

STUDY THE IMPACT OF CHEMICAL REACTION ON ELECTROMAGNETOHYDRODYNAMIC FLOW OF BLOOD IN A CHANNEL BOUNDED BY POROUS LAYERS

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ABSTRACT. In this paper contains some physiological model, to decorated the study the impact of chemical reaction on Electromagnetohydrodynamic (EMHD) flow of blood using generalized dispersion model to get dispersion of solute in couple stresses effects. Influence of chemical reaction, arising as a body couple in the governing equations is shown to decrease the dispersion coefficient. Expression for the velocity, dispersion coefficient and mean concentration are determined and compares with distinct parameters such as reaction rate, Hartmann number, electric number, porous parameter and couple stress parameter. Finally, we conclude with some interesting results in detail with the help of graphs.

1. INTRODUCTION

In biomechanics there is so many number of problem in blood flow with electromagnetic fields and its plays an vital role by interacting with the chemical reaction. Huge number of applications about the transport of drugs in physiological systems and pollutants in environment engineering, chemical engineering helps the dispersion of gaseous tracer and cancer tumor treatment. Many authors have utilized these model to study the dispersion of solute to [2, 13–17].

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Kolin was first introduced the concept of electromagnetic fields in medical research [11]. Later, Korchevskii and Marochnik [12] are discussed the idea of electromagnetohydrodynamic to reduces the blood flow for the treatment of brain-hemorrhage and hypertension etc. Rudraiah et al., [9] scrunized the unsteady convective diffusion in a couple-stress fluid flow by utilize the technique of [10].

Chiu-On Ng et al. [4, 5] have investigated the dispersion increase and decreases the concentration of RBCs with the impact of influence of electric field. Nirmala P.Ratchagar and VijayaKumar [8] investigated the unsteady connective diffusion of erythrocytes in the plasma flow with impact of magnetic field. Recently, Debnath et al. [6] studied the physiological impact in the blood flow through a artery.

In this paper, we focused on the dispersion of solute in blood flow for an incompressible couple stress fluid influenced by chemical reaction, electric field (EF) and magnetic field(MF) by utilizing the technique of [7]. In Section 2 consist the mathematical formulation of the problem with corresponding appropriate initial and boundary conditions. In Section 3 contains the method of solution of the mathematical model by the technique of generalized dispersion model. In section 4 the effect of Hartmann number(M) ,electric number, chemical reaction rate coefficient(α_1) and porous parameter σ on the dispersion coefficients and mean concentration is discussed. Section 5 consist of the final conclusions.

2. FORMULATION AND SOLUTION OF THE PROBLEM

A model of the problem is shown in Figure 1.

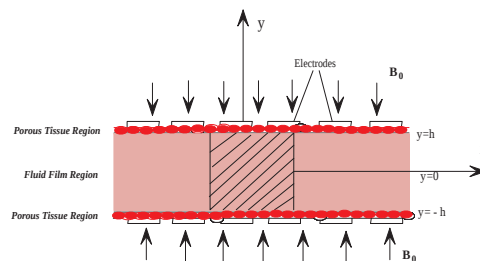


Figure 1. Geometry of the problem

Fluid Film Region

Let us consider the couple stress fluid with chemical reaction is to be steady,

fully developed, unidirectional and incompressible. The fluid region is bounded by porous layers and its distance is $2h$. The governing equations of motion for an electric and magnetic fields of couple stress fluid are given by,

$$(2.1) \quad \nabla \cdot \vec{q} = 0$$

$$(2.2) \quad \rho \left(\frac{\partial \vec{q}}{\partial t} + (\vec{q} \cdot \nabla) \vec{q} \right) = -\nabla p + \mu \nabla^2 \vec{q} - \lambda \nabla^4 \vec{q} + \rho_e E_i + J_i \times B$$

The conservation of species

$$(2.3) \quad \frac{\partial \vec{C}}{\partial t} + (\vec{q} \cdot \nabla) \vec{C} = D \nabla^2 \vec{C} + R_C$$

Porous Tissue Region

$$(2.4) \quad \nabla \cdot \vec{q}_p = 0$$

$$(2.5) \quad \rho \left(\frac{\partial \vec{q}_p}{\partial t} + (\vec{q}_p \cdot \nabla) \vec{q}_p \right) = -\nabla p + \mu \nabla^2 \vec{q}_p - \frac{\mu}{k} (1 + \beta) \vec{q}_p + \rho_e E$$

where, $\vec{q}$ and $\vec{q}_p$ are the velocity vector in fluid film and porous tissue region, p is the pressure, λ is the couple stress parameter, $\mathbf{B} = (0, B_0, 0)$ the magnetic field, k is the permeability parameter of porous medium, μ is the dynamic viscosity of the blood, $\vec{C}$ is the concentration, R_C is the sources or sink of the species concentration and D is the coefficient of mass diffusivity. The conservation of charges

$$\frac{\partial \rho_e}{\partial t} + (\vec{q} \cdot \nabla) \rho_e + \nabla \cdot J_i = 0.$$

The interaction between the fluid motion and the electromagnetic fields are expressed by Maxwell's equations. Using the electromagnetohydrodynamics (EMHD) approximation, these equations are written as:

$$\nabla \cdot \mathbf{B} = 0, \nabla \times \mathbf{B} = \mu_0 J_i,$$

$$\nabla \times E_i = -\frac{\partial B_0}{\partial t},$$

where, μ_0 magnetic permeability.

Put $J_i \times \mathbf{B} = -B_0^2 \sigma_0 u^*$ (small magnetic Reynolds number).

To find $\rho_e E_x$.

Conservation of charges:

$$(2.6) \quad \frac{\partial \rho_e}{\partial t} + \frac{\partial J_i}{\partial x_j} = 0.$$

$$(2.7) \quad J_i = \rho_e q_i + \sigma_c E_i.$$

Gauss law

$$\frac{\partial E_i}{\partial x_i} = \frac{\rho_e}{\epsilon_0},$$

where J_i , ρ_e , $\rho_e q_i$, $\sigma_c E_i$, σ_c , E_i and ϵ_0 are the current density, distribution of charge density, convective current, conduction current, the electrical conductivity, the electric field and dielectric constant for free space, respectively.

Faraday's law

$$\begin{aligned} \frac{\partial E_i}{\partial x_j} - \frac{\partial E_j}{\partial x_i} &= 0 \\ E_i &= -\frac{\partial \phi}{\partial x_i}, \end{aligned}$$

electric field is conservative, where, ϕ is the electric potential. From (2.6) by (2.7), gives

$$(2.8) \quad \left(\frac{\partial}{\partial t} + q_j \frac{\partial}{\partial x_j} \right) \rho_e + \frac{\partial(\sigma_c E_i)}{\partial x_i} = 0 \quad (\text{since } \rho_e q_i \ll \sigma_c E_i)$$

In cartesian form, using the above assumptions equations (2.1)-(2.5) becomes

Fluid Film Region

$$(2.9) \quad \begin{aligned} \frac{\partial u^*}{\partial x} &= 0, \\ 0 &= -\frac{\partial p^*}{\partial x} + \mu \frac{\partial^2 u^*}{\partial y^2} - \lambda \frac{\partial^4 u^*}{\partial y^4} + \rho_e E_x - B_0^2 \sigma_0 u^*, \\ 0 &= -\frac{\partial p^*}{\partial y} + \rho_e E_y. \end{aligned}$$

The concentration $C(t, x, y)$ with slug and it satisfies convective diffusion equation

$$(2.10) \quad \frac{\partial C}{\partial t} + u^* \frac{\partial C}{\partial x} = D \left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} \right) - K_1 C.$$

Porous Tissue Region

$$(2.11) \quad \begin{aligned} \frac{\partial u_p^*}{\partial x} &= 0, \\ 0 &= -\frac{\partial p^*}{\partial x} - \frac{\mu(1 + \beta_1)}{k} u_p^* + \rho_e E_x, \\ 0 &= -\frac{\partial p^*}{\partial y} + \rho_e E_y, \end{aligned}$$

with required boundary are

$$(2.12) \quad \frac{\partial u^*}{\partial y} = \frac{-\alpha}{\sqrt{k}}(u^* - u_p^*) \text{ at } y = h,$$

$$(2.13) \quad \frac{\partial u^*}{\partial y} = \frac{\alpha}{\sqrt{k}}(u^* - u_p^*) \text{ at } y = -h.$$

The couple stress conditions,

$$(2.14) \quad \frac{\partial^2 u^*}{\partial y^2} = 0 \text{ at } y = \pm h,$$

$$(2.15) \quad C(0, x, y) = \begin{cases} C_0, & |x| \leq \frac{x_s}{2} \\ 0, & |x| > \frac{x_s}{2} \end{cases},$$

$$(2.16) \quad \frac{\partial C}{\partial y}(t, x, -h) = \frac{\partial C}{\partial y}(t, x, h) = 0,$$

$$(2.17) \quad C(t, \infty, y) = \frac{\partial C}{\partial x}(t, \infty, y) = 0,$$

where, u^* represents the axial velocity of the blood, u_p^* is the Darcy velocity. It may be noted that (2.11) is Darcy equation, α is the slip parameter. Equations (2.12) and (2.13) is slip condition [3]. $-K_1 C$ is the volume rate of disappearance of the solute due to chemical reaction and K_1 represents the first order chemical reaction, C_0 is the initial concentration.

Introducing the non-dimensional variables

$$u = \frac{u^*}{\bar{u}}, u_p = \frac{u_p^*}{\bar{u}}, \eta = \frac{y}{h}, \xi' = \frac{x}{hPe}, \xi_s = \frac{x_s}{hPe}, \rho_e^* = \frac{\rho_e}{\left(\frac{\epsilon_0 V}{h^2}\right)}, Pe = \frac{\bar{u}h}{D},$$

$$\tau = \frac{tD}{h^2}, p = \frac{p^*}{\rho \bar{u}}, E_x^* = \frac{E_x}{\left(\frac{V}{h}\right)}, E_y^* = \frac{E_y}{\left(\frac{v}{h}\right)}, \phi = \frac{\phi^*}{v},$$

where h, v and ϕ are the characteristic length, applied constant, and electric potential.

Equations (2.9) and (2.10) in non-dimensional form are

Fluid Film Region

$$(2.18) \quad \frac{\partial^4 u}{\partial \eta^4} - a^2 \frac{\partial^2 u}{\partial \eta^2} + a^2 M^2 u = -Ka^2 P + l_1(1 - \alpha_c \eta)$$

and

$$\frac{\partial \theta}{\partial \tau} + u \frac{\partial \theta}{\partial \xi'} = \frac{1}{Pe^2} \left(\frac{\partial^2 \theta}{\partial \xi'^2} + \frac{\partial^2 \theta}{\partial \eta^2} \right) - \alpha_1^2 \theta.$$

We define the axial coordinate moving with the average velocity of flow as $x_1 = x - t\bar{u}$ which is in dimensionless form $\xi = \xi' - \tau$, where $\xi = \frac{x_1}{hPe}$. Then equation (2.25) becomes

$$(2.19) \quad \frac{\partial \theta}{\partial \tau} + U^* \frac{\partial \theta}{\partial \xi} = \frac{1}{Pe^2} \left(\frac{\partial^2 \theta}{\partial \xi^2} + \frac{\partial^2 \theta}{\partial \eta^2} \right) - \alpha_1^2 \theta,$$

with $U^* = \frac{u}{\bar{u}}$ (moving coordinate system of velocity) and $\alpha_1^2 = \frac{h^2 K_1}{D}$ is the chemical reaction rate coefficient.

Porous Tissue Region

$$u_p = \frac{Re}{\sigma^2(1 + \beta_1)} \left(-\frac{P}{Pe} + \frac{WeX_0\alpha_c(1 - \alpha_c\eta)Pe}{2} \right),$$

with required non-dimensional form of equations (2.12)-(2.17) gives

$$\frac{\partial u}{\partial \eta} = -\alpha\sigma(u - u_p) \text{ at } \eta = 1,$$

$$\frac{\partial u}{\partial \eta} = \alpha\sigma(u - u_p) \text{ at } \eta = -1,$$

$$\frac{\partial^2 u}{\partial \eta^2} = 0 \text{ at } \eta = \pm 1,$$

where $P = \frac{\partial p}{\partial \xi}$, $K = \frac{D}{\gamma}$, $\alpha_c = \alpha_h \left(\frac{\Delta T}{2} \right)$ is the conductivity variation parameter, $\sigma = \frac{h}{\sqrt{k}}$ is the porous parameter, and

$$(2.20) \quad \theta(0, \xi, \eta) = \begin{cases} 1, & |\xi| \leq \frac{\xi_s}{2} \\ 0, & |\xi| > \frac{\xi_s}{2} \end{cases},$$

$$(2.21) \quad \frac{\partial \theta}{\partial \eta}(\tau, \xi, -1) = \frac{\partial \theta}{\partial \eta}(\tau, \xi, 1) = 0,$$

$$(2.22) \quad \theta(\tau, \infty, \eta) = \frac{\partial \theta}{\partial \xi}(\tau, \infty, \eta) = 0.$$

From equation (2.8), poorly conducting fluid is $\sigma_c \ll 1$ and grows with temperature condition,

$$(2.23) \quad \sigma_c = \sigma_0 [1 + \sigma_h (T_b - T_0)].$$

Here σ_0 is that of σ_c at $T_b = T_0$, α_h is the volumetric expansion coefficient of σ_c .

The expression for conduction temperature(T_b) is obtained by solving non-dimensional equation

$$(2.24) \quad \frac{d^2 T_b}{d\eta^2} = 0,$$

where T_b is non-dimensional equation using $\frac{T_b}{\Delta T}$ with boundary condition in non-dimensional form

$$(2.25) \quad T_b = T_0 \text{ at } \eta = -1,$$

$$(2.26) \quad T_b = T_1 \text{ at } \eta = 1.$$

The solution to equation (2.24) using (2.25) and (2.26), we get

$$(2.27) \quad T_b = \frac{\Delta T}{2} \eta + \frac{\Delta T}{2} + T_0,$$

where $\Delta T = T_1 - T_0$.

Equation (2.27) using (2.23) becomes

$$\sigma_c = \sigma_0 [1 + \alpha_c (\eta + 1)] \approx \sigma_0 e^{\alpha_c (\eta + 1)} \text{ (because } \alpha_c \ll 1 \text{)}.$$

The electrical conductivity is

$$\frac{\partial^2 \phi^*}{\partial y^2} + \frac{1}{\sigma_c} \frac{\partial \phi^*}{\partial y} \frac{\partial \sigma_c}{\partial y} = 0.$$

The required boundary conditions are

$$\phi^* = \frac{xv}{h} \text{ at } y = -h,$$

$$\phi^* = \frac{(x - x_0)v}{h} \text{ at } y = h.$$

Introducing the non-dimensional quantities, we get

$$(2.28) \quad \frac{\partial^2 \phi}{\partial \eta^2} + \frac{1}{\sigma_c} \frac{\partial \phi}{\partial \eta} \frac{\partial \sigma_c}{\partial \eta} = 0,$$

with boundary conditions

$$\phi = XPe \text{ at } \eta = -1$$

$$\phi = \frac{Pe(X - X_0)}{h} \text{ at } \eta = 1$$

The solution of equation (2.28), we get

$$\phi = Pe \left[X - \frac{X_0(1 - e^{-\alpha_c y})}{e^{\alpha_c} - e^{-\alpha_c}} \right].$$

The expression for ρ_e is

$$\rho_e = -\frac{PeX_0\alpha_c^2 e^{-\alpha_c y}}{e^{\alpha_c} - e^{-\alpha_c}},$$

$$E_x = -1, E_y = \frac{PeX_0\alpha_c e^{-\alpha_c y}}{e^{\alpha_c} - e^{-\alpha_c}}.$$

Hence,

$$\rho_e E_x = \frac{PeX_0\alpha_c^2 e^{-\alpha_c \eta}}{e^{\alpha_c} - e^{-\alpha_c}} \approx \frac{PeX_0\alpha_c(1 - \alpha_c \eta)}{2} \text{ (because } \alpha_c \ll 1).$$

3. SOLUTION OF THE PROBLEM

3.1. Velocity distribution. From equation (2.18), we obtain the velocity of blood as

Region 1: **Fluid Film Region**

$$(3.1) \quad u = C_1 e^{m_1 \eta} + C_2 e^{-m_1 \eta} + C_3 e^{m_2 \eta} + C_4 e^{-m_2 \eta} - \frac{KP}{M^2} + \frac{l_1(1 - \alpha_c \eta)}{a^2 M^2}.$$

The normalized axial components of velocity is

$$U^* = \frac{u}{\bar{u}},$$

where the average velocity is

$$\bar{u} = \frac{1}{2} \int_{-1}^1 u(\eta) d\eta = \frac{(C_1 + C_2) \sinh(m_1)}{m_1} + \frac{(C_3 + C_4) \sinh(m_2)}{m_2} - \frac{2KP}{M^2} + \frac{2l_1}{a^2 M^2}.$$

3.2. Generalized Dispersion model. Now we introduce the generalized dispersion model of [7], formulated as

$$(3.2) \quad \theta(\tau, \xi, \eta) = \theta_m(\tau, \xi) + \sum_{k=1}^{\infty} f_k(\tau, \eta) \frac{\partial^k \theta_m}{\partial \xi^k},$$

where θ_m is average concentration

$$\theta_m(\tau, \xi) = \frac{1}{2} \int_{-1}^1 \theta(\tau, \xi, \eta) d\eta.$$

Integrating equation(2.19), we get

$$(3.3) \quad \frac{\partial \theta_m}{\partial \tau} = \frac{1}{Pe^2} \frac{\partial^2 \theta_m}{\partial \xi^2} + \frac{1}{2} \int_{-1}^1 \frac{\partial^2 \theta}{\partial \eta^2} d\eta - \frac{1}{2} \frac{\partial}{\partial \xi} \int_{-1}^1 U^* \theta d\eta - \alpha_1^2 \theta_m.$$

Substituting value of θ from equation (3.1) in (3.2), we obtain

$$(3.4) \quad \frac{\partial \theta_m}{\partial \tau} = \frac{1}{P_e^2} \frac{\partial^2 \theta_m}{\partial \xi^2} - \frac{1}{2} \frac{\partial}{\partial \xi} \int_{-1}^1 U^* \left(\theta_m(\tau, \xi) + f_1(\tau, \eta) \frac{\partial \theta_m}{\partial \xi}(\tau, \xi) + \dots \right) d\eta - \alpha_1^2 \theta_m.$$

In this model we write

$$(3.5) \quad \frac{\partial \theta_m}{\partial \tau} = \sum_{k=1}^{\infty} K_k(\tau) \frac{\partial^k \theta_m}{\partial \xi^k},$$

where the dispersion coefficient, $K_k(\tau)$. Substituting the Equation (3.4) in (3.3) we obtain

$$\begin{aligned} K_1 \frac{\partial \theta_m}{\partial \xi} + K_2 \frac{\partial^2 \theta_m}{\partial \xi^2} + K_3 \frac{\partial^3 \theta_m}{\partial \xi^3} + \dots &= \frac{1}{P_e^2} \frac{\partial^2 \theta_m}{\partial \xi^2} - \frac{1}{2} \frac{\partial}{\partial \xi} \int_{-1}^1 U^*(\theta_m(\tau, \xi) \\ &+ f_1(\tau, \eta) \frac{\partial \theta_m}{\partial \xi} + f_2(\tau, \eta) \frac{\partial^2 \theta_m}{\partial \xi^2}(\tau, \xi) + \dots) d\eta - \alpha_1^2 \theta_m. \end{aligned}$$

Comparing the coefficient $\frac{\partial \theta_m}{\partial \xi}, \frac{\partial^2 \theta_m}{\partial \xi^2} \dots$ we get,

$$(3.6) \quad K_i(\tau) = \frac{\delta_{ij}}{P_e^2} - \frac{1}{2} \int_{-1}^1 U f_{i-1}(\tau, \eta) d\eta,$$

$i = 1, 2, 3, \dots$ and $j = 2$, where, Kroneckar delta $\delta_{ij} = \begin{cases} 1, & \text{if } i = j \\ 0, & \text{if } i \neq j \end{cases}$.

Substituting equation (3.1) in (2.19), we get

$$\begin{aligned} &\frac{\partial}{\partial \tau} \left(\theta_m(\tau, \xi) + f_1(\tau, \eta) \frac{\partial \theta_m}{\partial \xi}(\tau, \xi) + f_2(\tau, \eta) \frac{\partial^2 \theta_m}{\partial \xi^2}(\tau, \xi) + \dots \right) \\ &+ U^* \frac{\partial}{\partial \xi} \left(\theta_m(\tau, \xi) + f_1(\tau, \eta) \frac{\partial \theta_m}{\partial \xi}(\tau, \xi) + f_2(\tau, \eta) \frac{\partial^2 \theta_m}{\partial \xi^2}(\tau, \xi) + \dots \right) \\ &= \frac{1}{P_e^2} \frac{\partial^2}{\partial \xi^2} \left(\theta_m(\tau, \xi) + f_1(\tau, \eta) \frac{\partial \theta_m}{\partial \xi}(\tau, \xi) + f_2(\tau, \eta) + \dots \right) \\ &+ \frac{\partial^2}{\partial \eta^2} \left(\theta_m(\tau, \xi) + f_1(\tau, \eta) \frac{\partial \theta_m}{\partial \xi} + \dots \right) \\ &- \alpha_1^2 \left(\theta_m(\tau, \xi) + f_1(\tau, \eta) \frac{\partial \theta_m}{\partial \xi} + \dots \right). \end{aligned}$$

Rearranging the terms and using

$$\frac{\partial^{k+1}\theta_m}{\partial\tau\partial\xi^k} = \sum_{i=1}^{\infty} K_i(\tau) \frac{\partial^{k+i}\theta_m}{\partial\xi^{k+i}}$$

we obtain

$$\begin{aligned} & \left[\frac{\partial f_1}{\partial\tau} - \frac{\partial^2 f_1}{\partial\eta^2} + U^* + K_1(\tau) + \alpha_1^2 f_1 \right] \frac{\partial\theta_m}{\partial\xi} \\ & + \left[\frac{\partial f_2}{\partial\tau} - \frac{\partial^2 f_2}{\partial\eta^2} + f_1 U^* + K_1(\tau) f_1 + K_2(\tau) - \frac{1}{P_e^2} + \alpha_1^2 f_2 \right] \frac{\partial^2\theta_m}{\partial\xi^2} \\ & + \sum_{k=1}^{\infty} \left[\frac{\partial f_{k+2}}{\partial\tau} - \frac{\partial^2 f_{k+2}}{\partial\eta^2} + f_{k+1} U^* + f_{k+1} K_1(\tau) + \left(K_2(\tau) - \frac{1}{P_e^2} \right) f_k \right. \\ & \left. + \sum_{i=3}^{k+2} K_i f_{k+2-i} + \alpha_1^2 f_{k+2} \right] \frac{\partial^{k+2}\theta_m}{\partial\xi^{k+2}} = 0, \end{aligned}$$

with $f_0 = 1$. Comparing the coefficients of $\frac{\partial^k\theta_m}{\partial\xi^k}$ ($k = 1, 2, 3, \dots$) in (3.6) and equating to zero, we get

$$\begin{aligned} \frac{\partial f_1}{\partial\tau} &= \frac{\partial^2 f_1}{\partial\eta^2} - U^* - K_1(\tau) - \alpha_1^2 f_1 \\ \frac{\partial f_2}{\partial\tau} &= \frac{\partial^2 f_2}{\partial\eta^2} - f_1 U^* - K_1(\tau) f_1 - K_2(\tau) + \frac{1}{P_e^2} - \alpha_1^2 f_2 \\ \frac{\partial f_{k+2}}{\partial\tau} &= \frac{\partial^2 f_{k+2}}{\partial\eta^2} - f_{k+1} U^* - K_1(\tau) f_{k+1} \\ &\quad - \left(K_2(\tau) - \frac{1}{P_e^2} \right) f_k - \sum_{i=3}^{k+2} K_i f_{k+2-i} - \alpha_1^2 f_{k+2}. \end{aligned}$$

To find K_i 's we know the f_k 's and its corresponding initial and boundary conditions are

$$\begin{aligned} f_k(0, \eta) &= 0 \\ \frac{\partial f_k}{\partial\eta}(\tau, -1) &= 0 \\ \frac{\partial f_k}{\partial\eta}(\tau, 1) &= 0 \\ \int_{-1}^1 f_k(\tau, \eta) d\eta &= 0. \end{aligned}$$

Put $i = 1$ in (3.5), we get $K_1(\tau) = 0$. Put $i = 2$ in (3.5), we get K_2 as,

$$(3.7) \quad K_2(\tau) = \frac{1}{P_e^2} - \frac{1}{2} \int_{-1}^1 U^* f_1 d\eta.$$

To evaluate $K_2(\tau)$, let

$$(3.8) \quad f_1 = f_{10}(\eta) + f_{11}(\tau, \eta),$$

where $f_{10}(\eta)$ is independent of τ and f_{11} is τ - dependent obeying

$$\frac{df_{10}}{d\eta} = 0 \quad \text{at } \eta = \pm 1,$$

$$\int_{-1}^1 f_{10} d\eta = 0.$$

Using the (3.9) in (3.7) gives

$$(3.9) \quad \frac{d^2 f_{10}}{d\eta^2} - \alpha_1^2 f_{10} = U^*(\eta),$$

$$(3.10) \quad \frac{\partial f_{11}}{\partial \tau} = \frac{\partial^2 f_{11}}{\partial \eta^2} - \alpha_1^2 f_{11}.$$

The solution of the equation (3.10), we get

$$\begin{aligned} f_{10} = & C_5 e^{\alpha_1 \eta} + C_6 e^{-\alpha_1 \eta} + \frac{1}{\bar{u}} \left(\frac{C_1 e^{m_1 \eta} + C_2 e^{-m_1 \eta}}{m_1^2 - \alpha_1^2} + \frac{C_3 e^{m_2 \eta} + C_4 e^{-m_2 \eta}}{m_2^2 - \alpha_1^2} \right) \\ & + \frac{1}{M^2 \alpha_1^2 \bar{u}} \left(KP + \frac{l_1(\alpha \eta - 1)}{a^2} \right). \end{aligned}$$

Equation (3.11) is heat conduction type and its solution satisfying condition $f_{11}(\tau, \eta) = -f_{10}(\eta)$ of the form

$$f_{11} = \sum_{n=1}^{\infty} B_n e^{-\lambda_n^2 \tau} \cos(\lambda_n \eta),$$

where

$$B_n = -2 \int_0^1 f_{10}(\eta) \cos(\lambda_n \eta) d\eta.$$

Equation (3.9) gives,

$$\begin{aligned} f_1 = & C_5 e^{\alpha_1 \eta} + C_6 e^{-\alpha_1 \eta} + \frac{1}{\bar{u}} \left(\frac{C_1 e^{m_1 \eta} + C_2 e^{-m_1 \eta}}{m_1^2 - \alpha_1^2} + \frac{C_3 e^{m_2 \eta} + C_4 e^{-m_2 \eta}}{m_2^2 - \alpha_1^2} \right) \\ & + \frac{1}{M^2 \alpha_1^2 \bar{u}} \left(KP + \frac{l_1(\alpha \eta - 1)}{a^2} \right) \\ & - 2 \sum_{n=1}^{\infty} \left(\frac{e^{-m_1} m_1 (\cos(\pi n) (C_1 e^{2m_1} - C_2) + e^{m_1} (C_2 - C_1))}{\bar{u} (m_1^2 - \alpha_1^2) (m_1^2 + \pi^2 n^2)} \right. \\ & + \frac{e^{-m_2} m_2 (\cos(\pi n) (C_3 e^{2m_2} - C_4) + e^{m_2} (C_4 - C_3))}{\bar{u} (m_2^2 - \alpha_1^2) (m_2^2 + \pi^2 n^2)} + \frac{\alpha l_1 (\cos(\pi n) - 1)}{\pi^2 n^2 \bar{u} (m_2^2 - \alpha_1^2)} \\ & \left. + \frac{\alpha_1 (\cos(\pi n) (e^{\alpha_1} C_5 - e^{-\alpha_1} C_6) - C_5 + C_6)}{\alpha_1^2 + \pi^2 n^2} \right) e^{-n^2 \pi^2 \tau} \cos(\pi n \eta), \end{aligned}$$

where, C_1, C_2, C_3, C_4, C_5 , and C_6 are constant and are given in the appendix.

Dispersion model equation (3.4) brings to

$$\frac{\partial \theta_m}{\partial \tau} = K_2 \frac{\partial^2 \theta_m}{\partial \xi^2}.$$

The exact solution of (3.12) satisfying the conditions (2.20)-(2.22) can be obtained using Fourier Transform [10] as

$$\theta_m(\xi, \tau) = \frac{1}{2} \left[\operatorname{erf} \left(\frac{\frac{\xi_s}{2} + \xi}{2\sqrt{T}} \right) + \operatorname{erf} \left(\frac{\frac{\xi_s}{2} - \xi}{2\sqrt{T}} \right) \right],$$

where $T = \int_0^\tau K_2(\eta) d\eta$ and $\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-z^2} dz$.

4. DISCUSSION OF THE RESULTS

A mathematical model of blood flow through a capillary bounded by porous beds has been developed is to investigate the dispersion of a solute in a couple stress fluid with impacts of chemical reaction, magnetic and electric field. Generalized dispersion model helps to analyze the unsteady convective diffusion process with help of MATHEMATICA 8.0.

The results are obtained to illustrate the influence of the Hartmann number ($M = 1.1, 1.2, 1.3, 1.4$), electric number ($We = 30, 40, 50$), chemical reaction rate coefficient $\alpha_1 = 0.01, 0.1, 0.15$ and porous parameter $\sigma = (60, 120, 180)$ on the velocity, dispersion coefficient and the concentration profiles, for $Pe = 100, \alpha = 0.1, \tau = 0.3, a = 5$ and $\beta_1 = 0.1$. The impact of the parameters M, We, α_1

and σ on velocity, dispersion coefficient and concentration can be demonstrates through graphs 2-10.

Using the Beavers and Joseph's boundary conditions, velocity distribution in fluid region are obtained and show in Figure 2. From this we conclude that the impact of Hartmann number(M)and electric number(We) is to increase the velocity whereas the influence of magnetic field is to flatten the velocity profiles of the blood flow, But it is seen that the effect of increasing electric number is to increases the velocity profile of the blood flow, found to be parabolic in nature.

The expression for $K_2(\tau) - Pe^{-2}$ are numerically evaluated using equation (3.8) and are shown in Figures 3 – 6. These Figure evident that $K_2(\tau) - Pe^{-2}$ decreases with an increase in the magnitude of the Hartmann number(M) ,electric number, chemical reaction rate coefficient(α_1) and porous parameter σ . This result is useful in the design of an artificial organs and it reduces the rapture of red blood cell . Also, it is noted that the $K_2(\tau) - Pe^{-2}$ has no changes on very long period. Impact of Hartmann number(M) ,electric number, chemical reaction rate coefficient(α_1) and porous parameter σ are used to reduces the rate of dispersion of the solute in the fluid flow. This result noticed previously by [1].

Figures 10 – 14 are illustrates the effect on θ_m mean concentration versus axial distance ξ for different values of M, We, σ and τ_1 . Clearly, it is depicts that θ_m mean concentration increases with an raise in the magnitude of the Hartmann number(M) ,electric number, chemical reaction rate coefficient(α_1) and porous parameter σ is to increase the peak value of the mean concentration. This implies that the concentration is more distribution in ξ -direction for larger and larger values of We . The curve are bell shaped and symmetrical about the origin. From these result it is very helpful to analyses the transport of solute at various times.

5. CONCLUSIONS

Finally the conclusion about the dispersion of a solute with chemical reaction in blood flow under the influence of electric field and magnetic field is discussed. This impact reveals that the decrease the dispersion coefficient with an raises the magnitude of time(τ). Also, it is found that the peak of the mean concentration increases with an increasing the value of α_1 . Generalised dispersion model is used to calculate the time dependent coefficients and it is valid

for all time. In general conclusion of Taylor's dispersion model are recovered as a particular case in the limit $\tau \rightarrow \infty$. The present study is significant effect on some physiological blood flow analysis in comparison to different works.

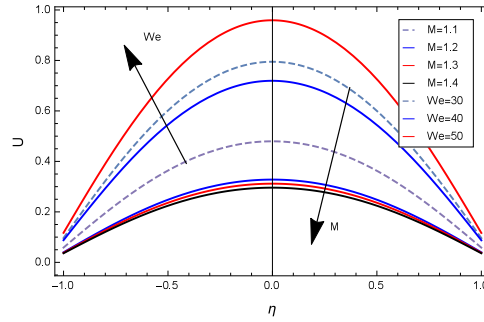


Figure 2. Velocity u on distinct values of (M) and (We) .

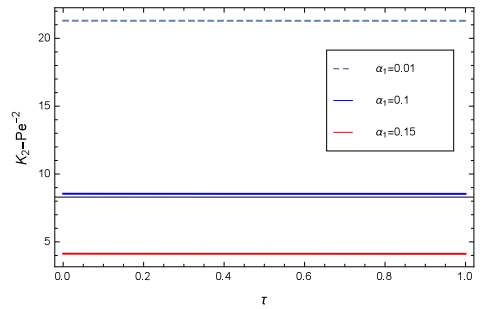


Figure 3. Dispersion coefficient $K_2(\tau) - Pe^{-2}$ on distinct values of (α_1) .

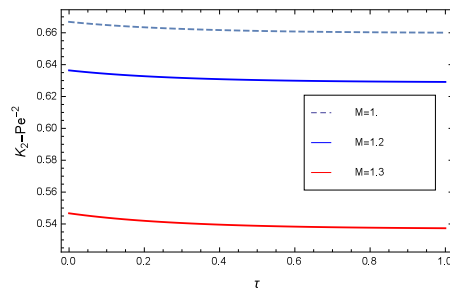


Figure 4. Dispersion coefficient $K_2(\tau) - Pe^{-2}$ on distinct values of (M) .

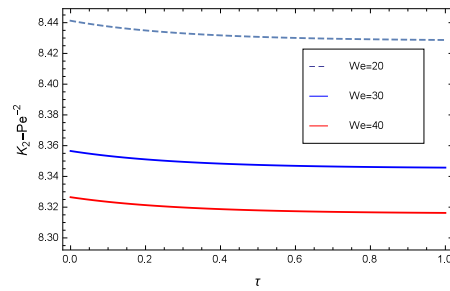


Figure 5. Dispersion coefficient $K_2(\tau) - Pe^{-2}$ on distinct values of (We).

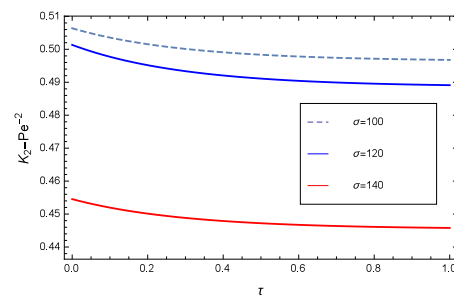


Figure 6. Dispersion coefficient $K_2(\tau) - Pe^{-2}$ on distinct values of (σ).

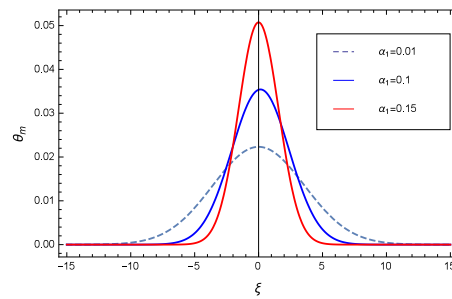


Figure 7. Mean concentration θ_m on distinct values of (α_1).

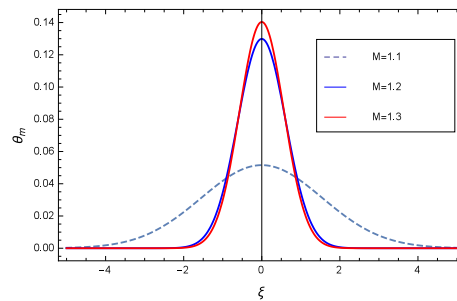
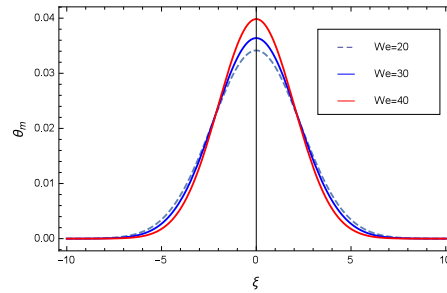
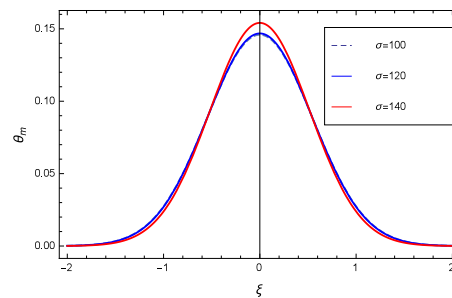


Figure 8. Mean concentration θ_m on distinct values of (M).

Figure 9. Mean concentration θ_m on distinct values of (We).Figure 10. Mean concentration θ_m on distinct values of (σ).

6. APPENDICES

$$C_1 = \frac{-1}{l_4} (a_3 a_5 a_7 l_2 - a_4 a_6 a_7 l_2 + a_4 a_5 a_8 l_2 - a_3 a_6 a_8 l_2 + a_1 a_8^2 l_2 - a_4 a_5 a_7 l_3 + a_3 a_6 a_7 l_3 + a_2 a_7^2 l_3 - a_3 a_5 a_8 l_3 + a_4 a_6 a_8 l_3 - a_2 a_8^2 l_3);$$

$$C_2 = \frac{-1}{l_4} (a_4 a_5 a_7 l_2 - a_3 a_6 a_7 l_2 - a_2 a_7^2 l_2 + a_3 a_5 a_8 l_2 - a_4 a_6 a_8 l_2 + a_2 a_8^2 l_2 - a_3 a_5 a_7 l_3 + a_4 a_6 a_7 l_3 + a_1 a_7^2 l_3 - a_4 a_5 a_8 l_3 + a_3 a_6 a_8 l_3 - a_1 a_8^2 l_3);$$

$$C_3 = \frac{-1}{l_4} (-a_3 a_5^2 l_2 + a_3 a_6^2 l_2 + a_1 a_5 a_7 l_2 + a_2 a_6 a_7 l_2 - a_2 a_5 a_8 l_2 - a_1 a_6 a_8 l_2 + a_4 a_5^2 l_3 - a_4 a_6^2 l_3 - a_2 a_5 a_7 l_3 - a_1 a_6 a_7 l_3 + a_1 a_5 a_8 l_3 + a_2 a_6 a_8 l_3);$$

$$C_4 = \frac{-1}{l_4} (-a_4 a_5^2 l_2 + a_4 a_6^2 l_2 + a_2 a_5 a_7 l_2 + a_1 a_6 a_7 l_2 - a_1 a_5 a_8 l_2 - a_2 a_6 a_8 l_2 + a_3 a_5^2 l_3 - a_3 a_6^2 l_3 - a_1 a_5 a_7 l_3 - a_2 a_6 a_7 l_3 + a_2 a_5 a_8 l_3 + a_1 a_6 a_8 l_3);$$

$$C_5 = \frac{1}{2} \left(\frac{l_7}{l_5} + \frac{l_8}{l_6} \right); \quad C_6 = \left(\frac{1}{2} \left(-\frac{l_7}{l_5} + \frac{l_8}{l_6} \right) \right);$$

$$m_1 = \frac{\sqrt{a^2 + \sqrt{a^4 - 4a^2 M^2}}}{\sqrt{2}}; \quad m_2 = \frac{\sqrt{a^2 - \sqrt{a^4 - 4a^2 M^2}}}{\sqrt{2}};$$

$$a_1 = e^{m_1} (\alpha \sigma + m_1); \quad a_2 = e^{-m_1} (m_1 - \alpha \sigma); \quad a_3 = e^{m_2} (\alpha \sigma + m_2);$$

$$a_4 = e^{-m_2} (m_2 - \alpha \sigma); \quad a_5 = m_1^2 e^{m_1}; \quad a_6 = m_1^2 e^{-m_1}; \quad a_7 = m_2^2 e^{m_2}; \quad a_8 = m_2^2 e^{-m_2};$$

$$l_1 = K a^2 W e P e^2 \alpha_c x_0; \quad l_2 = -\alpha \sigma \left(\frac{-K P}{M^2} + \frac{l_1 (1 - \alpha_c)}{a^2 M^2} - u_{p0} \right) + \frac{l_1 \alpha_c}{a^2 M^2};$$

$$\begin{aligned}
l_3 &= \alpha \sigma \left(\frac{-KP}{M^2} + \frac{l_1(1+\alpha_c)}{a^2 M^2} - u_{p1} \right) + \frac{l_1 \alpha_c}{a^2 M^2}; \\
l_4 &= a_1^2 a_7^2 - a_1^2 a_8^2 - 2a_1 a_3 a_5 a_7 + 2a_1 a_3 a_6 a_8 - 2a_1 a_4 a_5 a_8 + 2a_1 a_4 a_6 a_7 - a_2^2 a_7^2 + a_2^2 a_8^2 \\
&\quad + 2a_2 a_3 a_5 a_8 - 2a_2 a_3 a_6 a_7 + 2a_2 a_4 a_5 a_7 - 2a_2 a_4 a_6 a_8 + a_3^2 a_5^2 - a_3^2 a_6^2 - a_4^2 a_5^2 + a_4^2 a_6^2; \\
l_5 &= 2\alpha_1 \cosh \alpha_1; l_6 = 2\alpha_1 \sinh \alpha_1; \\
l_7 &= \frac{1}{u} \left[\frac{2(C_1 - C_2)m_1 \cosh m_1}{m_1^2 - \alpha_1^2} + \frac{2(C_3 - C_4)m_2 \cosh m_2}{m_2^2 - \alpha_1^2} \right]; \\
l_8 &= \frac{1}{u} \left[\frac{2(C_1 + C_2)m_1 \sinh m_1}{m_1^2 - \alpha_1^2} + \frac{2(C_3 + C_4)m_2 \sinh m_2}{m_2^2 - \alpha_1^2} + \frac{2l_1 \alpha_c}{a^2 M^2 \alpha_1^2} \right]; \\
u_{p0} &= \frac{Re}{\sigma^2(1+\beta_1)} \left(\frac{-P}{Pe} + \frac{WePe x_0 \alpha_c(1-\alpha_c)}{2} \right); u_{p1} = \frac{Re}{\sigma^2(1+\beta_1)} \left(\frac{-P}{Pe} + \frac{WePe x_0 \alpha_c(1+\alpha_c)}{2} \right)
\end{aligned}$$

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