function $f : (X, \alpha_1, \alpha_1^*) \to (Y, \alpha_2, \alpha_2^*)$ is said to be:*

(i) *double fuzzy $\mathcal{Z}$-open (resp. closed) [6] (briefly, $DF\mathcal{Z}O$ (resp. $DF\mathcal{Z}C$)) map if the image of every (l, k)-fo (resp. (l, k)-fc) set in X is (l, k)-$f\mathcal{Z}o$ (resp. (l, k)-$f\mathcal{Z}c$) set in Y.*

(ii) *pre double fuzzy $\mathcal{Z}$-open (resp. closed) (briefly, $pDF\mathcal{Z}O$ (resp. $pDF\mathcal{Z}C$)) map if the image of every (l, k)-$f\mathcal{Z}o$ (resp. (l, k)-$f\mathcal{Z}c$) set in X is (l, k)-$f\mathcal{Z}o$ (resp. (l, k)-$f\mathcal{Z}c$) set in Y.*

(iii) *double fuzzy $\mathcal{Z}g$-open (resp. closed) (briefly, $DF\mathcal{Z}gO$ (resp. $DF\mathcal{Z}gC$)) map if the image of every (l, k)-fo (resp. (l, k)-fc) set in X is (l, k)-$f\mathcal{Z}go$ (resp. (l, k)-$f\mathcal{Z}gc$) in Y. (iv) pre double fuzzy $\mathcal{Z}g$-open (resp. closed) (briefly, $pDF\mathcal{Z}gO$ (resp. $pDF\mathcal{Z}gC$)) map if the image of every (l, k)-$f\mathcal{Z}go$ (resp. (l, k)-$f\mathcal{Z}gc$) set in X is (l, k)-$f\mathcal{Z}go$ (resp. (l, k)-$f\mathcal{Z}gc$) in Y.*

Theorem 3.5. *Let $f : (X, \alpha_1, \alpha_1^*) \to (Y, \alpha_2, \alpha_2^*)$ be onto, $DF\mathcal{Z}Irr$ and $pDF\mathcal{Z}C$ mapping. If X is $DF\mathcal{Z}gT_{\frac{1}{2}}$-space, then $(Y, \alpha_2, \alpha_2^*)$ is $DF\mathcal{Z}gT_{\frac{1}{2}}$-space.*

Theorem 3.6. *If the bijective map $f : (X, \alpha_1, \alpha_1^*) \to (Y, \alpha_2, \alpha_2^*)$ is $DF\mathcal{Z}Irr$ & $pDF\mathcal{Z}O$ mapping then f is $DF\mathcal{Z}gIrr$.*

Theorem 3.7. *Let $u : (X, \alpha_1, \alpha_1^*) \to (Y, \alpha_2, \alpha_2^*)$. Then the statements*

(i) *f is $DF\mathcal{Z}gIrr$.*

(ii) *$\forall$ $f\mathcal{Z}gc$ set $\gamma \in Y$, $u^{-1}(\gamma)$ is $f\mathcal{Z}gc$ in X.*

(iii) *$\forall$ fp x_p of X & every $f\mathcal{Z}go$ set γ of $Y \ni f(x_p) \in \gamma$, $\exists$ a $f\mathcal{Z}go$ set $\ni x_p \in \sigma$ and $f(\sigma) \leq \gamma$.*

are equivalent.

Theorem 3.8. *Let* $u : (X, \alpha, \alpha^*) \to (Y, \beta, \beta^*)$, $v : (Y, \beta, \beta^*) \to (Z,\ \rho,\ \rho^*)$ *be two maps* $\ni v \circ u : (X, \alpha, \alpha^*) \to (Z,\ \rho,\ \rho^*)$ *is* $DF\mathcal{Z}gC$ *map. (i) If* u *is* $DFCts$ & *surjective, then* v *is* $DF\mathcal{Z}gC$ *map. (ii) If* v *is* $DF\mathcal{Z}gIrr$ *and injective, then* u *is* $DF\mathcal{Z}gC$ *map.*

Theorem 3.9. *Let* $u : (X, \alpha, \alpha^*) \to (Y, \beta, \beta^*)$, $v : (Y, \beta, \beta^*) \to (Z,\ \rho,\ \rho^*)$ *be two maps* $\ni v \circ u : (X, \alpha, \alpha^*) \to (Z,\ \rho,\ \rho^*)$ *is* $DF\mathcal{Z}^*gC$ *map.*

(i) *If* u *is* $DFCts$ & *surjective, then* v *is* $pDF\mathcal{Z}gC$ *map.*
(ii) *If* v *is* $DF\mathcal{Z}gIrr$ & *injective, then* u *is* $pDF\mathcal{Z}gC$ *map*

Theorem 3.10. *For the functions* $u : X \to Y$ & $v : Y \to Z$ *the following relations hold:*

(i) *If* $u : X \to Y$ *is* $DF\mathcal{Z}gCts$ & $v : Y \to Z$ *is* $DFCts$ *then* $v \circ u : X \to Z$ *is* $DF\mathcal{Z}gCts$.
(ii) *If* $u : X \to Y$ & $v : Y \to Z$ *are* $DF\mathcal{Z}gIrr$. *then* $v \circ u : X \to Z$ *is* $DF\mathcal{Z}gIrr$.
(iii) *If* $u : X \to Y$ *is* $DF\mathcal{Z}gIrr$ & $v : Y \to Z$ *is* $DF\mathcal{Z}Cts$ *then* $v \circ u : X \to Z$ *is* $DF\mathcal{Z}gCts$.

Theorem 3.11. *If* $u : (X, \alpha, \alpha^*) \to (Y, \beta, \beta^*)$ *is* $DF\mathcal{Z}gCts$ & $v : (Y, \beta, \beta^*) \to (Z,\ \rho,\ \rho^*)$ *is* $DF\mathcal{Z}Cts \ni Y$ *is* $DF\mathcal{Z}gT_{\frac{1}{2}}$*-space then* $v \circ u : (X, \alpha, \alpha^*) \to (Z,\ \rho,\ \rho^*)$ *is* $DF\mathcal{Z}gCts$.

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