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FUZZY $\mathcal{Z}$-OPEN MAPPINGS IN DOUBLE FUZZY TOPOLOGICAL SPACES

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ABSTRACT. We introduce and investigate some new class of mappings called double fuzzy $\mathcal{Z}$-open map and double fuzzy $\mathcal{Z}$-closed map in double fuzzy topological spaces. Also, some of their fundamental properties are studied. Moreover, we investigate the relationships between some double fuzzy open and their respective closed mappings.

1. INTRODUCTION AND PRELIMINARIES

In 1986, Atanassov [1] started 'Intuitionistic fuzzy sets' and Coker [2] in 1997, initiated Intuitionistic fuzzy topological space. The term 'double' instead of 'intuitionistic' coined by Garcia and Rodabaugh [4] in 2005. In the previous two decades many analysts accomplishing more applications on double fuzzy topological spaces. From 2011, $\mathcal{Z}$-open sets and maps were introduced in topological spaces by El-Maghrabi and Mubarki [5].

X denotes a non-empty set, $I_1 = [0,1)$, $I_0 = (0,1]$, $I = [0,1]$, $0 = \underline{0}(X)$, $1 = \underline{1}(X)$, $r \in I_0$ & $\kappa \in I_1$ and always $1 \geq r+\kappa$. I^X is a family of all fuzzy sets on X. In 2002, Double fuzzy topological spaces (briefly, dfts), (X, η, η^*), (r,κ)-fuzzy open (resp. (r,κ)-fuzzy closed) (briefly (r,κ)-fo (resp. (r,κ)-fc)) set were given by Samanta and Mondal [6].

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All other undefined notions are from [3, 6, 7] and cited therein.

2. On double fuzzy $\mathcal{Z}$ open (resp. closed) mappings

Definition 2.1. *A function f from a $dfts$ (X, ρ, ρ^*) to a $dfts$ (Y, ζ, ζ^*), is called as a double fuzzy $\mathcal{Z}$ open (resp. δ semiopen, δ preopen, preopen, semi open and b open) (briefly $DF\mathcal{Z}O$ (resp. $DF\delta sO$, $DF\delta pO$, $DFpO$, $DFsO$ and $DFbO$)) function if $f(\mu)$ is an (r, κ)-$f\mathcal{Z}o$ (resp. (r, κ)-$f\delta so$, (r, κ)-$f\delta po$, (r, κ)-fpo, (r, κ)-fso and (r, κ)-fbo) set in I^Y $\forall$ (r, κ)-fo set $\mu \in I^X$ for all $r \in I_0$ & $\kappa \in I_1$.*

Definition 2.2. *A function f from a $dfts$ (X, ρ, ρ^*) to a $dfts$ (Y, ζ, ζ^*), is called as a double fuzzy $\mathcal{Z}$ closed (resp. δ semiclosed, δ preclosed, preclosed, semi closed and b closed) (briefly $DF\mathcal{Z}C$ (resp. $DF\delta sC$, $DF\delta pC$, $DFpC$, $DFsC$ and $DFbC$)) function if $f(\mu)$ is an (r, κ)-$f\mathcal{Z}c$ (resp. (r, κ)-$f\delta sc$, (r, κ)-$f\delta pc$, (r, κ)-fpc, (r, κ)-fsc and (r, κ)-fbc) set in I^Y $\forall$ (r, κ)-fc set $\mu \in I^X$ for all $r \in I_0$ & $\kappa \in I_1$.*

Theorem 2.1. *Let $f : (X, \rho, \rho^*) \to (Y, \eta, \eta^*)$ be a mapping*

(i) *Every $DF\delta O$ mapping is DFO (resp. $DF\delta pO$ and $DF\delta sO$) mapping.*
(ii) *Every DFO mapping is $DFsO$ (resp. $DFpO$) mapping.*
(iii) *Every $DF\delta pO$ mapping is $DFeO$ mapping.*
(iv) *Every $DF\delta sO$ mapping is $DFeO$ (resp. $DF\mathcal{Z}O$) mapping.*
(v) *Every $DFpO$ mapping is $DF\mathcal{Z}O$ mapping.*
(vi) *Every $DFsO$ mapping is $DFbO$ mapping.*
(vii) *Every $DF\mathcal{Z}O$ mapping is $DFeO$ (resp. $DFbO$) mapping. But not conversely.*

$$\begin{tikzcd}
DF\delta O \arrow[rrr] \arrow[d] \arrow[dr] & & & DFO \arrow[dl] \arrow[d] \\
DF\delta pO \arrow[d] & DF\delta sO \arrow[dl] \arrow[d] & DFpO \arrow[dl] & DFsO \arrow[d] \\
DFeO & DF\mathcal{Z}O \arrow[l] \arrow[rr] & & DFbO
\end{tikzcd}$$

Figure 1

Example 1. *Let $X = Y = \{l, m, n\}$ and let the fuzzy sets α_1 to α_7 are defined as $\alpha_1(l) = 0.3,\ \alpha_1(m) = 0.4,\ \alpha_1(n) = 0.5$; $\alpha_2(l) = 0.6,\ \alpha_2(m) = 0.9,\ \alpha_2(n) = 0.5$; $\alpha_3(l) = 0.2,\ \alpha_3(m) = 0.2,\ \alpha_3(n) = 0.2$; $\alpha_4(l) = 0.4,\ \alpha_4(m) = 0.4,\ \alpha_4(n) = 0.5$; $\alpha_5(l) = 0.5,\ \alpha_5(m) = 0.5,\ \alpha_5(n) = 0.5$; $\alpha_6(l) = 0.2,\ \alpha_6(m) = 0.4,\ \alpha_6(n) = 0.4$ and $\alpha_7(l) = 0.3,\ \alpha_7(m) = 0.0,\ \alpha_7(n) = 0.4$. Consider the double fuzzy topologies (X, η, η^*), (Y, η_1, η_1^*), (Y, η_2, η_2^*), (Y, η_3, η_3^*), (Y, η_4, η_4^*), (Y, η_5, η_5^*) and (Y, η_6, η_6^*) with*

$$\eta(\gamma) = \eta^*(\gamma) = \begin{cases} 1, & \textit{if } \gamma \in \{\underline{0}, \underline{1}\}, \\ \frac{1}{2}, & \textit{if } \gamma \in \{\alpha_1, \alpha_2\} \\ 0, & \textit{o.w.} \end{cases}, \quad \eta_1(\gamma) = \eta_1^*(\gamma) = \begin{cases} 1, & \textit{if } \gamma \in \{\underline{0}, \underline{1}\}, \\ \frac{1}{2}, & \textit{if } \gamma = \alpha_2 \\ 0, & \textit{o.w.} \end{cases},$$

$$\eta_2(\gamma) = \eta_2^*(\gamma) = \begin{cases} 1, & \textit{if } \gamma \in \{\underline{0}, \underline{1}\}, \\ \frac{1}{2}, & \textit{if } \gamma = \alpha_3 \\ 0, & \textit{o.w.} \end{cases}, \quad \eta_3(\gamma) = \eta_3^*(\gamma) = \begin{cases} 1, & \textit{if } \gamma \in \{\underline{0}, \underline{1}\}, \\ \frac{1}{2}, & \textit{if } \gamma = \alpha_4 \\ 0, & \textit{o.w.} \end{cases},$$

$$\eta_4(\gamma) = \eta_4^*(\gamma) = \begin{cases} 1, & \textit{if } \gamma \in \{\underline{0}, \underline{1}\}, \\ \frac{1}{2}, & \textit{if } \gamma = \alpha_5 \\ 0, & \textit{o.w.} \end{cases}, \quad \eta_5(\gamma) = \eta_5^*(\gamma) = \begin{cases} 1, & \textit{if } \gamma \in \{\underline{0}, \underline{1}\}, \\ \frac{1}{2}, & \textit{if } \gamma = \alpha_6 \\ 0, & \textit{o.w.} \end{cases},$$

and

$$\eta_6(\gamma) = \begin{cases} 1, & \textit{if } \gamma \in \{\underline{0}, \underline{1}\}, \\ \frac{1}{2}, & \textit{if } \gamma = \alpha_7 \\ 0, & \textit{o.w.} \end{cases}, \quad \eta_6^*(\gamma) = \begin{cases} 0, & \textit{if } \gamma \in \{\underline{0}, \underline{1}\}, \\ \frac{1}{2}, & \textit{if } \gamma = \alpha_7 \\ 1, & \textit{o.w.}, \end{cases}.$$

Then the identity function (i) $f : (X, \eta_1, \eta_1^*) \to (Y, \eta, \eta^*)$ is a (i) DFO (resp. $DF\delta pO$) function but not a $DF\delta O$, (ii) $DF\mathcal{Z}O$ but not a $DF\delta sO$. (ii) $f : (X, \eta_2, \eta_2^*) \to (Y, \eta, \eta^*)$ is a $DFpO$ function but not a DFO. (iii) $f : (X, \eta_3, \eta_3^*) \to (Y, \eta, \eta^*)$ is a $DFsO$ (resp. $DF\delta sO$) function but not a DFO. (iv) $f : (X, \eta_4, \eta_4^*) \to (Y, \eta, \eta^*)$ is a $DF\mathcal{Z}O$ function but not a $DFpO$. (v) $f : (X, \eta_5, \eta_5^*) \to (Y, \eta, \eta^*)$ is a $DFbO$ function but not a $DFsO$ and (vi) $f : (X, \eta_6, \eta_6^*) \to (Y, \eta, \eta^*)$ is a $DFeO$ function but not a $DF\mathcal{Z}O$. From the above we have,

Definition 2.3. *A mapping $f : (X, \rho, \rho^*) \to (Y, \eta, \eta^*)$ is called $DF\mathcal{Z}O$ at a fpt x_r if the image of each (r, κ)-Q neighbourhood of x_r is an (r, κ)-$\mathcal{Z}Q$ neighbourhood of $f(x_r) \in I^Y$.*

Theorem 2.2. *A mapping $f : (X, \rho, \rho^*) \to (Y, \eta, \eta^*)$ is $DF\mathcal{Z}O$ iff it is $DF\mathcal{Z}O$ at every fpt $x_r \in I^X$.*

Theorem 2.3. *Let $(X,\ \rho,\ \rho^*)$ & $(Y,\ \eta,\ \eta^*)$ be $dfts$'s and $f : (X, \rho, \rho^*) \to (Y, \eta, \eta^*)$ be a mapping. Then statements the following statements are equivalent.*

(i) *f is $DF\mathcal{Z}O$ function.*
(ii) *$f(\gamma)$ is an (r, κ)-$f\mathcal{Z}o$ set in (Y, η, η^*) $\forall$ (r, κ)-fo set γ in (X, ρ, ρ^*).*
(iii) *f is $DF\mathcal{Z}C$ function.*
(iv) *$f(\gamma)$ is an (r, κ)-$f\mathcal{Z}c$ set in (Y, η, η^*) $\forall$ (r, κ)-fc set γ in (X, ρ, ρ^*).*
(v) *$\mathcal{Z}C_{\rho,\rho^*}(f(\gamma),\ r, \kappa) \leq f(C_{\eta,\eta^*}(\gamma,\ r, \kappa)),\ \forall\, \gamma \in I^X$.*
(vi) *$I_{\rho,\rho^*}(\delta C_{\rho,\rho^*}(f(\gamma),\ r, \kappa),\ r, \kappa) \wedge C_{\rho,\rho^*}(I_{\rho,\rho^*}(f(\gamma),\ r, \kappa),\ r, \kappa) \leq f(C_{\eta,\eta^*}(\gamma,\ r, \kappa))$, $\forall\, \gamma \in I^X$.*
(vii) *$f(I_{\rho,\rho^*}(\gamma,\ r, \kappa)) \leq C_{\eta,\eta^*}(\delta I_{\eta,\eta^*}(f(\gamma),\ r, \kappa),\ r, \kappa) \vee I_{\eta,\eta^*}(C_{\eta,\eta^*}(f(\gamma), r, \kappa), r, \kappa)$ $\forall\, \gamma \in I^X$.*
(viii) *$f(I_{\rho,\rho^*}(\gamma,\ r, \kappa)) \leq \mathcal{Z}I_{\eta,\eta^*}(f(\gamma),\ r, \kappa), \forall\, \gamma \in I^X$.*
(ix) *$I_{\rho_1,\rho_1^*}(f^{-1}(\gamma), r, \kappa) \leq f^{-1}(\mathcal{Z}I_{\rho_2,\rho_2^*}(\gamma, r, \kappa))\ \forall\, \gamma \in I^Y$.*

Theorem 2.4. *Let $(X,\ \rho, \rho^*)$ & $(Y,\ \eta, \eta^*)$ be $dfts$'s. Let $f : X \to Y$ be a $DF\mathcal{Z}C$ mapping iff f is surjective, then $\forall$ subset μ of Y and each (r, κ)-fo set α in X containing $f^{-1}(\mu)$, $\exists$ an (r, κ)-$f\mathcal{Z}o$ set β of Y containing $\mu \ni f^{-1}(\beta) \leq \alpha$.*

Theorem 2.5. *Let (X, ρ_1, ρ_1^*) & (Y, ρ_2, ρ_2^*) be $dfts$'s and $f : (X, \rho_1, \rho_1^*) \to (Y, \rho_2, \rho_2^*)$ be a $DF\mathcal{Z}O$ (resp. $DF\delta sO$ and $DFpO$) mapping. If $\mu \in I^Y$ and $\gamma \in I^X,\ \rho_1(\underline{1} - \gamma) \geq r,\ \rho_1^*(\underline{1} - \gamma) \leq \kappa,\ r \in I_0\ \kappa \in I_1 \ni f^{-1}(\mu) \leq \gamma$, $\exists$ an (r, κ)-$f\mathcal{Z}c$ (resp. (r, κ)-fδsc and (r, κ)-fpc) set ν of $Y \ni \mu \leq \nu, f^{-1}(\nu) \leq \gamma$.*

Theorem 2.6. *If $f : (X, \rho_1, \rho_1^*) \to (Y,\ \rho_2, \rho_2^*)$ be a $DF\mathcal{Z}O$ mapping. Then for each $\mu \in I^Y$, $f^{-1}(C_{\rho_2,\rho_2^*}\ (\delta I_{\rho_2,\rho_2^*}(\mu, r, \kappa),\ r, \kappa)) \wedge f^{-1}(I_{\rho_2,\rho_2^*}(C_{\rho_2,\rho_2^*}(\mu, r, \kappa), r, \kappa)) \leq C_{\rho_1,\rho_1^*}(f^{-1}(\mu), r, \kappa)$.*

Theorem 2.7. *If $f : (X, \rho_1, \rho_1^*) \to (Y, \rho_2, \rho_2^*)$ be a bijective mapping such that $f^{-1}(C_{\rho_2,\rho_2^*}(I_{\rho_2,\rho_2^*}(\mu, r, \kappa), r, \kappa)) \wedge f^{-1}(I_{\rho_2,\rho_2^*}(\delta C_{\rho_2,\rho_2^*}(\mu, r, \kappa), r, \kappa)) \leq C_{\rho_1,\rho_1^*}(f^{-1}(\mu), r, \kappa)$, for each $\mu \in I^Y$, then f is $DF\mathcal{Z}O$ map.*

Theorem 2.8. *Let (X, ρ, ρ^*) & (Y, η, η^*) be $dfts$'s. Let $f : X \to Y$ be a $DF\mathcal{Z}C$ mapping. Then (i) If f is a surjective map and $f^{-1}(\alpha)\overline{q}f^{-1}(\beta)$ in X, then there exists $\alpha, \beta \in I^Y$ such that $\alpha\overline{q}\beta$. (ii) $\mathcal{Z}I_{\eta,\eta^*}(\mathcal{Z}C_{\eta,\eta^*}(f(\gamma), r, \kappa), r, \kappa) \leq f(C_{\rho,\rho^*}(\gamma, r, \kappa))$, for each $\gamma \in I^X$ are hold.*

Proposition 2.1. *Let $f:(X,\rho_1,\rho_1^*)\to(Y,\rho_2,\rho_2^*)$ $DF\mathcal{Z}O$ mapping and if for any fuzzy subset γ of Y is (r,κ)-fuzzy nowhere dense then f is $df\delta pO$ map.*

Theorem 2.9. *If $f:(X,\rho_1,\rho_1^*)\to(Y,\rho_2,\rho_2^*)$ be a $DF\delta biCts$ mapping then the image of each (r,κ)-$f\mathcal{Z}o$ set in (X,ρ_1,ρ_1^*) under f is (r,κ)-$f\mathcal{Z}o$ set in (Y,ρ_2,ρ_2^*).*

Remark 2.1. *Let (X,ρ_1,ρ_1^*) & (Y,ρ_2,ρ_2^*) be $dfts$'s and $f:X\to Y$ be a mapping. The composition of two $DF\mathcal{Z}O$ mappings need not be $DF\mathcal{Z}O$ as shown by,*

Theorem 2.10. *Let (X,ρ_1,ρ_1^*), (Y,ρ_2,ρ_2^*) and (Z,ρ_3,ρ_3^*) be $dfts$'s. If*

$$f:(X,\rho_1,\rho_1^*)\to(Y,\rho_2,\rho_2^*)$$

and $g:(Y,\rho_2,\rho_2^*)\to(Z,\rho_3,\rho_3^*)$ are mappings, then $g\circ f$ is $DF\mathcal{Z}O$ mapping if (i) f is DFO & g is $DF\mathcal{Z}O$. (ii) f is $DF\mathcal{Z}O$ & g is $DF\delta biCts$ mapping.

Theorem 2.11. *Let (X,ρ_1,ρ_1^*), (Y,ρ_2,ρ_2^*) & (Z,ρ_3,ρ_3^*) be $dfts$'s. If $f:(X,\rho_1,\rho_1^*)\to(Y,\rho_2,\rho_2^*)$ and $g:(Y,\rho_2,\rho_2^*)\to(Z,\rho_3,\rho_3^*)$ are mappings, then (i) If $g\circ f$ is $DF\mathcal{Z}O$ mapping & f is a onto $DFCts$ map, then g is $DF\mathcal{Z}O$ map. (ii) If $g\circ f$ is DFO mapping & g is an 1-1 $DF\mathcal{Z}Cts$ map, then f is $DF\mathcal{Z}O$ map.*

3. Conclusion

In this paper, we introduced and investigated the classes of mappings called double fuzzy $\mathcal{Z}$-open map and double fuzzy $\mathcal{Z}$-closed map to the dfts's. Also, some fundamental properties were studied.

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