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# EFFECT OF COUPLE STRESS AND IMPACT OF CHEMICAL REACTION ON THE BEHAVIOUR OF ATMOSPHERIC AEROSOLS

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ABSTRACT. The model is developed as a course of improved understanding of interactions between aerosols in the couple stress fluid. Generalised dispersion model is used to precise unsteady convective diffusion for chemically reactive pollutant in the presence of electric field. The mean concentration scattering is attained as a function of axial distance, time and couple stress 'a'. The analytical results obtained for the dispersion coefficient and are illustrated by graphs for some values of electric number, couple stress parameter and reaction rate parameter. Using the obtained results conclusions are clearly depicted. It indicates that the effect of couple stress is predominating only for lesser values and the presence of chemical reaction minimises the concentration of aerosols.

# 1. INTRODUCTION

In recent years, most of the studies have focused on aerosol effects on earth environment and atmospheric pollution. This pollution has great influence on future of mankind, surpassing infectious diseases and nutritional disorders. Therefore, it is indispensible to develop new or improving the abilities of existing dispersion models to screen the atmospheric pollution due to aerosols. Many scientists and researchers are worked to study the effects of atmospheric aerosols in diverse situations. Anjusaini et al. [1] examined the effects of first order chemical reaction on the dispersion coefficient connected with laminar flow.

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Aparna et al. [2] observed that the solution of permeable sphere about its axis of symmetry in an incompressible couple stress fluid differs from the solution for the case of micro-polar fluids. Devarkar et al. [3] establishes the analytical solutions for the Poiseuille, Couette and generalized Couette flows under slip boundary conditions. Finlayson-Pitts [4] studied the principle way of dispersion of aerosols such as removal, migration, collision.

Jayaratne et al. [6] studied the electrical conductivity of atmospheric fluid is a function of temperature. Malkus et al. [7] studied the electric current produced by the cellular motions and atmosphere of the earth in which the electrical forces dominate in driving the fluid . Rudraiah et al. [8, 11] have investigated the effects of dispersion and concentration of aerosols under different assumptions. Meenapriya [9, 10] listed different analytical approaches to study the mechanisms of aerosols and splitting of aerosols. As a process of a better understanding of interactions between aerosol, their dispersion and concentration with and without chemical reaction are investigated in this study. The proposed model and analysis presented here finds the effect of couple stress fluid with aerosols in the atmosphere upon the rate of reaction rate parameter.

# 2. MATHEMATICAL FORMULATION

The considered system in this study is a two dimensional geometry as shown in Figure 1. It consists of an infinite horizontal channel extended to infinity in x-axis is and y-axis perpendicular to it. The channel containing couple stress fluid is bounded by electrodes which is electro-conducting impermeable rigid plates of different potentials at y = 0 and y = h. The electrical conductivity ( $\sigma$ )



FIGURE 1. Physical configuration

is negligibly small and hence the magnetic field is negligible, hence the electric field  $\vec{E}$  is conservative, i.e.,

$$\vec{E} = -\nabla\phi.$$

The basic equations are, Continuity equation

$$\nabla \cdot \vec{q} = 0.$$

Momentum equation

(2.1) 
$$\rho\left(\frac{\partial \vec{q}}{\partial t} + (\vec{q} \cdot \nabla)\vec{q}\right) = -\nabla p + \mu \nabla^2 \vec{q} - \lambda \nabla^4 \vec{q} + \rho_e \vec{E}.$$

Concentration equation

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} = D \left( \frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} \right) - KC.$$

The Conservation of charges

(2.2) 
$$\frac{\partial \rho e}{\partial t} + (\vec{q} \cdot \nabla)\rho e + \nabla \cdot \vec{J} = 0$$

Maxwell's equation

(2.3)  

$$\nabla \cdot \vec{E} = \frac{\rho e}{\epsilon_0} \quad (Gauss \ law),$$

$$\nabla \times \vec{E} = 0 \quad (Faraday's \ law),$$

$$\vec{J} = \sigma \vec{E} \quad (Ohm's \ law),$$

where  $\lambda$  is a couple stress parameter,  $\vec{q}$  the velocity,  $\rho$  the density, p the pressure,  $\vec{E}$  the electric field,  $\rho e$  the electric charge density,  $\sigma$  the current density.

All the above equations are solved using the following boundary conditions on velocity, couple stress, potential and concentration

(2.4) 
$$\begin{cases} u = 0 \text{ and } T = T_1 \text{ at } y = h \\ u = 0 \text{ and } T = T_0 \text{ at } y = 0 \\ \frac{d^2 u}{dy^2} = 0 \text{ at } y = 0 \text{ and } y = h \\ \begin{cases} \phi = \frac{V}{h}x \text{ at } y = 0 \\ \phi = \frac{V}{h}(x - x_0) \text{ at } y = h \end{cases}$$
$$C(0, X, y) = C_0 \text{ for } |x| \le \frac{1}{2}x_s,$$
$$C(0, X, y) = 0 \text{ for } |x| > \frac{1}{2}x_s,$$
$$\frac{\partial C}{\partial y}(t, X, h) = 0$$

$$\frac{\partial C}{\partial y}(t, X, -h) = 0$$
$$C(t, \infty, y) = \frac{\partial C}{\partial y}(t, \infty, y) = 0$$
$$C(t, X, y) = finite$$

Here  $x_s$  is the length of initial slug,  $C_0$  is the concentration of the slug. In cartesian form, equation (2.1) becomes

$$\mu \nabla^2 u - \frac{\partial p}{\partial x} + \rho e E_x - \nabla^4 u = 0$$

The electrical conductivity is assumed to vary linearly with temperature, since the couple stress fluid is poorly conducting. The velocity is independent of time and all physical quantities except pressure and concentration are independent of x. Using the following dimensionless quantities,

$$Y^* = \frac{y}{h}; u^* = \frac{u}{\left(\frac{V}{h}\right)}; E_x^* = \frac{E_x}{\left(\frac{V}{h}\right)}; \rho e^* = \frac{\rho e}{\left(\frac{\epsilon_0 V}{h^2}\right)}; x^* = \frac{x}{h};$$
$$Pe = \frac{\underline{u}h}{D}; \theta = \frac{C}{C_0}; \tau = \frac{tD}{h^2}; \beta^2 = \frac{kh^2}{D};$$

where V is electric potential, whose value is obtained through electrodes. We assume that the pollutants in the fluid is isotropic and homogenous so that viscosity  $\mu$  and the molecular diffusivity D are constants,

(2.5) 
$$\frac{d^4u}{dy^4} - a^2 \frac{d^2u}{dy^2} - a^2 W e \rho e E_x = a^2 P,$$

(2.6) 
$$\frac{\partial\theta}{\partial\tau} + U^* \frac{\partial\theta}{\partial\varsigma} = \frac{1}{Pe^2} \frac{\partial^2\theta}{\partial\varsigma^2} + \frac{\partial^2\theta}{\partial Y^2} - \beta^2\theta,$$

where  $a = \frac{h}{l}$  is the couple stress parameter,  $Pe = \frac{h\bar{u}}{D}$  is the Peclet number,  $We = \frac{\epsilon_0 V^2}{\mu}$ ,  $l = \sqrt{\frac{\lambda}{\mu}}$ , the dimensionless velocity in a moving coordinate is  $U^* = \frac{u-\bar{u}}{\bar{u}}$ ,  $X = x - \tau$  is the dimensionless axial coordinate moving with the average velocity  $\bar{u}$ .

The dimensionless initial and boundary conditions are

(2.7) 
$$u = 0$$
 at  $y = 0, 1;$ 

(2.8) 
$$\frac{d^2u}{dy^2} = 0$$
 at  $y = 0, 1;$ 

(2.9)  

$$\theta(0, X, Y) = 1 \quad \text{for} \quad |x| \le \frac{1}{2}x_s;$$

$$\theta(0, X, Y) = 0 \quad \text{for} \quad |x| > \frac{1}{2}x_s;$$

$$\frac{\partial \theta}{\partial \eta}(\tau, X, 1) = 0;$$

$$\frac{\partial \theta}{\partial \eta}(\tau, X, -1) = 0;$$

$$\theta(\tau, \infty, Y) = \frac{\partial \theta}{\partial y}(\tau, \infty, Y) = 0;$$
(2.10)  

$$\theta(\tau, X, Y) = finite;$$

$$\phi = x$$
 at  $y = 0$  and  $\phi = x - x_0$  at  $y = 1$ .

In a poorly conducting fluid,  $\sigma \ll 1$  and hence any perturbation on it is negligible and the value of  $\rho e E_x$  is calculated from (2.2) to (2.3) using (2.4) and (2)

$$\rho e = \nabla \cdot \vec{E} = -\nabla^2 \phi = \frac{-x_0 \alpha^2 e^{-\alpha y}}{1 - e^{-\alpha}}; E_x = -1.$$

Therefore,

$$\rho e E_x = \frac{x_0 \alpha^2 e^{-\alpha y}}{1 - e^{-\alpha}}.$$

The solution of equation (2.5) satisfying the condition (2.7) and (2.8) is

$$u = K_2 y - K_3 K_2 y + K_4 (y - y^2) - K_1 + K_1 e^{ay} + K_3 - K_3 e^{ay} - K_5 + K_5 e^{ay} - K_6 2 \sin h \sin hay,$$

where,  $K_1 = \left(\frac{a_0}{\alpha^2(\alpha^2 - a^2)}\right)$ ;  $K_2 = (1 - e^{-\alpha})$ ;  $K_3 = \left(\frac{a_0}{a^2(\alpha^2 - a^2)}\right)$ ;  $K_4 = \frac{P}{2}$ ;  $K_5 = \frac{P}{\alpha^2}$ ;  $K_6 = \frac{1}{\sinh \sinh a} \left(\frac{a_0(e^{-a} - e^a)}{a^2(\alpha^2 - a^2)} + \frac{a^2 P(e^a - 1)}{a^4}\right)$ ; The dispersion and concentration of atmospheric aerosols are discussed in

two separate cases in the proceeding subsections.

2.1. **Dispersion of aerosols in the presence of chemical reaction.** The solution of (2.6) is written as a series expansion in the form (Using Gill and Sankara-subramanian [5])

(2.11) 
$$\theta(\tau, X, Y) = f_0(\tau, Y) \theta_m(\tau, Y) + f_1(\tau, Y) \frac{\partial \theta_m}{\partial X}(\tau, X) + f_2(\tau, Y) \frac{\partial^2 \theta_m}{\partial X^2}(\tau, X) + \dots,$$

where  $\theta_m = \frac{1}{2} \int_{-1}^{1} \theta dY$ , is the non-dimensional cross sectional average concentration. using the definition of  $\theta_m$ , and integrating equation (2.6) with respect to Y in [-1, 1] we get

(2.12) 
$$\frac{\partial \theta_m}{\partial \tau} = \frac{1}{Pe^2} \frac{\partial^2 \theta_m}{\partial X^2} - \frac{1}{2} \frac{\partial}{\partial X} \int_{-1}^1 U^* \theta dY.$$

Now assume that the process of distributing  $\theta$  is diffusive in nature right from time, the Generalized dispersion model [7, 8] with time-dependent dispersion coefficient as

(2.13) 
$$\frac{\partial \theta_m}{\partial \tau} = K_1 \frac{\partial \theta_m}{\partial X} + K_2 \frac{\partial^2 \theta_m}{\partial X^2} + K_3 \frac{\partial^3 \theta_m}{\partial X^3} + \dots,$$

where  $K_i$ 's are given by

(2.14) 
$$K_i(\tau) = \frac{\delta_{i2}}{Pe^2} - \int_0^1 U^* f_{i-1}(\tau, Y) dY, \ (I = 1, 2, 3, \ldots).$$

Here f - 1 = 0 and  $\delta_{i2}$  is the Kronecker delta. Equation (2.13) is solved subject to the conditions

(2.15) 
$$\theta_m(0,X) = 1 \quad |X| \le \frac{1}{2}X_s; \quad \theta_m(0,X) = 0 \quad |X| \ge \frac{1}{2}X_s; \theta_m(\tau,\infty) = 0.$$

Introducing (2.12) in to (2.11), rearranging terms we get,

$$\frac{\partial^{k+1}\theta_m}{\partial \tau X^k} = \sum_{k=1}^{\infty} K_i(\tau) \frac{\partial^{i+k}\theta_m}{\partial X^{i+1}},$$

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$$\begin{split} & \left[\frac{f_1}{\partial \tau} - \frac{\partial^2 f_1}{\partial Y^2} + U^* + K_1(\tau) + \beta^2 f_1\right] \frac{\partial \theta_m}{\partial X} \\ & + \left[\frac{\partial f_2}{\partial \tau} - \frac{\partial^2 f_2}{\partial Y^2} + U^* f_1 + \beta^2 f_2 + k_1(\tau) f_1 + k_2(\tau) - \frac{1}{Pe^2}\right] \frac{\partial^2 \theta_m}{\partial X^2} \\ & + \sum_{k=1}^{\infty} \left[\frac{\partial f_{k+2}}{\partial \tau} - \frac{\partial^2 f_{k+2}}{\partial Y^2} + U^* f_{k+1} + k_1(\tau) f_{k+1} + \beta^2 f_{k+2} \right. \\ & + \left(k_2(\tau) - \frac{1}{Pe^2}\right) f_k + \sum_{i=3}^{k+2} k_i f_{k+2-i}\right] = 0, \end{split}$$

with  $f_0 = 1$ . Here,

$$(2.16) \quad U^* = \frac{1}{F_1} \Big[ (k_1 k_2 - k_3 k_2 + k_4) y - k_4 y^2 + k_1 e^{-\alpha y} + (k_5 - k_3 - k_6) e^{ay} + k_6 e^{-ay} + F_0 \Big]$$

and

$$F_{1} = \frac{K_{1}K_{2}}{4} - \frac{K_{3}K_{2}}{4} + \frac{K_{4}}{12} - K_{1}K_{7} + K_{3}K_{8} - K_{5}K_{8} - K_{6}K_{9},$$
  

$$F_{0} = K_{3} - K_{1}K_{7} + K_{5}K_{8} - K_{6}K_{9} - K_{1} - K_{3}K_{8} - K_{5} - \frac{1}{4}(K_{1}K_{2} - K_{3}K_{2}) + \frac{K_{4}}{12}$$

Equating the coefficients of  $\frac{\partial^k \theta_m}{\partial X^k}$ , where (k = 1, 2, 3, ...) in (2.15) to zero, we attain the infinite set of non-homogenous, parabolic partial differential equations

(2.17) 
$$\frac{\partial f_1}{\partial \tau} = \frac{\partial^2 f_1}{\partial Y^2} - U^* f_0 - K_1(\tau) f_0 - \beta^2 \tau$$
$$\frac{\partial f_2}{\partial \tau} = \frac{\partial^2 f_2}{\partial Y^2} - U^* f_1 - \beta^2 f_2 - K_1(\tau) f_1 - K_2(\tau) + \frac{1}{Pe^2}$$
$$\frac{\partial f_{k+2}}{\partial \tau} = \frac{\partial^2 f_{k+2}}{\partial Y^2} - U^* f_{k+1} - K_1(\tau) f_{k+1} - \left(K_2(\tau) - \frac{1}{Pe^2}\right)$$
$$f_k - \sum_{i=3}^{k+2} K_i f_{k+2-i}.$$

Since  $\theta$  has to satisfy the conditions (2.9) to (2.10)  $f_k(0,Y) = \delta_{k0}$  for  $k = 0, 1, 2, 3, \ldots$ 

$$\frac{\partial f_k}{\partial Y}(\tau, -1) = 0, \ \frac{\partial f_k}{\partial Y}(\tau, 1) = 0, \ k = 0, 1, 2, 3, \dots$$

Further

(2.18) 
$$\int_{-1}^{1} f_k(\tau, Y) dt = 0 \quad \text{for} \quad k = 1, 2, 3, \dots$$

From equation (2.14) for i = 1, using  $f_0 = 1$ , we get  $K_1$  as

$$K_1(\tau) = 0.$$

From (2.14) and (2.16) we get,

(2.19) 
$$K_1(\tau) = \frac{1}{P_e^2} - \int_0^1 U f_1 dY,$$

and we write

$$(2.20) f_1 = f_{10}(Y) + f_{11}(\tau, Y),$$

where  $f_{10}$  relates to an infinitely wide slug which is independent of  $\tau$  satisfying  $\frac{df_{10}}{dY} = 0$  at  $Y = \pm 1$  and

(2.21) 
$$\int_{-1}^{1} f_{10} dY = 0,$$

 $f_{11}$  is  $\tau$  -dependent. Then (2.17) using (2.20), becomes

(2.22) 
$$\frac{d^2 f_{10}}{\partial Y^2} = \frac{1}{F_1} \Big[ (K_1 K_2 - K_3 K_2 + K_4) Y - K_4 Y^2 + K_1 e^{-\alpha Y} + (K_5 - K_3 - K_6) e^{aY} + K_6 e^{-aY} + F_0 \Big] + \beta^2 f_{10}(\tau),$$

where  $F_0 = K_3 - K_1 K_7 + K_5 K_8 - K_6 K_9 - K_1 - K_3 K_8 - K_5 - \frac{1}{4} (K_1 K_2 - K_3 K_2) + \frac{K_4}{12}$ ,

(2.23) 
$$\frac{\partial f_{11}}{\partial \tau} = \frac{\partial^2 f_{11}}{\partial \eta^2}.$$

Solution of (2.22) satisfying (2.21) is

$$f_{10} = C_1 \cosh \cosh \beta Y + C_2 \sinh \sinh \beta Y - \frac{Y}{\beta^2} \left( \frac{K_1 K_2 - K_3 K_2 + K_4}{F_1} \right) + \frac{K_4 Y^2}{F_1 \beta^2} + \frac{K_1 e^{-\alpha Y}}{F_1 (\alpha^2 - \beta^2)} + \frac{(K_5 - K_3 - K_6)}{F_1 (a^2 - \beta^2)} e^{aY} + \frac{K_6 e^{-aY}}{F_1 (a^2 - \beta^2)} + F,$$

where

$$C_{1} = \frac{1}{\cosh\cosh\beta} \left[ \frac{K_{1}K_{2} - K_{3}K_{2} + K_{4}}{F_{1}\beta^{2}} \left( 1 - \frac{\sinh\sinh\beta}{\beta} \right) - \frac{\alpha K_{1}\sinh\sinh\beta}{\beta F_{1}(\alpha^{2} - \beta^{2})} + \frac{a(K_{5} - K_{3})\sinh\sinh\beta}{\beta F_{1}(a^{2} - \beta^{2})} - \frac{K_{4}}{F_{1}\beta^{2}} \left( 1 + \frac{2}{\beta^{2}} \right) - \frac{K_{1}e^{-\alpha}}{F_{1}(\alpha^{2} - \beta^{2})} - \frac{(K_{5} - K_{3} - K_{6})e^{a}}{F_{1}(a^{2} - \beta^{2})} - \frac{K_{6}e^{-a}}{F_{1}(a^{2} - \beta^{2})} + \frac{F_{0}}{F_{1}\beta^{2}} \right],$$

$$C_{2} = \left[ \frac{K_{1}K_{2} - K_{3}K_{2} + K_{4}}{F_{1}\beta^{3}} + \frac{\alpha K_{1}}{\beta F_{1}(\alpha^{2} - \beta^{2})} - \frac{a(K_{5} - K_{3})}{\beta F_{1}(a^{2} - \beta^{2})} \right],$$

$$F = \frac{2K_4}{F_1\beta^4} - \frac{F_0}{F_1\beta^2}.$$

Equation (2.23) is the well-known heat conduction type and its solution satisfying (2.18) is of the form

$$f(x) = a_0 + \sum_{n=1}^{\infty} \left( a_n \cos \cos \frac{n\pi x}{L} + b_n \sin \sin \frac{n\pi x}{L} \right)$$
$$f_{11} = \sum_{n=1}^{\infty} \left( A_n e^{-\lambda_n^2 X} \cos \cos \lambda_n Y \right),$$

where

$$A_n = (C_1\beta + C_2\beta\cosh\cosh\beta)\frac{\cos\cos\lambda_n}{\lambda_n^2} - \frac{C_2\beta}{\lambda_n^2}$$

and  $\lambda_n = n\pi$ ,

$$f_{1}(Y) = C_{1} \cosh \cosh \beta Y + C_{2} \sinh \sinh \beta Y - \frac{Y}{\beta^{2}} \left( \frac{K_{1}K_{2} - K_{3}K_{2} + K_{4}}{F_{1}} \right) + \frac{K_{4}}{F_{1}\beta^{2}} \left( Y^{2} + \frac{2}{\beta^{2}} \right) + \frac{K_{1}e^{-\alpha Y}}{F_{1}(\alpha^{2} - \beta^{2})} + \frac{(K_{5} - K_{3} - K_{6})}{F_{1}(a^{2} - \beta^{2})} e^{aY} + \frac{K_{6}e^{-aY}}{F_{1}(a^{2} - \beta^{2})} - \frac{F_{0}}{F_{1}\beta^{2}} + e^{-\beta^{2}\tau} \sum_{n=1}^{\infty} A_{n}e^{-(\lambda_{n}^{2}\tau)}Y.$$

Substituting (2.24) in to (2.19) and integrating we get  $K_2(\tau) = \frac{1}{P_e^2} - \int_0^1 U f_1 dY$ . Thus  $\frac{\partial \theta_m}{\partial \tau} = K_2 \frac{\partial^2 \theta_m}{\partial \tau}$ , which is the usual dispersion equation. The exact solution

of  $\theta_m$  is given by

$$\theta_m(X,\tau) = \frac{1}{2} \left[ \left( \frac{\frac{X_s}{2} + X}{2\sqrt{T}} \right) + \left( \frac{\frac{X_s}{2} - X}{2\sqrt{T}} \right) \right],$$
$$T = \int_0^\tau K_2(z) dz, \quad \text{and} \quad erf(x) = \frac{1}{\pi} \int_0^x e^{-z^2} dz.$$

The analytical results are depicted graphically through figures.

2.2. Dispersion of aerosols without chemical reaction. Suppose that there is no chemical reaction takes place inside the channel (i.e.,  $\beta = 0$ ) then, (2.6) becomes

$$\frac{\partial \theta}{\partial \tau} + U^* \frac{\partial \theta}{\partial \varsigma} = \frac{1}{Pe^2} \frac{\partial^2 \theta}{\partial \varsigma^2} + \frac{\partial^2 \theta}{\partial \eta^2}.$$

Using [5] and proceeding as in section 2.1 we obtain the following results

$$\begin{split} f_{10} &= \frac{1}{F_1} \Big[ \frac{Y^3}{6} \Big( \frac{K_1 K_2 - K_3 K_2 + K_4}{F_1} \Big) - \frac{K_4 Y^4}{12} + \frac{K_1 e^{-\alpha Y}}{\alpha^2} \\ &- \frac{(K_5 - K_3 - K_6)}{a^2} e^{aY} + \frac{K_6 e^{-aY}}{a^2} + \frac{F_0 Y^2}{2} \\ &+ \frac{1}{F_1} \Big[ \frac{K_1}{\alpha} - \frac{(K_5 - K_3 - K_6)}{a} + \frac{K_6}{a} \Big] \end{split}$$

$$f_{1}(Y) = -\frac{1}{F_{1}} \Big[ (K_{1}K_{2} - K_{3}K_{2} + K_{4}) + K_{4} \Big( \frac{Y^{4}}{12} \Big) \\ + \frac{K_{1}e^{-\alpha Y}}{\alpha^{2}} + \frac{(K_{5} - K_{3} - K_{6})}{(a^{2})} e^{aY} + \frac{K_{6}e^{-aY}}{(a^{2})} - \frac{F_{0}y^{2}}{2} \Big] + F_{2}y$$

$$(2.25) \qquad + F_{3} + \sum_{n=1}^{\infty} A_{n}e^{-(\lambda_{n}^{2}\tau)}Y) \\ F_{0} = (K_{3} + K_{1}K_{7} - K_{5}K_{8} + K_{6}K_{9} - K_{1} - K_{3}K_{8} - K_{5}) \\ - \frac{1}{4} \Big( \frac{K_{4}}{12} - K_{3}K_{2} + K_{1}K_{2} \Big);$$

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$$\begin{split} F_1 &= \frac{K_4}{12} - (K_1K_7 - K_5K_8 + K_6K_9 - K_3K_8) - \frac{1}{4}(K_3K_2 - K_1K_2); \\ F_2 &= \frac{1}{F_1} \Big[ \frac{K_1}{\alpha} - \frac{(K_5 - K_3 - K_6)}{a} + \frac{K_6}{a} \Big]; \\ F_3 &= \frac{1}{F_1} \Big( \frac{1}{6}(K_1K_2 - K_3K_2 + K_4) - K_4 \Big( \frac{1}{12} \Big) + \frac{K_1e^{-\alpha Y}}{(\alpha^2)} - \frac{(K_5 - K_3 - K_6)}{(a^2)} e^a \\ &- \frac{F_0}{2} + \frac{K_1}{\alpha} - \frac{(K_5 - K_3 - K_6)}{a} + \frac{K_6}{a}. \end{split}$$

Substituting (2.25) in to (2.19) and integrating we get dispersion co-efficient of aerosols in the absence of chemical reaction  $K_2^*(\tau) = \frac{1}{P_e^2} - \int_0^1 U f_1 dY$ . Thus  $\frac{\partial \theta_m}{\partial \tau} = K_2 \frac{\partial^2 \theta_m}{\partial \tau}$ , which is the usual dispersion equation. The exact solution of mean concentration of aerosols without chemical reaction is given by

$$\begin{aligned} \theta_m(X,\tau) &= \frac{1}{2} \Big[ \Big( \frac{\frac{X_s}{2} + X}{2\sqrt{T}} \Big) + \Big( \frac{\frac{X_s}{2} - X}{2\sqrt{T}} \Big) \Big], \\ T &= \int_0^\tau K_2(z) dz, \quad \text{and} \quad erf(x) = \frac{1}{\pi} \int_0^x e^{-z^2} dz. \end{aligned}$$

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The analytical results are illustrated graphically through figures which are discussed in next section.

## 3. RESULTS AND DISCUSSION

A mathematical modeling of velocity profile, dispersion coefficient and mean concentration distribution of atmospheric aerosols with and without chemical reaction under the influence of electric field are discussed. The graphical illustrations are presented .The velocity profile is depicted in figure 2. which shows that, when electric number increases the velocity in the presence of couple stress increases.

3.1. Dispersion and mean concentration in the presence of chemical reaction. The dispersion coefficient  $K_2(\tau)$  values are calculated for some values of couple stress a, reaction rate parameter  $\beta$ , dimensionless time  $\tau$  and electric number We and are plotted in figures 3, 4 and 5. It denotes that the dispersion coefficient increases with increase in electric number and couple stress parameter but decreases with increase in reaction rate parameter. The variation of mean concentration  $\theta_m$  with axial distance x are plotted in figures 6 and 7.

These figures reveals that increase in couple stress parameter and reaction rate parameter the concentration distribution increases.

3.2. Dispersion and concentration in the absence of chemical reaction. The dispersion coefficient  $K_2^*(\tau)$  values are computed for various values of couple stress a are plotted against dimensionless time  $\tau$  and the axial distance x in figures 8 and 9. It indicates that increase in couple stress values increases the dispersion coefficient. Figures 10 and 11 deals with mean concentration  $\theta_m^*$ against dimensionless time varying couple stress and axial distance. It shows that increase in couple stress and axial distance decreases mean concentration of atmospheric aerosols.



Figure 2. Plots of velocity u verus yfor different values of electric number We

dimensionless time  $\tau$ 



Figure 3. Plots of electric number on dispersion coefficient  $K_2(\tau)$ in the presence of couple stress.



parameter  $K_2(\tau)$ versus dimensionless time

4. CONCLUSION

The axial dispersion in a couple stress fluid bounded by electrodes at boundaries with and without chemical reaction is studied using generalized dispersion model. It concludes that the enhancement of electric field and couple stress increases the dispersion and mean concentration of aerosols. Also by enhancing the rate of the reaction the dispersion of atmospheric aerosols is reduced. In the

absence of chemical reaction by rising couple stress value, the aerosol dispersion minimizes.



Figure 6. Plots of mean concentration Figure 7. Plots of mean concentration  $\theta_m$  versus x for different values of couple stress parameter



Figure 8. Effect of Couple Stress on Dispersion Coefficient  $K_2^*(\tau)$ versus dimensionless time  $\tau$ 



Figure 10. Mean concentration  $\theta_m^*$  varying along  $\tau$ for different values of a.



 $\theta_m$  versus x for different values of reaction rate parameter



Figure 9. Plots of mean concentration  $\theta_m^*$  versus versus x



Figure 11. Mean concentration  $\theta_m^*$  varying along  $\tau$ for different values of x.

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