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ON FUZZY $\tilde{G}$-CLOSED SETS AND IT'S PROPERTIES

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ABSTRACT. New category of sets, specifically fuzzy $\tilde{g}$-closed sets is to launch and argue of fuzzy topological spaces. Further, compare with related sets are investigated. More over, the properties of fuzzy $\tilde{g}$-closed sets are given of this paper.

1. INTRODUCTION

In 1965, L. A. Zadeh [13] was introduced and discussed the novel model of a fuzzy subsets. The consequent research behavior in this field and linked have originate relevance in various bough of Modern sciences. In 1968, C. L. Chang [4] by the idea of generalized fuzzy spaces. Another researchers similar to K. K. Azad [1], S. P. Sinha [3], C. K. Wong [11] and any more authors donate to the growth of fuzzy topological spaces and so on.

New category of sets, specifically fuzzy $\tilde{g}$-closed sets is to launch and argue of fuzzy topological spaces. Further, compare with related sets are investigated. More over, the properties of fuzzy $\tilde{g}$-closed sets are given of this paper.

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2. Preliminaries

During this paper (X, F_τ) (briefly, X) will denote fuzzy topological spaces or space (X, F_τ). A fuzzy subset A of a fuzzy topological space (X, F_τ) is called a fuzzy semi-open [1], α-open [3] and regular open [10], the complement of open sets are called closed in (X, F_τ). The operators namely, fuzzy semi-closure [12], fuzzy α-closure [6], fuzzy semi-preclosure [12] in (X, F_τ). Further some fuzzy generalized closed sets are indicated [resp. shortly denotes fg-closed [2], (fsg-closed, $fgsp$-closed [5], $fpsg$-closed [8], $f\omega$-closed [9] and $f\psi$-closed [7]], the complement of closed sets are called open.

3. Fuzzy $\tilde{g}$-closed sets

Definition 3.1. *A fuzzy subset H of a space (X, F_τ) is said to be a*

1. *fuzzy $\tilde{g}$-closed set (shortly denotes $f\tilde{g}$-closed): condition says that $cl(H) \leq S$ each time $H \leq S$ and S is fsg-open.*
2. *fuzzy $\tilde{g}_\alpha$-closed set (shortly denotes $f\tilde{g}_\alpha$-closed): condition says that $\alpha cl(H) \leq S$ each time $H \leq S$ and S is fsg-open.*
3. *fuzzy αgs-closed set: condition says that $\alpha cl(H) \leq S$ each time $H \leq S$ and S is fuzzy semi-open.*

The complement of above closed set is called an open.

Proposition 3.1. *Entire fuzzy closed set is $f\tilde{g}$-closed but not converse.*

Proof. Let H be a fuzzy closed set of a space (X, F_τ) with K is a fsg-open set such that $H \leq K$, then $K \geq H = cl(H)$. Thus H is $f\tilde{g}$-closed. □

As shown from the follows.

Example 1. *Let a set be $X = \{m, n\}$ with $F_\tau = \{0_X, \eta, 1_X\}$ whereas η is a fuzzy subset in X, it's elected by $\eta(m) = 1, \eta(n) = 0$. In the space (X, F_τ), then α defined by $\alpha(m) = 0, \alpha(n) = 0.5$ is $f\tilde{g}$-closed set but not fuzzy closed set.*

Proposition 3.2. *Entire $f\tilde{g}$-closed set is $fgsp$-closed but not converse.*

Proof. Let H be a $f\tilde{g}$-closed subset of (X, F_τ) and K be a fuzzy open set such that $K \geq H$, then $K \geq cl(H) \geq spcl(H)$. Thus H is $fgsp$-closed in (X, F_τ). □

As shown from the follows.

Example 2. *Let a set be $X = \{m, n\}$ with F_τ = {0_X, η, 1_X} whereas η is a fuzzy subset in X, it's elected by $\eta(m) = \eta(n) = 0.4$. In the space (X, F_τ), then α defined by $\alpha(m) = \alpha(n) = 0.5$ is $fgsp$-closed set but not $f\tilde{g}$-closed set.*

Proposition 3.3. *Entire $f\tilde{g}$-closed set is fuzzy ω-closed but not converse.*

Proof. Assuming that $H \leq K$ and K is fuzzy semi-open subsets of (X, F_τ), we have $cl(H) \leq K$. Thus H is fuzzy ω-closed. □

As shown from the follows.

Example 3. *Let a set be $X = \{m, n\}$ with $F_\tau = \{0_X, \mu, \eta, 1_X\}$ where μ, η are fuzzy subsets in X, it's elected by $\mu(m) = \mu(n) = 0.4$ and $\eta(m) = \eta(n) = 0.6$. In the space (X, F_τ), then α defined by $\alpha(m) = \alpha(n) = 0.3$ is fuzzy ω-closed but not $f\tilde{g}$-closed set.*

Proposition 3.4. *Entire $f\tilde{g}$-closed set is fg-closed but not converse.*

Proof. Let H be a $f\tilde{g}$-closed set with K is a fuzzy open set of (X, F_τ) such that $K \geq H$, then $K \geq cl(H)$. Hence H is fg-closed. □

As shown from the follows.

Example 4. *Let a set be $X = \{m, n\}$ with $F_\tau = \{0_X, \eta, 1_X\}$ where η is a fuzzy subset in X, it's elected by $\eta(m) = \eta(n) = 0.5$. In the space (X, F_τ), then α defined by $\alpha(m) = \alpha(n) = 0.4$ is fg-closed but not $f\tilde{g}$-closed set.*

Proposition 3.5. *Entire $f\tilde{g}$-closed set is fαg-closed but not converse.*

Proof. Let H be a $f\tilde{g}$-closed set K be a fuzzy open set of a space (X, F_τ) such that $K \geq H$, then $K \geq cl(H) \geq \alpha cl(H)$. Thus H is $f\alpha g$-closed. □

As shown from the follows.

Example 5. *Let a set be $X = \{m, n\}$ with $F_\tau = \{0_X, \eta, 1_X\}$ where η is a fuzzy subset in X can be represented as $\eta(m) = \eta(n) = 0.5$. In a space (X, F_τ), then μ defined by $\mu(m) = \mu(n) = 0.4$ is $f\alpha g$-closed but not $f\tilde{g}$-closed.*

Proposition 3.6. *Entire $f\tilde{g}$-closed set is fgs-closed but not converse.*

Proof. Let H is a $f\tilde{g}$-closed set and K be a fuzzy open set of a space (X, F_τ) such that $K \geq H$, then $K \geq cl(H) \geq scl(H)$. Thus H is fgs-closed. □

As shown from the follows.

Example 6. *Let a set be $X = \{m, n\}$ with $F_\tau = \{0_X, \eta, 1_X\}$ where η is a fuzzy subset in X defined by $\eta(m) = \eta(n) = 0.5$. In a space (X, F_τ), then α defined by $\alpha(m) = \alpha(n) = 0.4$ is fgs-closed but not $f\tilde{g}$-closed.*

Proposition 3.7. *Entire $f\tilde{g}$-closed set is $f\tilde{g}_\alpha$-closed but not converse.*

Proof. Let H be a $f\tilde{g}$-closed subset of (X, F_τ) with K is a fsg-open set such that $K \geq H$ then we have $K \geq cl(H) \geq \alpha cl(H)$. Thus H is $f\tilde{g}_\alpha$-closed. □

As shown from the follows.

Example 7. *Let a set be $X = \{m, n\}$ with $F_\tau = \{0_X, \eta, \mu, \eta \vee \mu, 1_X\}$ where η, μ are fuzzy sets in X can be represented as $\eta(m) = 0.6, \eta(n) = 0$ and $\mu(m) = 0, \mu(n) = 0.3$. In a space (X, F_τ) , then α defined by $\alpha(m) = \alpha(n) = 0.3$ is $f\tilde{g}_\alpha$-closed but not $f\tilde{g}$-closed.*

Proposition 3.8. *Entire fuzzy α-closed set is $f\tilde{g}_\alpha$-closed but not converse.*

Proof. Let K be a fsg-open set so as to $K \geq H$, then $K \geq H = \alpha cl(H)$. Thus $f\tilde{g}_\alpha$-closed. □

As shown from the follows.

Example 8. *Let a set be $X = \{m, n\}$ and $F_\tau = \{0_X, \eta, 1_X\}$ whereas η is a fuzzy subset in X, it's elected by $\eta(m) = 1, \eta(n) = 0$. In the space (X, F_τ), then α defined by $\alpha(m) = 0, \alpha(n) = 1$ is $f\tilde{g}_\alpha$-closed but not fuzzy α-closed.*

Proposition 3.9. *Entire $f\tilde{g}$-closed set is $f\psi$-closed but not converse.*

Proof. Let H be a $f\tilde{g}$-closed set of a space (X, F_τ) with K is a fsg-open set such that $K \geq H$ then $H \geq cl(H) \geq scl(H)$. Thus H is $f\psi$-closed. □

As shown from the follows.

Example 9. *Let a set be $X = \{m, n\}$ with $F_\tau = \{0_X, \eta, 1_X\}$ whereas η is a fuzzy subset in X, it's elected by $\eta(m) = 1, \eta(n) = 0$. In a space (X, F_τ), then α defined by $\alpha(m) = 0, \alpha(n) = 0.5$ is $f\psi$-closed but not $f\tilde{g}$-closed.*

Proposition 3.10. *Entire $f\psi$-closed set is fsg-closed but not converse.*

Proof. Assuming that $H \leq K$ and K is fuzzy semi-open in (X, F_τ), then $scl(H) \leq K$. Thus H is fsg-closed.

As shown from the follows. □

Example 10. *Let a set be $X = \{m, n\}$ with $F_\tau = \{0_X, \eta, 1_X\}$ whereas η is a fuzzy subset in X, it's elected by $\eta(m) = \eta(n) = 0.5$. In the space (X, F_τ), then α defined by $\alpha(m) = \alpha(n) = 0.4$ is fsg-closed but not $f\psi$-closed.*

Proposition 3.11. *Entire fuzzy semi-closed set is $f\psi$-closed but not converse.*

Proof. Let H be a fuzzy semi-closed subset of a space (X, F_τ) such that $K \geq H$, then we have $K \geq H = scl(H)$. Thus H is $f\psi$-closed. □

As shown from the follows.

Example 11. *Let a set be $X = \{m, n\}$ with $F_\tau = \{0_X, \eta, 1_X\}$ whereas η is a fuzzy subset in X, it's elected by $\eta(m) = 1, \eta(n) = 0$. In the space (X, F_τ), then α defined by $\alpha(m) = 0, \alpha(n) = 0.5$ is $f\psi$-closed but not fuzzy semi-closed.*

Proposition 3.12. *Entire $f\omega$-closed set is fuzzy αgs-closed but not converse.*

Proof. Let H be a fuzzy ω-closed subset and K be a fuzzy semi-open set of a space (X, F_τ) such that $K \geq H$, then we have $K \geq cl(H) \geq \alpha cl(H)$. Thus H is fuzzy αgs-closed. □

As shown from the follows.

Example 12. *Let a set be $X = \{m, n\}$ with $F_\tau = \{0_X, \eta, 1_X\}$ whereas η is a fuzzy subset in X, it's elected by $\eta(m) = 1, \eta(n) = 0$. In the space (X, F_τ), then α defined by $\alpha(m) = 0, \alpha(n) = 0.5$ is fuzzy αgs-closed but not $f\omega$-closed.*

Proposition 3.13. *Entire $f\tilde{g}$-closed set is fuzzy αgs-closed but not converse.*

Proof. Let H be a $f\tilde{g}$-closed subset with K is a fuzzy semi-open set of a space (X, F_τ). Such that $K \geq H$, then $K \geq cl(H) \geq \alpha cl(H)$. Thus H is fuzzy αgs-closed. □

As shown from the follows.

Example 13. *Let a set be $X = \{m, n\}$ with $F_\tau = \{0_X, \eta, 1_X\}$ whereas η is a fuzzy subset in X, it's elected by $\eta(m) = 1, \eta(n) = 0$. In the space (X, F_τ), then α defined by $\alpha(m) = 0, \alpha(n) = 0.5$ is fuzzy αgs-closed but not $f\tilde{g}$-closed.*

Proposition 3.14. *Entire fuzzy αgs-closed set is fuzzy αg-closed but not converse.*

Proof. Assuming that $H \leq K$ and K is fuzzy open in (X, F_τ) such that $H \leq K$. Then $K \geq \alpha cl(H)$. Thus H is fuzzy αg-closed. ☐

As shown from the follows.

Example 14. *Let a set be $X = \{m, n\}$ with $F_\tau = \{0_X, \eta, 1_X\}$ whereas η is a fuzzy subset in X, it's elected by $\eta(m) = 0.3, \eta(n) = 0.6$. In the space (X, F_τ), then α defined by $\alpha(m) = \alpha(n) = 0.4$ is fuzzy αg-closed but not fuzzy αgs-closed set.*

Proposition 3.15. *Entire $f\omega$-closed set is fg-closed but not converse.*

Proof. Assuming that $H \leq K$ and K is fuzzy open set such that $K \geq H$. then $K \geq cl(H)$. Therefore H is fg-closed. ☐

As shown from the follows.

Example 15. *Let a set be $X = \{m, n\}$ with $F_\tau = \{0_X, \eta, 1_X\}$ whereas η is a fuzzy subset in X, it's elected by $\eta(m) = \eta(n) = 0.5$. In the space (X, F_τ), then α defined by $\alpha(m) = \alpha(n) = 0.4$ is fg-closed but not $f\omega$-closed.*

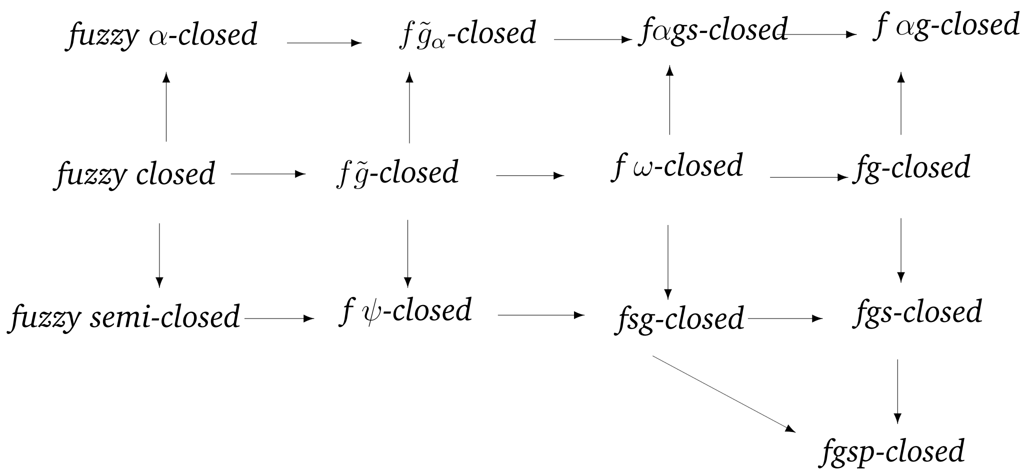
Remark 3.1. *The concepts of $f\tilde{g}$-closed sets are independent of the concepts of fuzzy α-closed sets and the concepts of fuzzy semi-closed sets.*

As shown from the follows.

Example 16. *Let a set be $X = \{m, n\}$ with $F_\tau = \{0_X, \eta, 1_X\}$ whereas η is a fuzzy subset in X, it's elected by $\eta(m) = \eta(n) = 0.5$. In the space (X, F_τ), then μ defined by $\mu(m) = \mu(n) = 0.4$ is $f\tilde{g}$-closed set but it is neither fuzzy α-closed nor fuzzy semi-closed.*

Example 17. *Let a set be $X = \{m, n\}$ with $F_\tau = \{0_X, \eta, 1_X\}$ whereas η is a fuzzy subset in X, it's elected by $\eta(m) = 1, \eta(n) = 0$. In the space (X, F_τ), then μ defined by $\mu(m) = 0.5, \mu(n) = 0$ is fuzzy α-closed as it's fuzzy semi-closed but not $f\tilde{g}$-closed.*

Remark 3.2. *Above outcomes are obtain as shown in the following diagram.*



None of the implications are reversible.

4. Some more properties of fuzzy $\tilde{g}$-closed sets

Proposition 4.1. *In a space (X, F_τ), if H and K are $f\tilde{g}$-closed sets $\Rightarrow H \vee K$ is $f\tilde{g}$-closed sets.*

Proof. If $H \vee K \leq A$ and H is fsg-open, then $H \leq A$ and $K \leq A$. Since H and K are $f\tilde{g}$-closed, $A \geq cl(H)$ and $A \geq cl(K)$ and therefore $cl(H) \vee cl(K) = cl(H \vee K) \leq A$. Hence $H \vee K$ is $f\tilde{g}$-closed set. □

Theorem 4.1. *In a space (X, F_τ), if H is $f\tilde{g}$-closed and $H \leq K \leq cl(H) \Rightarrow K$ is $f\tilde{g}$-closed.*

Proof. Let $K \leq G$ where G is fsg-open in (X, F_τ). Since $H \leq K$ and $H \leq G$. Since H is $f\tilde{g}$-closed in (X, F_τ), $cl(H) \leq G$. Since $K \leq cl(H), cl(K) \leq cl(H) \leq G$. Hence K is $f\tilde{g}$-closed. □

Proposition 4.2. *In a space (X, F_τ), H is a fsg-open and $f\tilde{g}$-closed $\Rightarrow H$ is fuzzy closed.*

Proof. Since H is fsg-open and $f\tilde{g}$-closed, $cl(H) \leq H$ and hence H is fuzzy closed in (X, F_τ). □

Theorem 4.2. *Let H be a $f\tilde{g}$-closed set of a space (X, F_τ), then H is fuzzy regular open $\Rightarrow scl(H)$ is also $f\tilde{g}$-closed set.*

Proof. Since H is fuzzy regular open in (X, F_τ), $H = int(cl(H))$. Then $scl(H) = H \vee int(cl(H)) = H$. Thus, $scl(H)$ is $f\tilde{g}$-closed. □

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