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M GENERALIZED OPEN SETS IN DOUBLE FUZZY TOPOLOGICAL SPACES

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ABSTRACT. In this paper we introduce some new class of generalized sets namely (r,κ)-fuzzy M generalized closed (resp. open) sets, (r,κ)-fuzzy M generalized closure (resp. interior) and (r,κ)-fuzzy M generalized q neighbourhood in double fuzzy topological spaces. Also, some of their fundamental properties and characterizations are discussed.

1. INTRODUCTION AND PRELIMINARIES

In 1986, Atanassov [1] started 'Intuitionistic fuzzy sets' and Coker [2] in 1997, initiated Intuitionistic fuzzy topological space. The term 'double' instead of 'intuitionistic' coined by Garcia and Rodabaugh [7] in 2005. In the previous two decades many analysts [9–11, 16] accomplishing more applications on dou ble fuzzy topological spaces. From 2011, M-open sets and maps were introduced in topological spaces by El-Maghrabi and Al - Johany [3–6].

X denotes a non-empty set, $I_1 = [0,1)$, $I_0 = (0,1]$, $I = [0,1]$, $0 = \underline{0}(X)$, $1 = \underline{1}(X)$, $r \in I_0$ *and* $\kappa \in I_1$ and always $1 \geq r + \kappa$. I^X is a family of all fuzzy sets on X. In 2002, Double fuzzy topological spaces (briefly, dfts), (X,η,η^*), (r,κ)-fuzzy open (resp. (r,κ)-fuzzy closed) (briefly (r,κ)-fo (resp. (r,κ)-fc)) set were given by Samanta and Mondal [13]. All other undefined notions are from [8–16] and cited therein.

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2. (r, κ)-FUZZY M-GENERALIZED CLOSED SETS

In this section, we define the concept of (r, κ)-fuzzy M-generalized closed sets. Some interesting properties and characterizations of them are also investigated.

Definition 2.1. *A fuzzy subset γ in a dfts (X, ρ, ρ^*) is called an*

(i) *(r, κ)-fuzzy M-generalized closed (briefly, (r, κ)-fMgc) set if*

$$MC_{\rho,\rho^*}(\gamma, r, \kappa) \leq \mu,$$

whenever $\gamma \leq \mu$ and μ is (r, κ)-fMo set.

(ii) *(r, κ)-fuzzy M-generalized open (briefly, (r, κ)-fMgo) set if*

$$\delta \leq MI_{\rho,\rho^*}(\gamma, r, \kappa))$$

whenever $\delta \leq \gamma$ and μ is (r, κ)-fMc set.

Also the complement of an (r, κ)-fMgc set is called as (r, κ)-fMgo set.

Definition 2.2. *Let (X, ρ, ρ^*) be a dfts. For $\gamma,\ \delta \in I^X$, the intersection of all (r, κ)-fMgc sets containing γ is called the (r, κ)-fuzzy M-generalized closure of γ and is denoted by $MgC_{\rho,\rho^*}(\gamma, r, \kappa) = \bigwedge\{\delta \in I^X : \delta \geq \gamma,\ \mu$ is a (r, κ)-fMgc set $\}$ and the union of all (r, κ)-fMgo sets contained in γ is called the (r, κ)-fuzzy M-generalized interior of γ and is denoted by $MgI_{\rho,\rho^*}(\gamma, r, \kappa) = \bigvee\{\delta \in I^X : \delta \leq \gamma,\ \mu$ is a (r, κ)-fMgo set $\}$.*

Example 1. *Let γ, μ, be fuzzy subsets of $X = \{l, m, n\}$ and defined as, $\gamma(l) = 0.3$, $\gamma(m) = 0.4$, $\gamma(n) = 0.5$; $\mu(l) = 0.6$, $\mu(m) = 0.5$, $\mu(n) = 0.5$. Consider the dfts (X, ρ, ρ^*) with*

$$\rho(\gamma) = \begin{cases} 1, & \text{if } \gamma = \underline{0} \text{ or } \underline{1}\ , \\ \frac{1}{2}, & \text{if } \gamma = \gamma,\ \mu, \\ 0, & o.w. \end{cases} \qquad \rho^*(\gamma) = \begin{cases} 0, & \text{if } \gamma = \underline{0} \text{ or } \underline{1}\ , \\ \frac{1}{2}, & \text{if } \gamma = \gamma,\ \mu, \\ 1, & o.w. \end{cases}$$

The fuzzy set, $\underline{0.4}$ is $(\frac{1}{2}, \frac{1}{2})$-f$M$gc set in (X, ρ, ρ^*).

Theorem 2.1. *Every (r, κ)-fc (resp. (r, κ)-fMc) set in (X, ρ, ρ^*) is (r, κ)-fMgc set, but not conversely.*

Example 2. *In Example 1, the fuzzy set $\underline{0.4}$ is $(\frac{1}{2}, \frac{1}{2})$-f$M$gc set in (X, ρ, ρ^*) but not an $(\frac{1}{2}, \frac{1}{2})$-fc set.*

Theorem 2.2. *Let γ be any fuzzy subset of X. Then*

(1) *γ is (r,κ)-fMgc if $\gamma = MgC_{\rho,\rho^*}(\gamma, r, \kappa)$.*
(2) *$MgC_{\rho,\rho^*}(\gamma, r, \kappa)$ is (r,κ)-fMgc in X.*

Theorem 2.3. *A finite union of (r,κ)-fMgo sets is an (r,κ)-fMgo set.*

Remark 2.1. *Union of two (r,κ)-fMgc sets need not be an (r,κ)-fMgc set.*

Example 3. *Let α, β and $\gamma \in I^X$, $X = \{l, m\}$, $\alpha(l) = 0.3$, $\alpha(m) = 0.5$; $\beta(l) = 0.2$, $\beta(m) = 0.7$; $\gamma(l) = 0.6$, $\gamma(m) = 0.5$. Consider the dfts (X, ρ, ρ^*) with*

$$\rho(\gamma) = \begin{cases} 1, & \textit{if } \gamma = \underline{0} \textit{ or } \underline{1}\ , \\ \frac{1}{3}, & \textit{if } \gamma = \alpha, \\ 0, & \textit{o.w.} \end{cases} \qquad \rho^*(\gamma) = \begin{cases} 0, & \textit{if } \gamma = \underline{0} \textit{ or } \underline{1}\ , \\ \frac{2}{3}, & \textit{if } \gamma = \alpha, \\ 1, & \textit{o.w.} \end{cases}$$

The fuzzy sets β and γ are $(\frac{1}{3}, \frac{2}{3})$-f$Mc$ sets and hence β and γ are $(\frac{1}{3}, \frac{2}{3})$-f$Mgc$ sets in (X, ρ, ρ^*). But $\beta \vee \gamma = (0.6_a,\ 0.7_b)$ is not $(\frac{1}{3}, \frac{2}{3})$-f$Mgc$ set in X.

Theorem 2.4. *A finite intersection of (r,κ)-fMgc sets is an (r,κ)-fMgc set.*

Remark 2.2. *Intersection of two (r,κ)-fMgo sets need not be an (r,κ)-fMgo set.*

Example 4. *In Example 1, The intersection of two $(\frac{1}{2}, \frac{1}{2})$-f$Mgc$ sets, $\beta = \underline{0.5}$, $\gamma(l) = 0.4,\ \gamma(m) = 0.3,\ \gamma(n) = 0.6$ then $\beta \wedge \gamma = (0.4_l,\ 0.3_m,\ 0.5_n)$ is not $(\frac{1}{2}, \frac{1}{2})$-f$Mgc$ set in X.*

Theorem 2.5. *If γ is (r,κ)-fMgc set and (r,κ)-fMo set in (X, ρ, ρ^*), then γ is (r,κ)-fMc in (X, ρ, ρ^*).*

Theorem 2.6. *If γ is (r,κ)-fMgc set in (X, ρ, ρ^*) and $\gamma \leq \delta \leq MC_{\rho,\rho^*}(\gamma, r, \kappa)$, then μ is (r,κ)-fMgc set in (X, ρ, ρ^*).*

Theorem 2.7. *If γ is (r,κ)-fMgo set in (X, ρ, ρ^*) and $MI_{\rho,\rho^*}(\gamma, r, \kappa) \leq \delta \leq \gamma$, then μ is (r,κ)-fMgo set in (X, ρ, ρ^*).*

Theorem 2.8. *Let (X, ρ, ρ^*) be the dfts and γ be a fuzzy set of X. Then γ is (r,κ)-fMgc set iff $\gamma\ \overline{q}\ \mu$ implies $MC_{\rho,\rho^*}(\gamma, r, \kappa)\ \overline{q}\ \mu$, $\forall (r,\kappa)$-fMc set μ of X.*

Theorem 2.9. *Let γ be (r,κ)-fMgc set in (X, ρ, ρ^*) and x_p be a fuzzy point of $(X, \rho, \rho^*) \ni x_p\ q\ MC_{\rho,\rho^*}(\gamma, r, \kappa)$ then $MC_{\rho,\rho^*}(x_p, r, \kappa)\ q\ \gamma$.*

Theorem 2.10. *Let $(Y,\ \rho_Y, \rho_Y^*)$ be a subspace of (X, ρ, ρ^*) and γ be a fuzzy set of Y. If γ is (r, κ)-fMgc set in X, then γ is (r, κ)-fMgc set in Y.*

Definition 2.3. *Let γ be a fuzzy set in dfts X and x_p be a fpt of X, then γ is called (r, κ)-fuzzy M-generalized neighbourhood (briefly, (r, κ)-fMg-nbhd) of x_p if there exists a (r, κ)-fMgo set μ of $X \ni x_p \in \delta \leq \gamma$.*

Definition 2.4. *Let $\gamma \in I^X$ and x_p be a fpt of X, then γ is called (r, κ)-fuzzy M-generalized q-neighbourhood (briefly, (r, κ)-fMgq-nbhd) of x_p if there exists a (r, κ)-fMgo set μ of X such that $x_p\ q\ \delta \leq \gamma$.*

Theorem 2.11. *γ is (r, κ)-fMgo set in X if and only if for each fuzzy point $x_p \in \gamma$, γ is a (r, κ)-fMg-nbhd of x_p.*

Theorem 2.12. *If γ and μ are (r, κ)-fMg-nbhd of x_p, then $\gamma \wedge \mu$ is also a (r, κ)-fMg-nbhd of x_p.*

Theorem 2.13. *Let γ be a fs of a dfts X. Then a fuzzy point $x_p \in MC_{\rho,\rho^*}(\gamma, r, \kappa)$ if and only if every (r, κ)-fMgq-nbhd of x_p is quasi-coincident with γ.*

Definition 2.5. *A dfts X is dfM-generalized $T_{\frac{1}{2}}$-space (briefly, $dfMgT_{\frac{1}{2}}$-space) if every (r, κ)-fMgc set in X is (r, κ)-fMc.*

Theorem 2.14. *A dfts (X, ρ, ρ^*) is $dfMT_{\frac{1}{2}}$-space iff every (r, κ)-fMgo is (r, κ)-fMo set in (X, ρ, ρ^*).*

3. CONCLUSION

It is interesting to work on (r, κ)-fMg closed sets. As applications of these sets we may try several mappings in dfts.

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