

DISPERSION OF A SOLUTE IN A COUPLE STRESS FLUID WITH CHEMICAL REACTION USING GENERALIZED DISPERSION MODEL

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ABSTRACT. The present work was carried out to investigate the dispersion of a solute in a non-Newtonian fluid flow in an inclined channel bounded by porous beds using Generalized Dispersion Model. This model is used to analysis the dispersion of solute in blood flow and also externally and internally there is an effect of chemical reaction and magnetic field. It found that the dispersion coefficient decreases due the chemical reaction but accumulating when there is no chemical reaction. The impacts of couple stress parameter, Reynolds number, Froude number, Hartmann number on the velocity profile, dispersion coefficient and mean concentration are discussed in detail with the help of graphs.

1. INTRODUCTION

In biological science, to study blood as a non-Newtonian fluid for the dispersion of solute with the effect of chemical reaction in the fluid film region bounded by porous beds. Several biological applications to study the dispersion of a soluble matter in fluids plays vital role in blood circulation. Recently, there has been a growing interest in the fluid dynamical studies of various characteristics of dispersion of solute in blood flow under different conditions see [1, 2, 4, 6, 11, 15]. The dispersion of soluble matter in fluid flow has been intensively researched, since the classic papers on the subject by [13, 14].

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Annapurna [3] utilized the generalized dispersion model to study the dispersion of a solute with the impact of transverse magnetic field. Pal [10] assuming the chemical reaction to study the unsteady dispersion of a solute in a liquid and its packed porous medium by utilizing the model of [7].

Jaafar et al., [8] considering the chemical reaction on dispersion of solute in blood flow and utilized the model by [7]. Umavathi et al., [16] investigated the dispersion of the solute in an electrically conducting immiscible channel flow. In present paper is to established the unsteady convective diffusion in blood flow for couple stress fluid with under impact of chemical reaction and magnetic field by using the generalized dispersion model of [7]. It helps to study about the transport of nutrients (drug) through fluid region into tissue region and also it plays an vital role in cardiovascular flow.

2. FORMULATION OF THE PROBLEM

The physical configuration of the problem is shown in Figure 1. Assuming the fluid is to be steady, fully developed, unidirectional and incompressible. The Fluid Film Region is bounded by Porous Tissue Region and its distance $2h$. The solute diffuses in a fully developed flow through a parallel plate inclined channel bounded by porous beds.

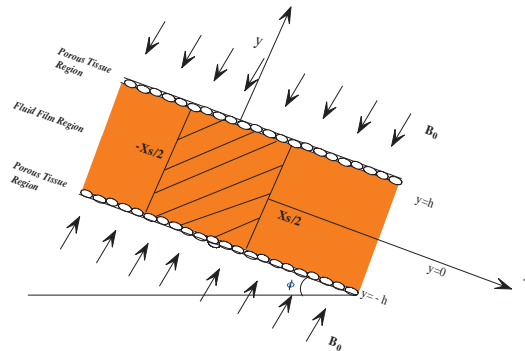


Figure 1: Physical model of the problem

The continuity and momentum equations which govern the steady incompressible blood flow subjected to external applied electric and magnetic fields are given by

Fluid Film Region

$$\frac{\partial u^*}{\partial x} = 0$$

$$(2.1) \quad -\frac{\partial p^*}{\partial x} + \mu \frac{\partial^2 u^*}{\partial y^{*2}} - \lambda \frac{\partial^4 u^*}{\partial y^{*4}} - B_0^2 \sigma_0 u^* + \rho g \sin \phi = 0$$

$$-\frac{\partial p^*}{\partial y^*} - \rho g \cos \phi = 0.$$

The concentration C satisfying the convective diffusion equation is

$$\frac{\partial C}{\partial t} + u^* \frac{\partial C}{\partial x} = D \left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^{*2}} \right) - K_1 C.$$

Porous Tissue Region

$$\frac{\partial u_p^*}{\partial x} = 0$$

$$(2.2) \quad -\frac{\partial p^*}{\partial x} - \frac{\mu}{k}(1 + \beta_1)u_p^* + \rho g \sin \phi = 0$$

$$(2.3) \quad -\frac{\partial p^*}{\partial y^*} - \rho g \cos \phi = 0,$$

with required initial and boundary conditions are

$$(2.4) \quad \frac{\partial u^*}{\partial y^*} = \frac{-\alpha}{\sqrt{k}}(u^* - u_p^*) \text{ at } y^* = h$$

$$(2.5) \quad \frac{\partial u^*}{\partial y^*} = \frac{\alpha}{\sqrt{k}}(u^* - u_p^*) \text{ at } y^* = -h$$

$$(2.6) \quad \frac{\partial^2 u^*}{\partial y^{*2}} = 0 \text{ at } y^* = \pm h$$

$$C = \begin{cases} C_0, |x| \leq \frac{x_s}{2} \\ 0, |x| > \frac{x_s}{2} \end{cases} \quad \text{at } t = 0$$

$$\frac{\partial C}{\partial y^*} = 0 \text{ at } y^* = \pm h$$

$$(2.7) \quad C = \frac{\partial C}{\partial x} = 0 \quad \text{at } x \rightarrow \infty,$$

where u^* is the x component of velocity, p^* is the pressure, μ is the viscosity of the fluid, λ is the couple stress parameter, B_0 is the applied magnetic field, σ_0 is the electrical conductivity, t is the time, D is the molecular diffusivity. Equation (2.2) is the Darcy equation, incompressible couple stress parameter β_1 in to the

Darcy equation, k is the permeability of the porous medium and u_p^* is the Darcy velocity, α is the slip parameter, C_0 is the reference concentration and K_1 is the first order reaction rate parameter.

Equations (2.4) and (2.5) are slip condition [5]. Equation (2.6) specifies the vanishing of the couple stress.

We define the non-dimensional quantities

$$U = \frac{u^*}{\bar{u}}; \quad U_p = \frac{u_p^*}{\bar{u}}; \quad \xi_s = \frac{x_s}{hPe}; \quad \eta = \frac{y}{h}; \quad p = \frac{p^*}{\rho \bar{u}^2}; \quad \xi' = \frac{x}{hPe};$$

$$\tau = \frac{Dt}{h^2}; \quad Pe = \frac{h\bar{u}}{D}; \quad \theta = \frac{C}{C_0}.$$

Equations (2.1) to (2.3) in dimensionless form are

Fluid Film Region

$$(2.8) \quad \frac{\partial^4 U}{\partial \eta^4} - a^2 \frac{\partial^2 U}{\partial \eta^2} + a^2 M^2 U + \frac{Re \, a^2}{Fr} \sin \phi = P \, a^2 \, Re$$

and

$$(2.9) \quad \frac{\partial \theta}{\partial \tau} + U \frac{\partial \theta}{\partial \xi'} = \frac{1}{Pe^2} \left(\frac{\partial^2 \theta}{\partial \xi'^2} + \frac{\partial^2 \theta}{\partial \eta^2} \right) - \alpha_1^2 \theta$$

We define the axial coordinate moving with the average velocity of flow as $x_1 = x - \tau \bar{u}$ which is in dimensionless form $\xi = \xi' - \tau$, where $\xi' = \frac{x_1}{hPe}$. Then equation (2.13) becomes

$$(2.10) \quad \frac{\partial \theta}{\partial \tau} + U' \frac{\partial \theta}{\partial \xi} = \frac{1}{Pe^2} \left(\frac{\partial^2 \theta}{\partial \xi^2} + \frac{\partial^2 \theta}{\partial \eta^2} \right) - \alpha_1^2 \theta,$$

with $U' = \frac{U - \bar{U}}{\bar{U}}$.

Porous Tissue Region

$$U_p = \frac{Re}{\sigma^2(1 + \beta_1)} \left(P + \frac{1}{Fr} \right)$$

The initial and boundary conditions (2.4) to (2.7) in dimensionless form

$$(2.11) \quad \frac{\partial U}{\partial \eta} = -\alpha \sigma (U - U_p) \text{ at } \eta = 1$$

$$(2.12) \quad \frac{\partial U}{\partial \eta} = \alpha \sigma (U - U_p) \text{ at } \eta = -1$$

$$(2.13) \quad \frac{\partial^2 U}{\partial \eta^2} = 0 \text{ at } \eta = \pm 1$$

$$\theta = \begin{cases} 1, & |\xi| \leq \frac{\xi_s}{2} \\ 0, & |\xi| > \frac{\xi_s}{2} \end{cases} \text{ at } \tau = 0$$

$$\frac{\partial \theta}{\partial \eta} = 0 \text{ at } \eta = \pm 1$$

$$(2.14) \quad \theta = \frac{\partial \theta}{\partial \xi} = 0 \text{ at } \xi \rightarrow \infty,$$

where $a = \frac{h}{l}$ is the couple stress parameter, $l = \sqrt{\frac{\lambda}{\mu}}$ is the material constant characterizing the couple stress property of the fluid, $M^2 = \frac{B_0^2 \sigma_0 h^2}{\mu}$ is the square of the Hartmann number, $\alpha_1^2 = \frac{K_1 h^2}{D}$ is the chemical reaction rate coefficient, $P = -\frac{1}{Pe} \frac{\partial p}{\partial \xi'}$, $Re = \frac{\rho \bar{u} h}{\mu}$ is the Reynolds number, $Pe = \frac{\bar{u} h}{D}$ is the Peclet number, $\sigma = \frac{h}{\sqrt{k}}$ is the porous parameter, $Fr = \frac{\bar{u}^2}{hg}$ is the Froude number.

3. SOLUTION OF THE PROBLEM

The solution to equation (2.8) which is a fourth order differential equation with constant coefficient gives $U(\eta)$ as

$$(3.1) \quad U(\eta) = C_1 e^{m_1 \eta} + C_2 e^{-m_1 \eta} + C_3 e^{m_3 \eta} + C_4 e^{-m_3 \eta} + \frac{1}{M^2} Re \left(P - \frac{1}{Fr} \sin \phi \right).$$

Applying the boundary conditions (2.10) - (2.12) in (3.1), we obtain the velocity of blood as

$$U(\eta) = 2C_1 \cosh m_1 \eta + 2C_3 \cosh m_3 \eta + \frac{1}{M^2} Re \left(P - \frac{1}{Fr} \sin \phi \right),$$

where C_1, C_2, C_3 and C_4 are constants given in Appendix 1. Next, we get the axial velocity components is

$$U' = \frac{U - \bar{U}}{\bar{U}} = \frac{2 \left(C_1 \cosh m_1 \eta + C_3 \cosh m_3 \eta - \frac{C_1 \sinh m_1}{m_1} - \frac{C_3 \sinh m_3}{m_3} \right)}{\frac{2C_1 \sinh m_1}{m_1} + \frac{2C_3 \sinh m_3}{m_3} + \frac{Re}{M^2} \left(P - \frac{1}{Fr} \sin \phi \right)},$$

where

$$\bar{U} = \frac{1}{2} \int_{-1}^1 U(\eta) d\eta = \frac{2C_1 \sinh m_1}{m_1} + \frac{2C_3 \sinh m_3}{m_3} + \frac{1}{M^2} Re \left(P - \frac{1}{Fr} \sin \phi \right).$$

3.1. Generalized Dispersion Model. To obtaining the mean concentration valid for all time, we introduce the generalized dispersion model of [7], formulated as

$$(3.2) \quad \theta(\tau, \xi, \eta) = \theta_m(\tau, \xi) + \sum_{k=1}^{\infty} f_k(\tau, \eta) \frac{\partial^k \theta_m}{\partial \xi^k},$$

where θ_m is average concentration

$$(3.3) \quad \theta_m(\tau, \xi) = \frac{1}{2} \int_{-1}^1 \theta(\tau, \xi, \eta) d\eta.$$

Equation is obtained by integrating equation (2.9) gives

$$(3.4) \quad \frac{\partial \theta_m}{\partial \tau} = \frac{1}{Pe^2} \frac{\partial^2 \theta_m}{\partial \xi^2} + \frac{1}{2} \int_{-1}^1 \frac{\partial^2 \theta}{\partial \eta^2} d\eta - \frac{1}{2} \frac{\partial}{\partial \xi} \int_{-1}^1 U' \theta d\eta - \alpha_1^2 \theta_m.$$

Substituting equation (3.2) in (3.4), we obtain

$$(3.5) \quad \frac{\partial \theta_m}{\partial \tau} = \frac{1}{Pe^2} \frac{\partial^2 \theta_m}{\partial \xi^2} - \frac{1}{2} \frac{\partial}{\partial \xi} \int_{-1}^1 U' \left(\theta_m(\tau, \xi) + f_1(\tau, \eta) \frac{\partial \theta_m}{\partial \xi}(\tau, \xi) + \dots \right) d\eta - \alpha_1^2 \theta_m.$$

In this model we write

$$(3.6) \quad \frac{\partial \theta_m}{\partial \tau} = \sum_{i=1}^{\infty} K_i(\tau) \frac{\partial^i \theta_m}{\partial \xi^i}.$$

Substituting the equation (3.6) in (3.5) we obtain

$$\begin{aligned} K_1 \frac{\partial \theta_m}{\partial \xi} + K_2 \frac{\partial^2 \theta_m}{\partial \xi^2} + K_3 \frac{\partial^3 \theta_m}{\partial \xi^3} + \dots &= \frac{1}{Pe^2} \frac{\partial^2 \theta_m}{\partial \xi^2} - \frac{1}{2} \frac{\partial}{\partial \xi} \int_{-1}^1 U' \left(\theta_m(\tau, \xi) \right. \\ &\quad \left. + f_1(\tau, \eta) \frac{\partial \theta_m}{\partial \xi} + f_2(\tau, \eta) \frac{\partial^2 \theta_m}{\partial \xi^2}(\tau, \xi) + \dots \right) d\eta - \alpha_1^2 \theta_m. \end{aligned}$$

Equating the coefficients of $\frac{\partial \theta_m}{\partial \xi}, \frac{\partial^2 \theta_m}{\partial \xi^2} \dots$ we get,

$$(3.7) \quad K_i(\tau) = \frac{\delta_{ij}}{P_e^2} - \frac{1}{2} \int_{-1}^1 U' f_{i-1}(\tau, \eta) d\eta, \quad (i = 1, 2, 3, \dots \text{ and } j = 2)$$

$$\text{where, Kronecker delta } \delta_{ij} = \begin{cases} 1, & \text{if } i = j \\ 0, & \text{if } i \neq j \end{cases}.$$

Substituting equation (3.2) in (2.9), we get

$$\begin{aligned} & \frac{\partial}{\partial \tau} \left(\theta_m(\tau, \xi) + f_1(\tau, \eta) \frac{\partial \theta_m}{\partial \xi}(\tau, \xi) + f_2(\tau, \eta) \frac{\partial^2 \theta_m}{\partial \xi^2}(\tau, \xi) + \dots \right) \\ & + U' \frac{\partial}{\partial \xi} \left(\theta_m(\tau, \xi) + f_1(\tau, \eta) \frac{\partial \theta_m}{\partial \xi}(\tau, \xi) + f_2(\tau, \eta) \frac{\partial^2 \theta_m}{\partial \xi^2}(\tau, \xi) + \dots \right) \\ & = \frac{1}{P_e^2} \frac{\partial^2}{\partial \xi^2} \left(\theta_m(\tau, \xi) + f_1(\tau, \eta) \frac{\partial \theta_m}{\partial \xi}(\tau, \xi) + f_2(\tau, \eta) + \dots \right) \\ & \quad + \frac{\partial^2}{\partial \eta^2} \left(\theta_m(\tau, \xi) + f_1(\tau, \eta) \frac{\partial \theta_m}{\partial \xi} + \dots \right) \\ & \quad - \alpha_1^2 \left(\theta_m(\tau, \xi) + f_1(\tau, \eta) \frac{\partial \theta_m}{\partial \xi} + \dots \right) \end{aligned}$$

Rearranging the terms and using

$$\frac{\partial^{k+1} \theta_m}{\partial \tau \partial \xi^k} = \sum_{i=1}^{\infty} K_i(\tau) \frac{\partial^{k+i} \theta_m}{\partial \xi^{k+i}}$$

we obtain

$$\begin{aligned} (3.8) \quad & \left[\frac{\partial f_1}{\partial \tau} - \frac{\partial^2 f_1}{\partial \eta^2} + U' + K_1(\tau) + \alpha_1^2 f_1 \right] \frac{\partial \theta_m}{\partial \xi} \\ & + \left[\frac{\partial f_2}{\partial \tau} - \frac{\partial^2 f_2}{\partial \eta^2} + U' f_1 + K_1(\tau) f_1 + K_2(\tau) - \frac{1}{P_e^2} + \alpha_1^2 f_2 \right] \frac{\partial^2 \theta_m}{\partial \xi^2} \\ & + \sum_{k=1}^{\infty} \left[\frac{\partial f_{k+2}}{\partial \tau} - \frac{\partial^2 f_{k+2}}{\partial \eta^2} + U' f_{k+1} + K_1(\tau) f_{k+1} + \left(K_2(\tau) - \frac{1}{P_e^2} \right) f_k \right. \\ & \left. + \sum_{i=3}^{k+2} K_i f_{k+2-i} + \alpha_1^2 f_{k+2} \right] \frac{\partial^{k+2} \theta_m}{\partial \xi^{k+2}} = 0, \end{aligned}$$

with $f_0 = 1$. Comparing the coefficients of $\frac{\partial^k \theta_m}{\partial \xi^k}$ ($k = 1, 2, 3, \dots$) in (3.8) and equating to zero, we get

$$(3.9) \quad \frac{\partial f_1}{\partial \tau} = \frac{\partial^2 f_1}{\partial \eta^2} - U' - K_1(\tau) - f_1 \alpha_1^2$$

$$\begin{aligned}\frac{\partial f_2}{\partial \tau} &= \frac{\partial^2 f_2}{\partial \eta^2} - U' f_1 - K_1(\tau) f_1 - K_2(\tau) + \frac{1}{P_e^2} - f_2 \alpha_1^2 \\ \frac{\partial f_{k+2}}{\partial \tau} &= \frac{\partial^2 f_{k+2}}{\partial \eta^2} - (U' - K_1(\tau)) f_{k+1} - \left(K_2(\tau) - \frac{1}{P_e^2} \right) f_k \\ &\quad - \sum_{i=3}^{k+2} K_i f_{k+2-i} - f_{k+2} \alpha_1^2.\end{aligned}$$

Since θ_m is chosen in such a way to satisfy the initial and boundary conditions on θ , conditions (2.13) to (2.14) on f_k function becomes

$$(3.10) \quad f_k = 0 \text{ at } \tau = 0,$$

$$(3.11) \quad \frac{\partial f_k}{\partial \eta} = 0 \text{ at } \eta = -1,$$

$$(3.12) \quad \frac{\partial f_k}{\partial \eta} = 0 \text{ at } \eta = 1.$$

Also, from equation (3.3) we have

$$(3.13) \quad \int_{-1}^1 f_k(\tau, \eta) d\eta = 0,$$

for $k = 1, 2, 3, \dots$. To find $K_2(\tau)$, the f_1 are evaluated using (3.10) - (3.13). Equation (3.7) for $i = 1$, we get

$$K_1(\tau) = 0.$$

Equation (3.7) for $i = 2$, we get K_2 as

$$K_2(\tau) = \frac{1}{P_e^2} - \frac{1}{2} \int_{-1}^1 U' f_1 d\eta$$

$$(3.14) \quad \text{put } f_1 = f_{10}(\eta) + f_{11}(\tau, \eta),$$

where $f_{10}(\eta)$ is independent of τ and f_{11} is τ -dependent satisfying

$$(3.15) \quad \frac{df_{10}}{d\eta} = 0 \text{ at } \eta = \pm 1$$

$$(3.16) \quad \int_{-1}^1 f_{10} d\eta = 0.$$

Using the (3.14) in (3.9) implies that

$$(3.17) \quad \frac{d^2 f_{10}}{d\eta^2} - \alpha_1^2 f_{10} = \frac{2 \left(C_1 \cosh m_1 \eta + C_2 \cosh m_3 \eta - \frac{C_1 \sinh m_1}{m_1} - \frac{C_3 \sinh m_3}{m_3} \right)}{\frac{2C_1 \sinh m_1}{m_1} + \frac{2C_3 \sinh m_3}{m_3} + \frac{Re}{M^2} \left(P - \frac{1}{Fr} \sin \phi \right)}$$

$$(3.18) \quad \frac{\partial f_{11}}{\partial \tau} = \frac{\partial^2 f_{11}}{\partial \eta^2} - \alpha_1^2 f_{11}$$

Solving the equation (3.17) with conditions (3.15) and (3.16) is

$$(3.19) \quad f_{10} = 2C_6 \sinh \alpha \eta + \frac{2 \left[C_1 \frac{\cosh m_1 \eta}{m_1^2 - \alpha_1^2} + C_3 \frac{\cosh m_3 \eta}{m_3^2 - \alpha_1^2} + \frac{C_1 \sinh m_1}{m_1 \alpha_1^2} + \frac{C_3 \sinh m_3}{m_3 \alpha_1^2} \right]}{\frac{2C_1 \sinh m_1}{m_1} + \frac{2C_3 \sinh m_3}{m_3} + \frac{Re}{M^2} \left(P - \frac{1}{Fr} \sin \phi \right)}.$$

Equation (3.18) is heat conduction type and its solution satisfying condition $f_{11}(\tau, \eta) = -f_{10}(\eta)$ of the form

$$(3.20) \quad f_{11} = \sum_{n=1}^{\infty} B_n e^{-(\lambda_n^2 - \alpha_1^2)\tau} \cos(\lambda_n \eta),$$

$$\text{where } B_n = -2 \int_0^1 f_{10}(\eta) \cos(\lambda_n \eta) d\eta$$

and $\lambda_n = n\pi$. Substituting (3.19) and (3.20) in equation (3.14) we get,

$$(3.21) \quad f_1 = 2C_6 \sinh \alpha \eta + \frac{2 \left[C_1 \frac{\cosh m_1 \eta}{m_1^2 - \alpha_1^2} + C_3 \frac{\cosh m_3 \eta}{m_3^2 - \alpha_1^2} + \frac{C_1 \sinh m_1}{m_1 \alpha_1^2} + \frac{C_3 \sinh m_3}{m_3 \alpha_1^2} \right]}{\frac{2C_1 \sinh m_1}{m_1} + \frac{2C_3 \sinh m_3}{m_3} + \frac{Re}{M^2} \left(P - \frac{1}{Fr} \sin \phi \right)} + \sum_{n=1}^{\infty} B_n e^{-(\lambda_n^2 - \alpha_1^2)\tau} \cos(\lambda_n \eta).$$

Substituting f_1 into equation (3.21) and performing the integration, we get solution of dispersion coefficient with help of MATHEMATICA 8.0.

Similarly, $K_3(\tau)$, $K_4(\tau)$ and so on are negligibly small compared to dispersion coefficient $K_2(\tau)$.

Dispersion model (3.6) brings to

$$(3.22) \quad \frac{\partial \theta_m}{\partial \tau} = K_2 \frac{\partial^2 \theta_m}{\partial \xi^2}.$$

The exact solution of (3.22) satisfying the condition (2.13) to (2.14) can be obtained using Fourier Transform [9] as

$$\theta_m(\xi, \tau) = \frac{1}{2} \left[\operatorname{erf} \left(\frac{\frac{\xi_s}{2} + \xi}{2\sqrt{T}} \right) + \operatorname{erf} \left(\frac{\frac{\xi_s}{2} - \xi}{2\sqrt{T}} \right) \right],$$

where $T = \int_0^\tau K_2(\eta) d\eta$ and $\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-z^2} dz$.

4. DISCUSSION OF THE RESULTS

In this segment, the dispersion of a solute in a couple stress fluid(blood) flow through a inclined channel with impact of Hartmann number ($M = 1, 1.5, 2$), Froude number ($Fr = 0.5, 1.5, 2.5$) and couple stress parameter ($a = 1, 5, 10$), on the velocity, dispersion coefficient and mean concentration profiles for $Pe = 100$, $\alpha = 0.1$ and $\beta_1 = 0.1$. The generalized dispersion model of [7] is to study dispersion of solute.

We have extracted interesting insights regarding the influence of all the parameters that govern this problem. The influence of the parameters M , Fr and a on horizontal velocity, dispersion coefficient and concentration profiles can be analyzed from Figures 2-10.

Figure 2 plots of the velocity u versus η for different values of M . This figure depict that the velocity profile decreases with the increase of the Hartmann number (M). The effect of the magnetic field parameter M on the velocity profile is displayed. Impact of magnetic field acts normal to the blood flow introduces a Lorentz force, it resist to flow. Blood flow become slow while comparing with the normal flow, the behavior of boundary layer reduces with the increase of the magnetic field.

The effects of the velocity profiles for different values of the Froude number and couple stress parameter(a) is shown in Figures 3 and 4. It is seen that the effect of increasing Froude number and couple stress parameter increases the velocity profile of the blood flow, found to be parabolic in nature.

Dispersion coefficient is evaluated for various values of M , Fr and a . The results obtained are plotted in Figures 5-7. From these Figures, the axial dispersion coefficient decreases $K_2(\tau) - Pe^{-2}$ with the increase of Froude number, couple stress and porous parameter. This phenomena is noticed to control the rapture of red blood cell and it helps to design an artificial organs free from impurities and in the effective removable of liquid particles in industrial problems.

Figures 8 – 10 represented the θ_m mean concentration were plotted versus axial distance ξ with changes in M , Fr and a . From figure 10, it is depict that θ_m mean concentration raises with an increase in the magnitude of the Hartmann number (M), Froude number(Fr) and couple stress parameter(a). From these figures that the impact of increasing We , a , and Fr is to increase the peak value of the θ_m . It shows that the concentration is large distribution along the ξ -direction for huge values of We . The curve are bell shaped and symmetrical about the origin. From these result it is very helpful to analyses the transport of solute at various times.

5. CONCLUSIONS

Transport of a solute in blood flow with impact of magnetic field(M), Froude number(Fr) and couple stress parameter(a) are discussed. The influence of different values of M is to reduce the dispersion coefficient and increase the mean concentration. Also, it is found that the dispersion coefficient decrease due to chemical reaction but increases in the absence of chemical reaction along the mean concentration increases with an increasing the value of Hartmann number. $K_2 - Pe^{-2}$ are obtained by generalised dispersion model and its valid for all time. In general conclusion couple stress ($a \rightarrow \infty$) the flow become a Newtonian fluid and also $\tau \rightarrow \infty$ is particular case of Taylor's dispersion model.

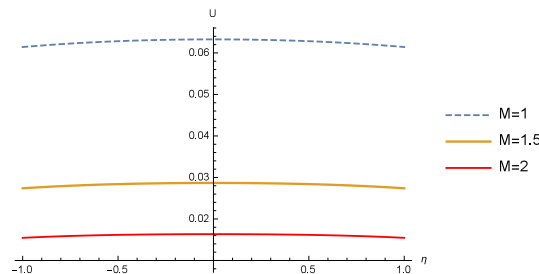


Figure 2: Velocity u on distinct values of (M)
with $a = 20$, $\sigma = 120$, $\beta_1 = 0.1$ and $\alpha = 0.1$

The above results are pertinent to application of dispersion of nutrients, blood oxygenators, branching and curvature pulsatile flow and various complexities in the human circulatory system [11, 12].

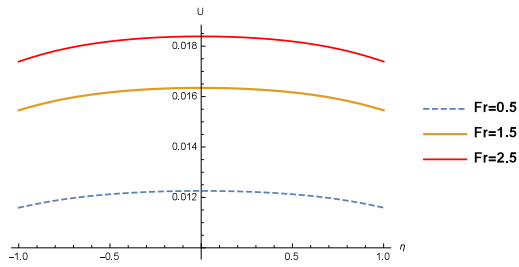


Figure 3: Velocity u on distinct values of Fr

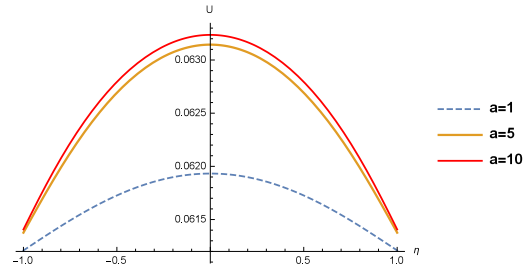


Figure 4: Velocity u on distinct values of a

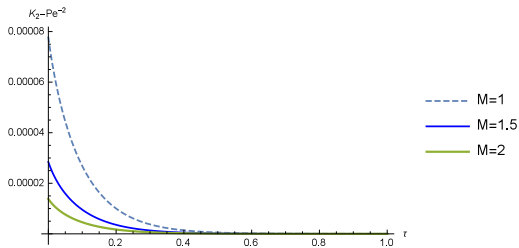


Figure 5: $K_2 - Pe^{-2}$ on distinct values of M

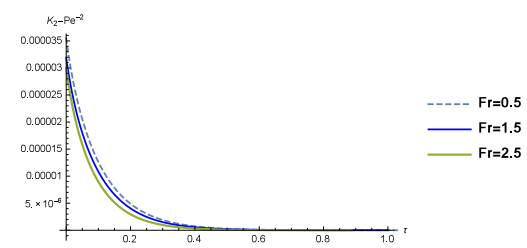


Figure 6: $K_2 - Pe^{-2}$ on distinct values of Fr

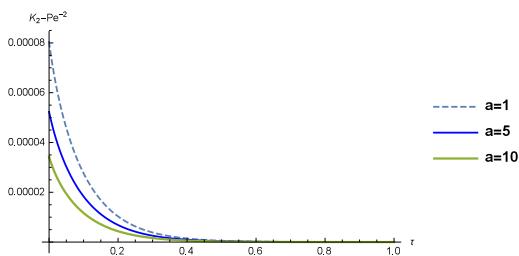


Figure 7: $K_2 - Pe^{-2}$ on distinct values of a

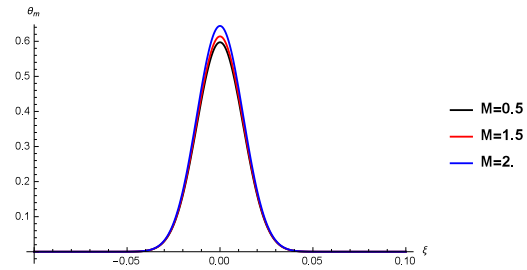


Figure 8: θ_m on distinct values of M

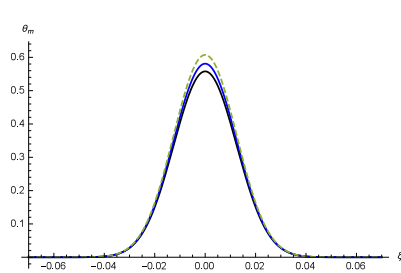


Figure 9: θ_m on distinct values of Fr

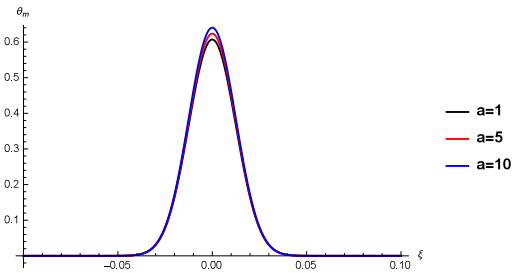


Figure 10: θ_m on distinct values of a

6. APPENDICES

$$\begin{aligned}
 m_1 &= \frac{\sqrt{a^2 + \sqrt{a^4 - 4a^2M^2}}}{\sqrt{2}}; & m_3 &= \frac{\sqrt{a^2 - \sqrt{a^4 - 4a^2M^2}}}{\sqrt{2}}; \\
 a_3 &= e^{m_1}(\alpha\sigma + m_1); & a_4 &= e^{-m_1}(m_1 - \alpha\sigma); & a_5 &= e^{m_3}(\alpha\sigma + m_3); \\
 a_6 &= e^{-m_3}(m_3 - \alpha\sigma); & a_7 &= -\frac{Re}{M^2}\left(P - \frac{\sin\phi}{Fr}\right) + \frac{Re}{\sigma^2(1 + \beta_1)}\left(P + \frac{1}{Fr}\right); \\
 a_8 &= m_1^2 e^{m_1}; & a_9 &= m_1^2 e^{-m_1}; & a_{10} &= m_3^2 e^{m_3}; \\
 a_{11} &= m_3^2 e^{-m_3}; & a_{12} &= 2\alpha_1 \cosh \alpha_1; & a_{13} &= 2\alpha_1 \sinh \alpha_1; \\
 a_{14} &= 4C_3 m_3 \frac{\cosh m_3}{m_3^2 - \alpha_1}; & a_{15} &= 4C_1 m_1 \frac{\sinh m_1}{m_1^2 - \alpha_1}; \\
 C_6 &= -\frac{a_{13}a_{14} + a_{12}a_{15}}{2a_{12}a_{13}}; \\
 C_1 = C_2 &= \frac{-a_7a_{10} - a_7a_{11}}{a_5a_8 - a_6a_8 + a_5a_9 - a_6a_9 - a_3a_{10} + a_4a_{10} - a_3a_{11} + a_4a_{11}}; \\
 C_3 = C_4 &= \frac{a_7a_8 + a_7a_9}{a_5a_8 - a_6a_8 + a_5a_9 - a_6a_9 - a_3a_{10} + a_4a_{10} - a_3a_{11} + a_4a_{11}}.
 \end{aligned}$$

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